A data-driven wall law for the mean velocity in adverse-pressure gradient and modification of the SSG/LRR-w model

Tobias Knopp

NASA 2022 Symposium on Turbulence Modeling: Roadblocks, and the Potential for Machine Learning
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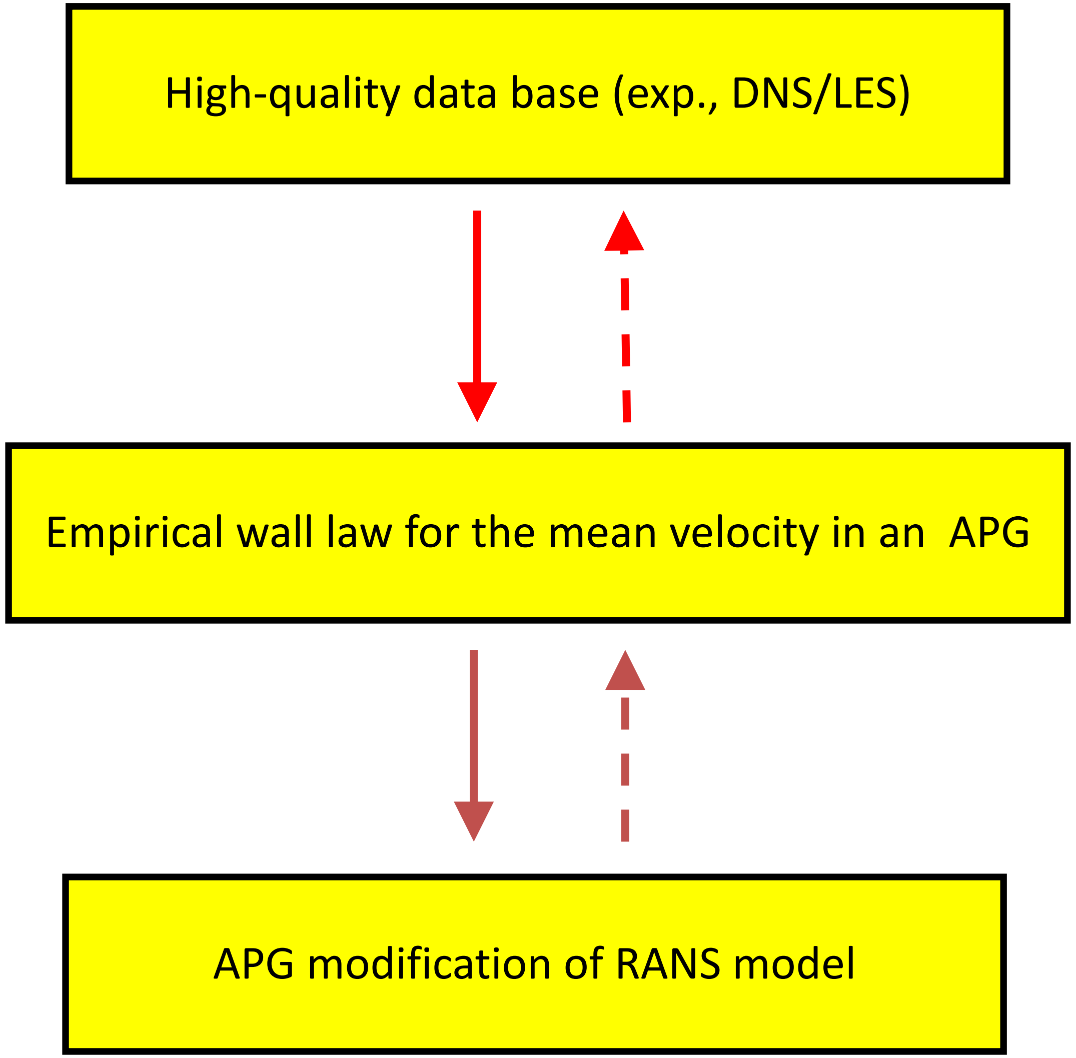
Tobias Knopp (DLR)

