Sutherland's Law

in CFL3D / FUN3D

Sutherland's Law:

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T + S}\right)$$

$T_0 = 491.6 R$
$S = 198.6 R$
$\mu_0 = 1.716 \times 10^{-5}\ kg/ms$

For CFL3D & FUN3D, the T_0 and μ_0 do not matter, as they cancel out as follows:

We want: $\frac{\mu}{\mu_{ref}} \Rightarrow$ where μ_{ref} is the "reference (or freestream) viscosity" in the code

Use Sutherland's Law to write

$$\frac{\mu_{ref}}{\mu_0} = \left(\frac{T_{ref}}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T_{ref} + S}\right)$$

T_{ref} is the corresponding reference (or freestream) temperature in the code

So

$$\frac{\mu}{\mu_{ref}} = \frac{\mu}{\mu_0} \cdot \frac{\mu_0}{\mu_{ref}}$$

$$= \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T + S}\right) \cdot \left(\frac{T_0}{T_{ref}}\right)^{3/2} \left(\frac{T_{ref} + S}{T_0 + S}\right)$$

$$\therefore \boxed{\frac{\mu}{\mu_{ref}} = \left(\frac{T}{T_{ref}}\right)^{3/2} \left(\frac{T_{ref} + S}{T + S}\right)}$$

in the code, local $\frac{\gamma p'}{\rho'} \equiv a'^2 = \frac{a^2}{a_{ref}^2} = \frac{T}{T_{ref}}$

So $\frac{\mu}{\mu_{ref}}$ can be found via $\left(\frac{\gamma p'}{\rho'}\right)^{3/2} \left(\frac{T_{ref} + S}{T + S}\right)$

$$= \left(\frac{\gamma p'}{\rho'}\right)^{3/2} \frac{1 + \frac{S}{T_{ref}}}{\frac{T}{T_{ref}} + \frac{S}{T_{ref}}}$$

$$= \left(\frac{\gamma p'}{\rho'}\right)^{3/2} \frac{1 + S/T_{ref}}{\left(\frac{\gamma p'}{\rho'}\right) + S/T_{ref}}$$

← This is what is used in the codes