Verification of SA-QCR2000 implementation Supersonic Flow in Square Duct

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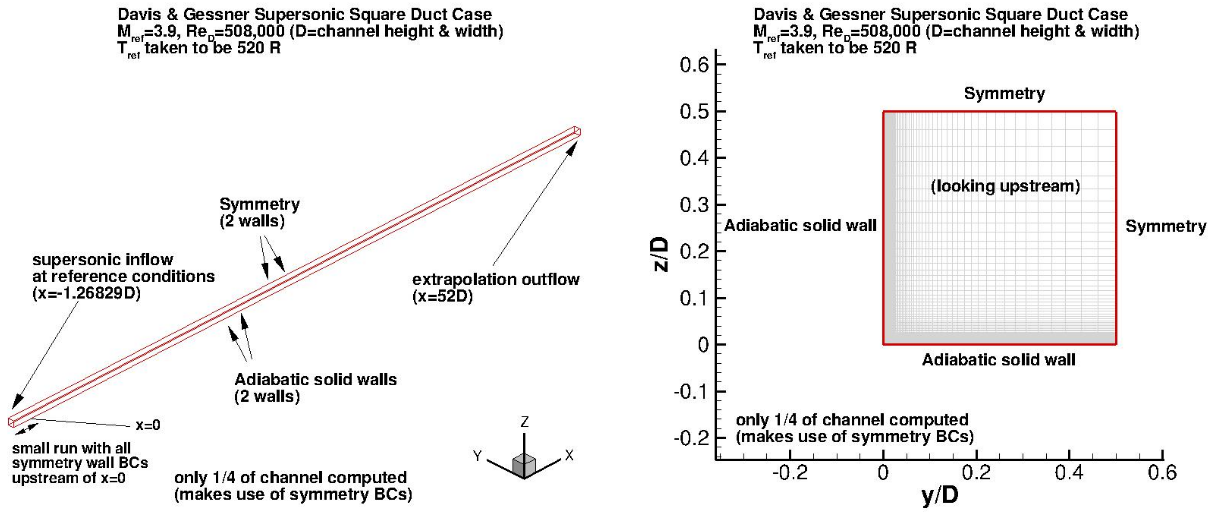
1 Introduction

Reference solutions computed by FUN3D and USM3D for the 3D Modified Supersonic Square Duct Validation Case are reported in the "Cases and Grids for Turbulence Model Numerical Analysis" section of the TMR website [Rumsey (2019); Rumsey *et al.* (2010)]. The reference solutions are used for verification of RANS solvers with the SA-QCR2000 turbulence model [Spalart (2000)]. An experimental investigation of this case is reported in [Davis *et al.* (1986)]. The negative version (SA-neg) of the SA model [Allmaras *et al.* (2012)] is chosen due to its superior iterative convergence properties.

2 Flow Parameters, Geometry, and Boundary Conditions

The experiment uses a constant area square duct of height and width $D = 25.4mm$ and length $52D$. The primary feature of this case is the flow in the corners. In such cases, the anisotropy of the Reynolds stress tensor is important because normal stress differences induce flow behavior that cannot be captured with models based on the Boussinesq assumption. The flow conditions corresponding to the experiment are as follows. The Reynolds number per D is $Re = 508,000$, the reference Mach number is $M_{ref} = 3.9$, and the reference temperature is $T_{ref} = 520°$ Rankine. The experimental measurements were conducted at the stations located $40D$ and $50D$ downstream of the duct inlet.

Figure 1(a) illustrates the geometry and boundary conditions. Figure 1(b) shows a cross-section upstream view. The coordinate system origin is set at the lower-left corner of the front section of the duct. The positive direction of the x-axis is the streamwise direction along the duct. The positive direction of the y-axis is to the right looking upstream. The positive direction of the z-axis is upward. The duct dimensions are $0 \le x \le 52D; 0 \le y \le 0.5D; 0 \le z \le 0.5D$. Adiabatic-wall conditions are set at the duct left and bottom boundaries corresponding to $0 \le x \le 52D; y = 0; 0 \le z \le 0.5D$ and $0 \le x \le 52D; 0 \le y \le 0.5D; z = 0$. The computational domain is extended upstream to allow for a small run of symmetry boundary at $-1.26829D \le x \le 0$. Supersonic inflow conditions are set at $x = -1.26829D$ corresponding to the reference conditions. Supersonic outflow conditions are set at $x = 52D$. Symmetry conditions are used on the right ($y = 0.5D$) and top ($z = 0.5D$) boundaries and on all boundaries at $-1.26829D \le x \le 0$. Due to use of symmetry boundary conditions, only one quarter of the duct is computed.