Outline. Strategy for RANS model improvement

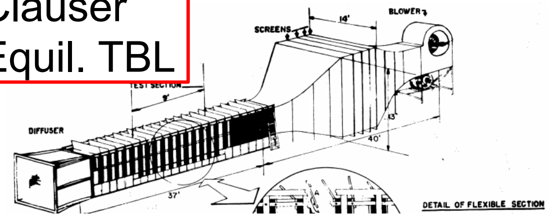


Database and Parameter Space

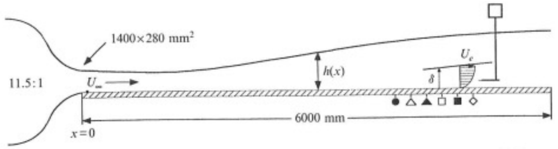
High-dim. feature space of TBL@APG

Flows in/near equilibrium

Clauser Equil. TBL

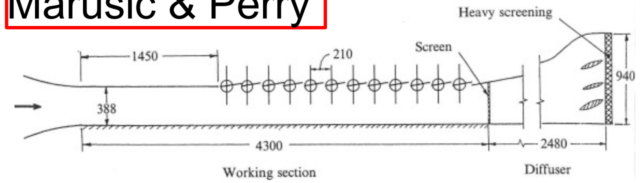


Skare & Krogstad Equilibrium TBL

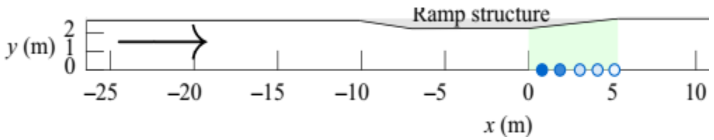


Flows in mild APG

Marusic & Perry

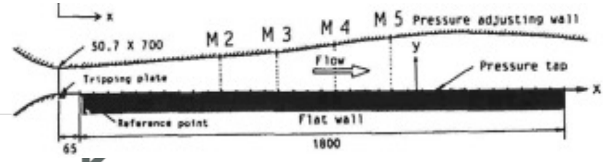


UNH Romero & Klewicki

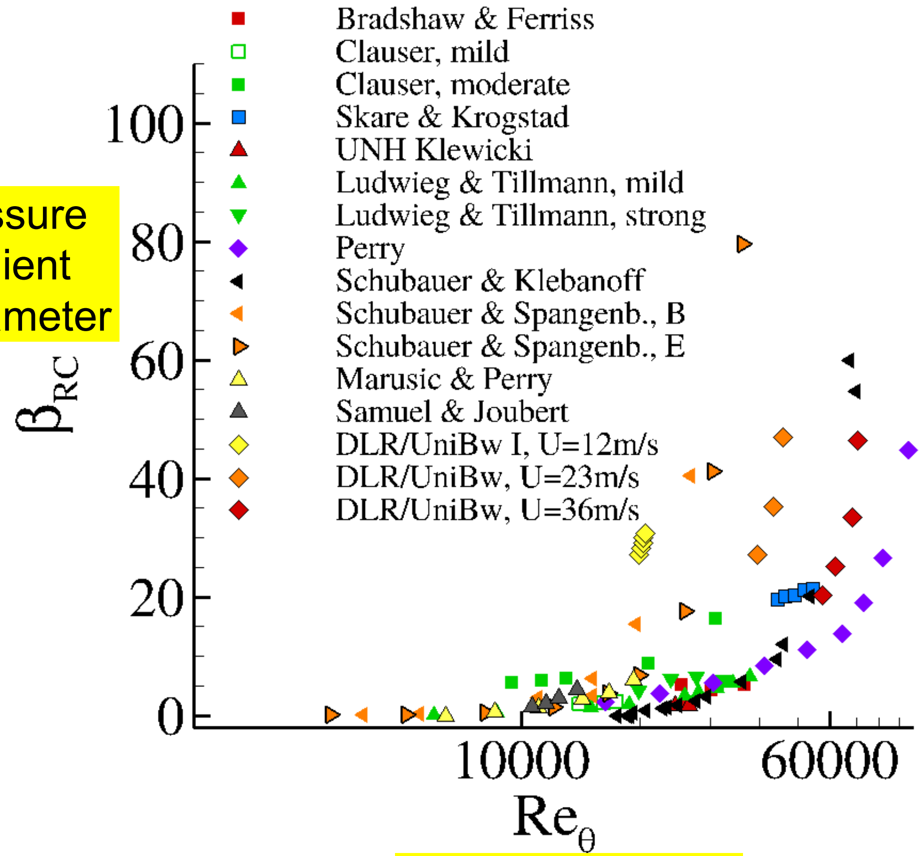


Flows at low Re

Nagano et al.



