Transonic Inviscid Flow through a Channel with a Smooth Gaussian Bump

Andrew Corrigan* and Matthew J. Zahr†

1 Summary

The goal of this problem is to assess the ability of high-order methods to preserve consistency and high-order accuracy in the presence of an attached curved shock. This problem is a transonic Mach-0.7 variation of the *VI2 Smooth Gaussian bump* that involved smooth, subsonic flow at the *HiOCFD5 5th International Workshop on High-Order CFD Methods*[1] and earlier workshops[2]. In the present configuration, an attached shock forms over the bump, as shown in Figure 1. Therefore, the entropy is no longer constant throughout the domain and instead accuracy can be measured in terms of the stagnation enthalpy.

2 Specification

The governing equations are the 2D Euler equations with a constant ratio of specific heats $\gamma = 1.4$. The inflow plane is located at $x = -1.5$ and outflow plane is located at $x = +1.5$. The top of the channel is located at $y = 0.8$, while the bottom of the channel is located at

$$y = 0.0625e^{-25x^2}.$$

A set of quadrilateral and triangle grids can be generated from the Python code that remains available from the website of the previous workshop[3].

The freestream Mach number is $M_\infty = 0.7$. The top and bottom of the channel are both inviscid walls. The left boundary condition is specified as a subsonic inflow condition, which requires specification of the stagnation temperature T_0 and stagnation pressure p_0 that are defined in terms of freestream reference quantities as

*U.S. Naval Research Laboratory
†University of Notre Dame

[1] `https://how5.cenaero.be/content/vi2-smooth-gaussian-bump`

[2] Wang et al., "High-Order CFD Methods: Current Status and Perspective," IJNMF, 2013. `http://dx.doi.org/10.1002/fld.3767`

[3] `https://how5.cenaero.be/sites/how5.cenaero.be/files/VI2_gridGeneration.tgz`

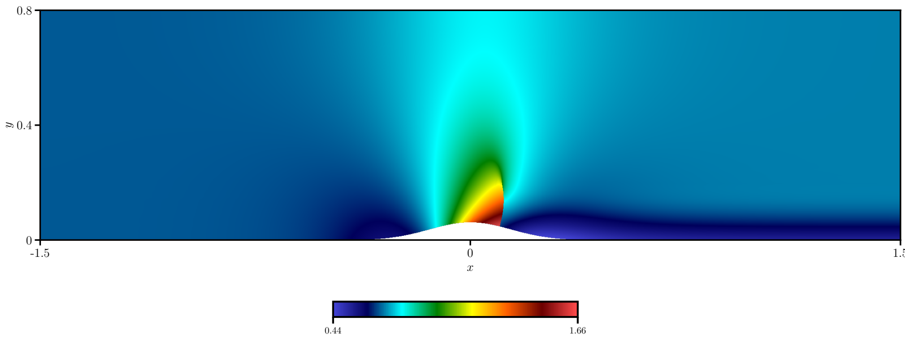

Figure 1: Mach-0.7 Transonic Inviscid Flow through a Channel with a Smooth Gaussian Bump

$$\frac{T_0}{T_\infty} = 1 + \frac{1}{2}(\gamma - 1) M_\infty^2,$$

$$\frac{p_0}{p_\infty} = \left(\frac{T_0}{T_\infty}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}.$$

Here we define the freestream temperature $T_\infty = 1$, freestream pressure $p_\infty = 1/\gamma$, where γ =1.4. The stagnation enthalpy remains constant throughout the domain with value given by

$$H_\infty = c_p T_0 = \frac{\gamma R}{\gamma - 1} T_0 = 2.745.$$

The right boundary condition is a subsonic outflow condition with back pressure $p = p_\infty$.

3 Mandatory Campaign

Measure the L_2 stagnation enthalpy error over the domain on a sequence of successively refined meshes,

$$\|H - H_\infty\|_{L_2(\Omega)} = \sqrt{\int_\Omega (H - H_\infty)^2}.$$

Report the L_2 error, the number of DOFs and/or the number of cells, along with a succinct description of the method/mesh/element type used (e.g. "DG, Roe-flux, isoparametric, quadratic triangles").

4 Optional Campaign

1. Report the work units required to converge the solution on each mesh according to the workshop guidelines[4].
2. Report the shock attachment point computed on each mesh.
3. Report the shock geometry computed on each mesh. The shock geometry can be specified as a piecewise linear curve, consisting of a list of point coordinates. Alternatively, if a higher-order representation is used, provide a small, stand-alone script that can be used to sample the coordinates of the curve as a function of a parameter $t \in (0, 1)$.
4. Repeat the study for the case of Mach number 0.86.

5 Preliminary Results

1. AIAA-2019-3207

6 Contact

For questions regarding the product setup or reporting requirements, contact Matthew J. Zahr (mzahr@nd.edu).

[4] https://how5.cenaero.be/content/guidelines