(a) Far view (b) Cross-section

Figure 1: 3D duct: geometry and boundary conditions.

3 Grids

Grid families can be generated with a grid-generation code that is posted on the TMR website together with recommended input files. The FUN3D-FV and USM3D computations reported at the TMR website performed on four nested hexahedral grids with $2688 \times 768 \times 768$ cells (Grid 1), $1344 \times 384 \times 384$ cells (Grid 2), $672 \times 192 \times 192$ cells (Grid 3), and $386 \times 96 \times 96$ cells (Grid 4). The grid dimensions represent cells in the streamwise, spanwise, and vertical directions, respectively. The grids are stretched in the wall-normal directions with a constant stretching factor. Grid 1 has the minimum non-dimensional spacing at the wall of approximately 0.0000085 and the average $y^+ = 0.022$.

4 Reference solutions

The following is a summary of the parameters used by FUN3D and USM3D in computing reference solutions:

- Unit length: $D = 1$.
- Length of duct: $L = 52$.
- Width and height of duct: $D = 1$.
- Reynolds number per unit length: $Re_D = 508,000$.
- Reference Mach number: $M_{ref} = 3.9$.
- Reference temperature: $T_{ref} = 520°$ Rankine.
- Reference area: $A_{ref} = 1$.
- Turbulence model: SA-neg-QCR2000.

- Clipping method for $\hat{S}$: USM3D uses method "a" with cut-off 0.00001; FUN3D uses method "c". The clipping methods are described in Note 1 at link Clipping $\hat{S}$.
- Dynamic viscosity: Sutherland's law (see link Sutherland's law for implementation details)
- SA-model diffusion term: Full Navier-Stokes, second order.
- SA-model advection term: Non-conservative approximation [Allmaras *et al.* (2012)], first order.
- SA-model variable in farfield: $\hat{\nu}_{farfield} = 3\times$ laminar kinematic viscosity in the freestream.
- Prandtl number for diffusion fluxes: 0.72 for meanflow, 0.90 for SA model.
- Initial conditions: Based on reference conditions.

5 Optional setting

The setting with the upstream symmetry run and without downstream symmetry run is the target verification setting for which the reference solutions have been produced. However, an optional setting can be used as long as the produced solutions do not deviate from the reference solutions. The grid generation code can generate grids with and without symmetry runs upstream and downstream of the duct. The run of symmetry boundary can be added downstream of the duct $52D \leq x \leq 53.268290D$. In this optional setting, supersonic outflow is set at $x = 53.268290D$ and symmetry conditions are used on all boundaries at $52D \leq x \leq 53.268290D$. In preliminary computations conducted with FUN3D-FV and USM3D on, presence of the downstream symmetry run had no noticeable effects on the computed solutions within the duct, but significantly increased computational time to converge residuals to the machine-zero level.

6 Expected outputs

The contributors to the workshop are expected to compare and contrast solutions computed with the SA-neg and SA-neg-QCR2000 turbulence models. To minimize iterative and discretization errors, workshop contributions should demonstrate sufficient residual convergence and solution convergence in grid refinement.

The following results should be reported:

- Residual convergence history (vs. iteration and/or CPU time and/or work units (TauBench?)) on all grids
- Grid convergence of scalar solution quantities
  - drag coefficients (pressure drag and viscous drag) computed over the solid-wall boundary.
  - center-line velocity normalized by the reference speed of sound U_{CL}/a_{ref} at $x/D = 40$ and $x/D = 50$
- Grid convergence of line plots
  - center-line velocity normalized by the reference speed of sound vs x/D.
  - skin-friction coefficient vs. $2z/D$ at $x/D = 40$ and $x/D = 50$

  - vertical and diagonal cuts of the stream-wise velocity normalized by the reference speed of sound (U/a_{ref}) vs. $2z/D$ at $x/D = 40$ and $x/D = 50$

- Contour plots of ($\sqrt{V^2 + W^2}/a_{ref}$) at $x/D = 50$

References

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