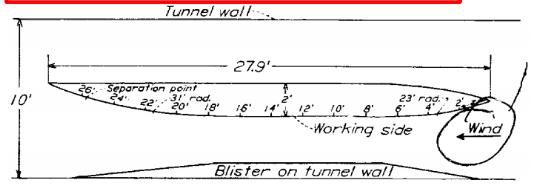
Pressure gradient parameter



Reynolds number

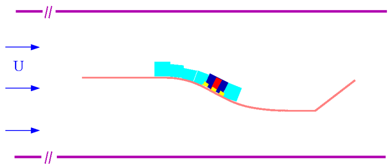
Flows in strong APG with separation

Schubauer & Klebanoff

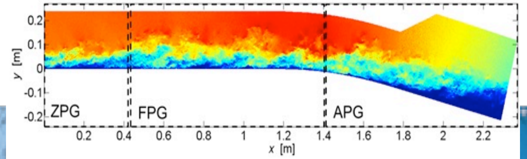


Flows with history effects

DLR/UniBw exp. I

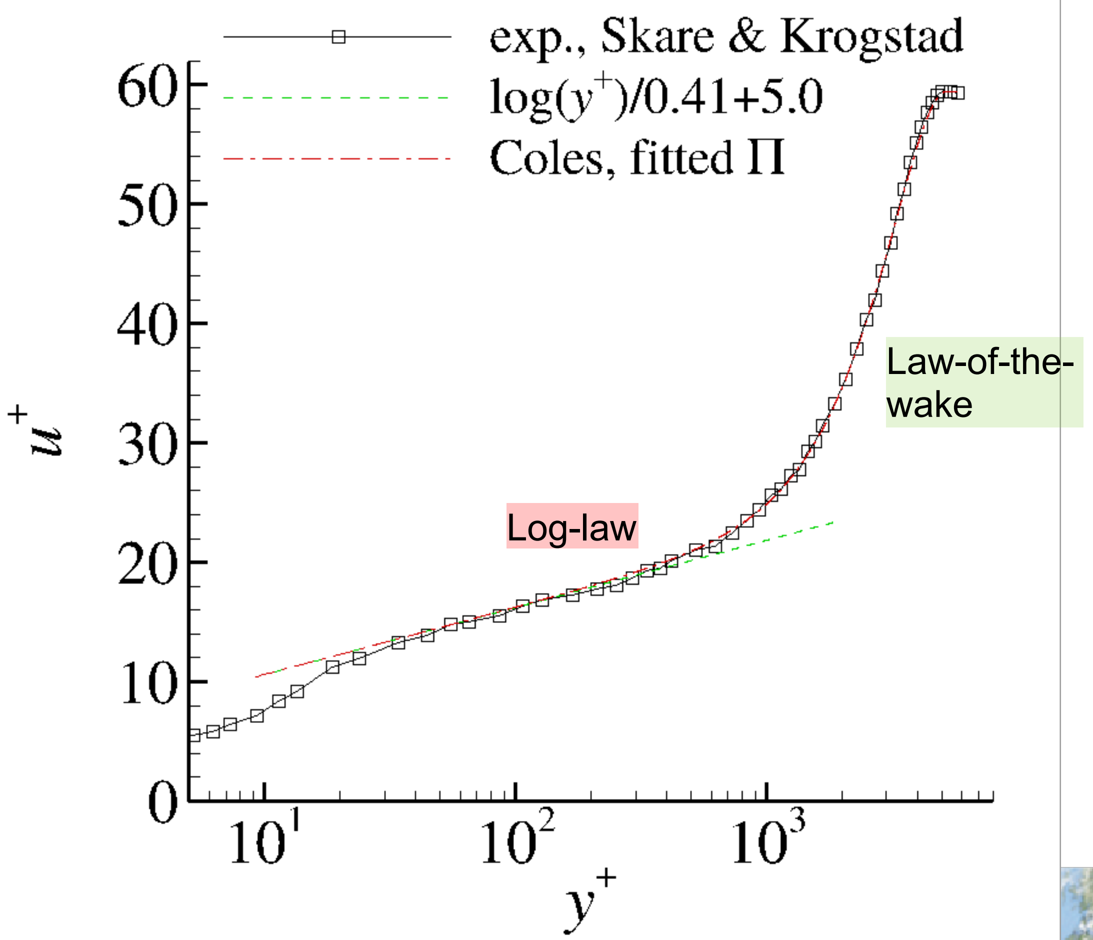


DLR/UniBw exp. II



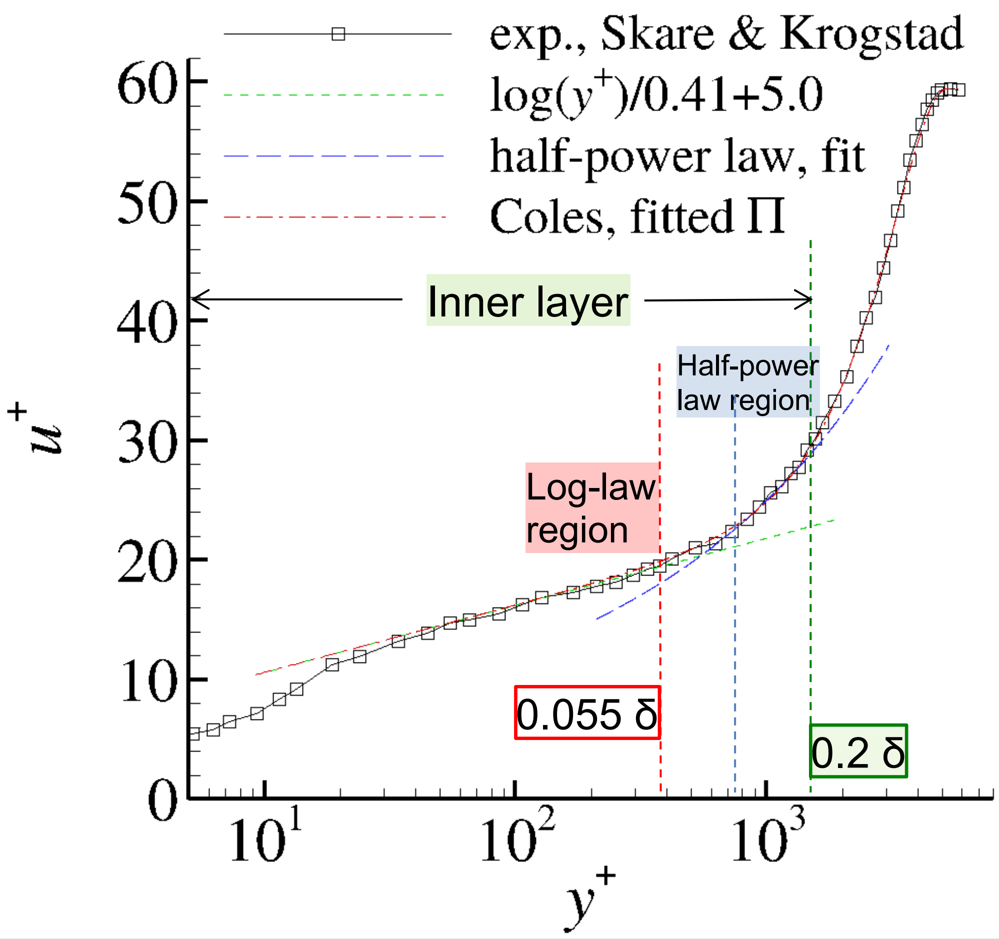
The classical view of the mean velocity profile of TBL in adverse pressure grad.

- Resilience of the log-law in APG
- Law of the wake with an empirical relation for the wake factor $\Pi = f(\beta_{RC}, \ldots)$, e.g. by Perry
- However: This has not been used for the modification of RANS turbulence models so far



Alternative view of the mean velocity profile of TBL in adverse pressure grad.

- Resilience of the log-law in APG
- Half-power law above the log-law (Perry, Bell & Joubert)
- Attemps to use a pure half-power law to modify RANS models (Rao & Hassan 1998, Aupoix & Catris 2000)



Resilience of the log-law in the mean-velocity:
- Coles, Coles & Hirst (1968)
- Galbraith et al. (1977),
- Granville (1985),
- Skare & Krogstad (1994)
- Alving & Fernholz (1995)
- Spalart & Coleman (2010)

Half-power law above the log-law
- Perry, Bell & Joubert (1966)
- Kader & Yaglom (1978)
- Durbin & Belcher (1992)

Alternative view of the mean velocity profile of TBL in adverse pressure grad.

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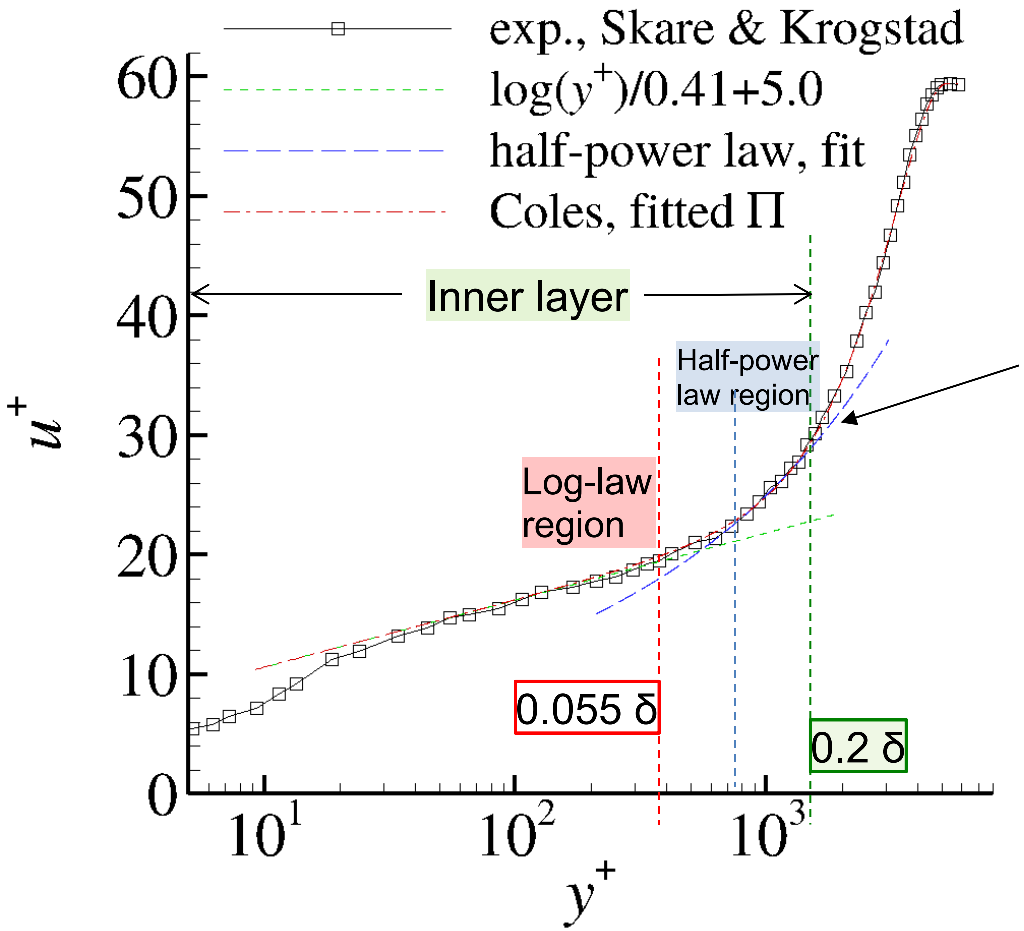

Slope of the half-power law depends on
- the local shear-stress gradient

$$\frac{du^+}{dy^+} = \frac{\sqrt{1+\alpha^+ y^+}}{K y^+}$$

$$\frac{\partial \tau^+}{\partial y^+} = \alpha^+ , \qquad \alpha^+ = \lambda \Delta p_s^+$$

Approximation by a **local model**

$$\frac{\partial \tau^+}{\partial y^+} = \Delta p_s^+ + \text{H.O. local terms} + \text{History effects}$$

DLR

Alternative view of the mean velocity profile of TBL in adverse pressure grad.

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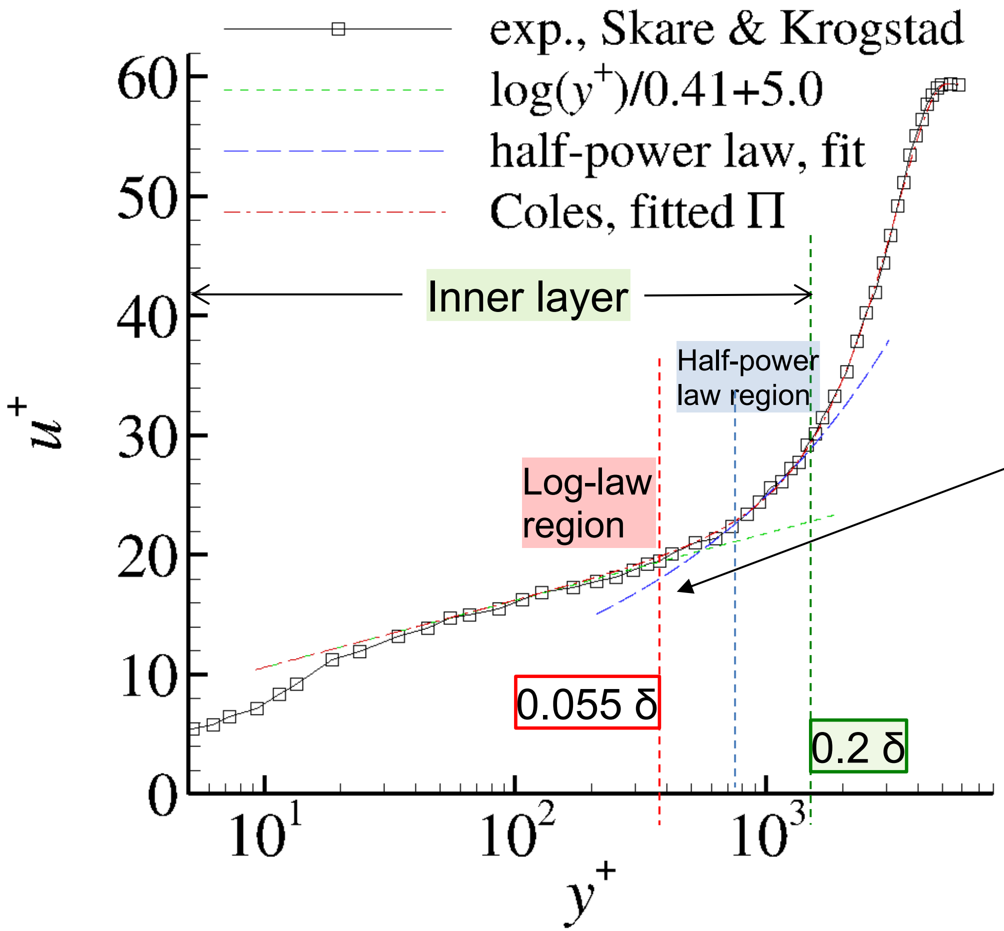


Conjecture of a local wall law:
u^+ is a function of y^+ and local parameters

$y^+_{log,max} = f(\Delta p_s^+, \delta^+)$

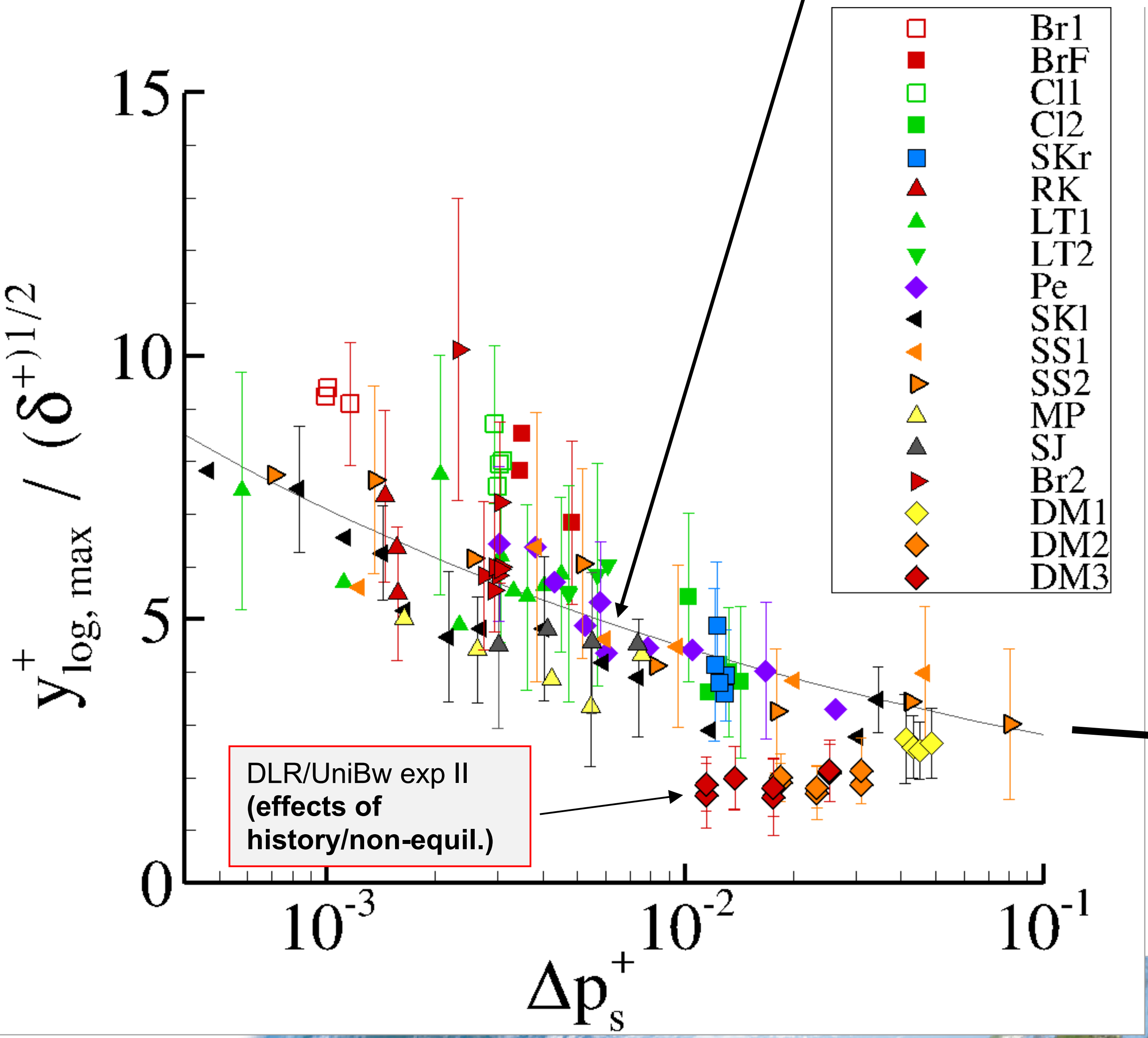
Database study to find a correlation

Calibration of the wall law

- Goal: Find an empirical correlation $y^+_{log,max} = f(\Delta p_s^+, \delta^+)$

$y^+_{log,max} = 1.78(\Delta p_s^+)^{-0.2}(\delta^+)^{1/2}$

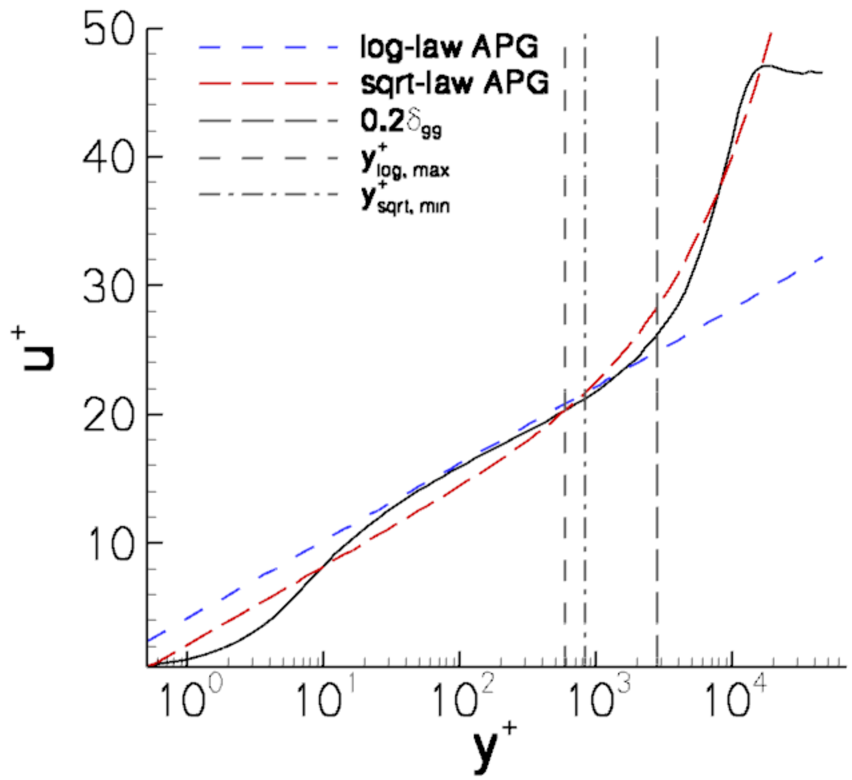
Re-effects
Motivated from self-similarity analysis of the boundary layer equations (Hartree parameter)
and by work for ZPG by Wei & Klewicki



Towards separation (Stratford 1959)

Goal: Adjustment of dU / dy in the half-power law region

- Analysis of the omega-equation in boundary-layer approximation (➔ see below)
- HGR01 airfoil at high Rec=25Mio, incidence angle α=10°



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- Analysis of the omega-equation in boundary-layer approximation (➔ see below)
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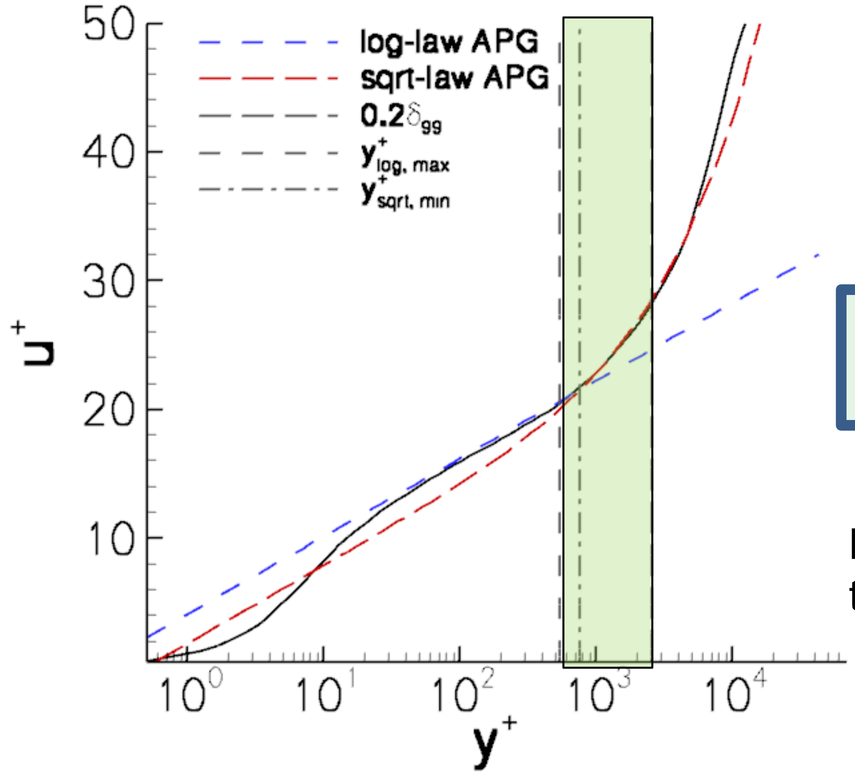


APG modification only in the half-power law region

Motivated using boundary layer theory for the ω-equation

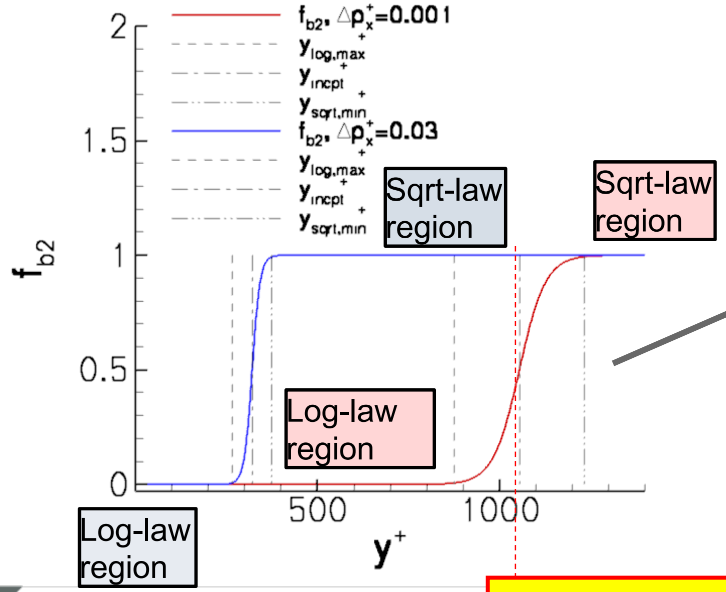
Blending functions for sqrt-law modification

- Modifications should be activated only in the half-power law region

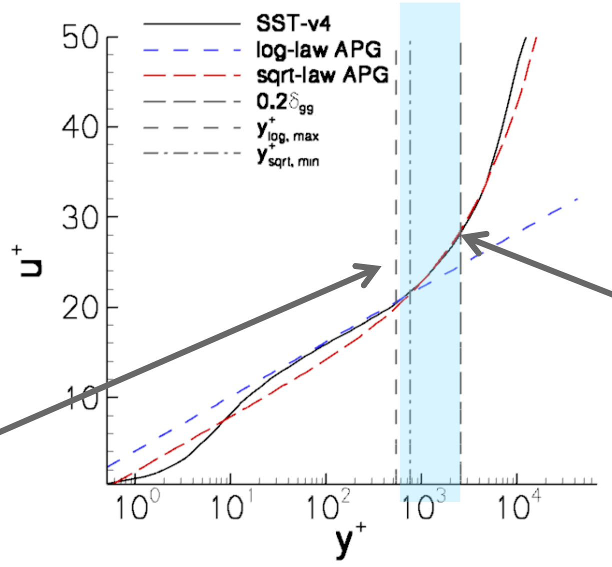
P_* motivated from analysis of the BL eq. for omega

$$\frac{\partial \omega}{\partial t} + \vec{\nabla} \cdot (\vec{U}\omega) - D_{\omega,t} - f_{b2} f_{b3} P_* = P_\omega - \epsilon_\omega$$

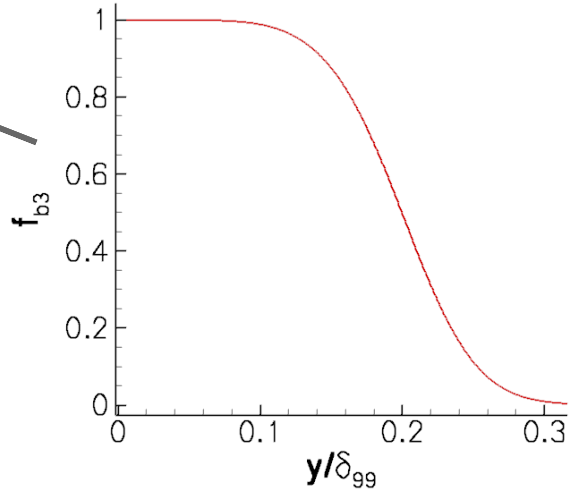
Inner blending given by $y^+_{log,max}$
- Depending on Δp_x^+ and Re_τ



$y^+_{incpt} = f(\Delta p_s^+, \delta^+)$



Outer blending: $y<0.2\delta_{99}$

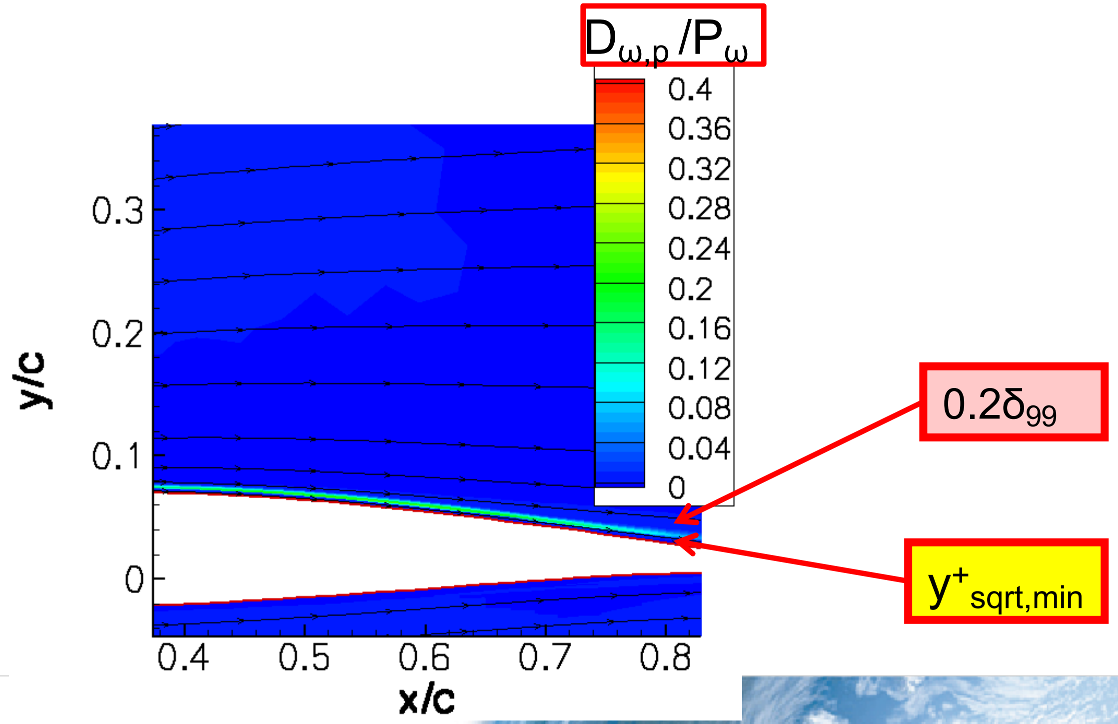


Blending functions for sqrt-law modification

- Modifications should be activated only in parts of the boundary layer

$$\frac{\partial \omega}{\partial t} + \vec{\nabla} \cdot (\vec{U}\omega) - D_{\omega,t} - f_{b2} f_{b3} P_* = P_\omega - \epsilon_\omega$$

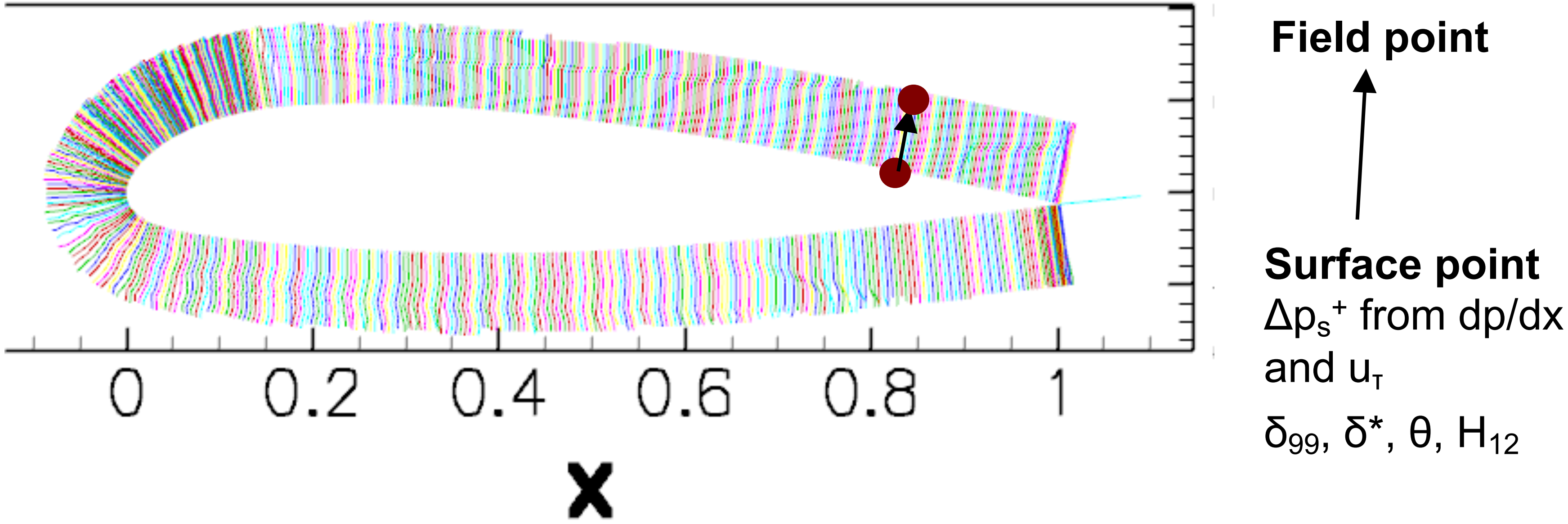
HGR01 airfoil at Re=0.65Mio, α=12°



Data structure of wall-normal lines for Δp_x^+

- Extension of unstructured flow solver DLR TAU code
  - Data structure for wall-normal lines
  - Method to determine $\Delta p_s = \nu/(\rho u_\tau^3)\, dp/ds = (dp/ds)^+$, δ_{99}, δ^*, θ, H_{12}

Wall normal lines for HGR01 airfoil



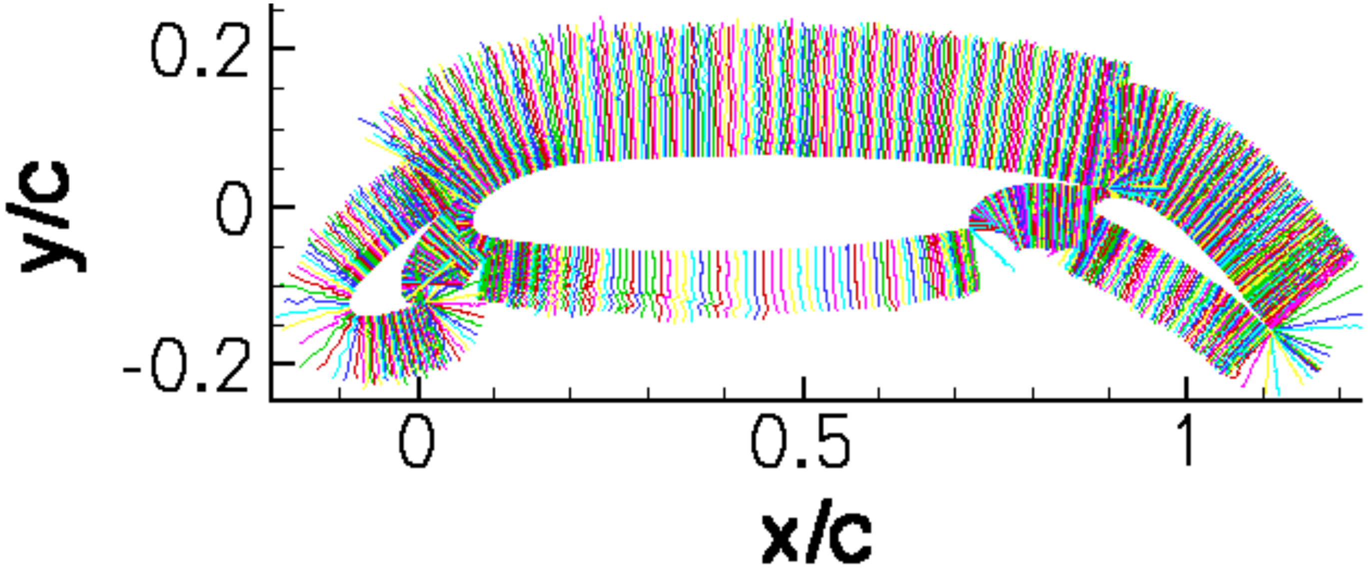
Δp_s^+ from dp/dx and u_τ

δ_{99}, δ^*, θ, H_{12}

Data structure of wall-normal lines for Δp_x^+

- Extension of unstructured flow solver DLR TAU code (working also for 3D aircraft configuration in high-lift)
  - Data structure for wall-normal lines
  - Method to determine δ_{99}, δ^*, θ, H_{12}

Wall normal lines for DLR F15 3-element airfoil



Field point

Surface point

Δp_s^+ from dp/dx and u_τ

δ_{99}, δ^*, θ, H_{12}

RANS model augmentation of the SSG/LRR-ω model

- Transport equation for the mean velocity

$$\frac{\partial}{\partial x_j}(U_i U_j) + \frac{1}{\rho}\frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\nu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \langle u_i' u_j' \rangle\right) = 0$$

- Transport equation for the Reynods stress tensor

$$U_k \frac{\partial}{\partial x_k}\overline{u_i' u_j'} = \mathcal{P}_{ij} + D_{ij}^{\nu} + D_{ij}^{t} + D_{ij}^{p} + \Pi_{ij} - \epsilon_{ij}$$

- Transport equation for the dissipation rate ω

$$U_j \frac{\partial \omega}{\partial x_j} = P_\omega - \epsilon_\omega + D_\omega^{\nu} + D_\omega^{t}$$

RANS model augmentation

- Transport equation for the mean velocity

$$\frac{\partial}{\partial x_j}(U_i U_j) + \frac{1}{\rho}\frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\nu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \langle u_i' u_j' \rangle\right) = 0$$

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- Transport equation for the dissipation rate ω → ε = 0.09*k*ω (k : TKE)

$$U_j \frac{\partial \omega}{\partial x_j} = P_\omega - \epsilon_\omega + D_\omega^{\nu} + D_\omega^{t}$$

„Net effect“:
The sum of all modelled terms determines <u_i‘u_j‘> and hence U

RANS model augmentation

- Transport equation for the mean velocity

$$\frac{\partial}{\partial x_j}(U_i U_j) + \frac{1}{\rho}\frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\nu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \langle u_i' u_j' \rangle\right) = 0$$

- Transport equation for the Reynods stress tensor

$$U_k \frac{\partial}{\partial x_k}\overline{u_i' u_j'} = \mathcal{P}_{ij} + D^{\nu}_{ij} + D^{t}_{ij} + D^{p}_{ij} + \Pi_{ij} - \epsilon_{ij}$$

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$$U_j \frac{\partial \omega}{\partial x_j} = P_\omega - \epsilon_\omega + D^{\nu}_{\omega} + D^{t}_{\omega}$$

The sum of all modelled terms determines $\langle u_i' u_j' \rangle$ and hence U

Step 1: Boundary layer approximation

Take into account only dominant terms and derivatives in wall-normal direction

$$U_j \frac{\partial \omega}{\partial x_j} = \gamma \left(\frac{\partial U_i}{\partial x_j} \right)^2 - \beta_\omega \omega^2 + \frac{\partial}{\partial x_j} \left[(\nu + \sigma_\omega \nu_t) \frac{\partial \omega}{\partial x_j} \right]$$

Non-dimensionalize (= scale) the equation to inner viscous units

$$-\frac{\mathrm{d}}{\mathrm{d}y^+} \left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+} \right) = \gamma \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+} \right)^2 - \beta_\omega \left(\omega^+ \right)^2$$

Step 2: Substitute of wall-law into the ω-equation

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D_{\omega,t}^+ \quad = \quad P_\omega^+ \quad - \quad \epsilon_\omega^+$$

Step 2: Substitute wall-law into the ω-eq.

Following ideas by Rao & Hassan,
Catris & Aupoix

Part 2: Wall law assumptions:
- Half-power law → dU/dy
- Linear total shear stress → $\tau = 1 + \lambda \Delta p_s^+ y^+$
- Derived relations for ν_t and ω

$\kappa y^+ \sqrt{1+\alpha y^+}$ → ν_t^+; $\frac{\sqrt{1+\alpha y^+}}{a_1 \kappa y^+}$ → ω^+ (in $\frac{d\omega^+}{dy^+}$); $\frac{\sqrt{1+\alpha y^+}}{\kappa y^+}$ → $\frac{du^+}{dy^+}$; $\frac{\sqrt{1+\alpha y^+}}{a_1 \kappa y^+}$ → ω^+

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D_{\omega,t}^+ = P_\omega^+ - \epsilon_\omega^+$$

Step 2: Substitute wall-law into the ω-equation

$\kappa y^+\sqrt{1+\alpha y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{\kappa y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) \neq \gamma\left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \qquad -D^+_{\omega,t} \neq P^+_\omega - \epsilon^+_\omega$$

$$\Leftrightarrow \qquad -\frac{\sigma_\omega}{a_1}\frac{1}{\left(y^+\right)^2} \neq \frac{1}{\kappa^2}\left(\gamma - \frac{\beta_\omega}{a_1^2}\right)\frac{1+\alpha y^+}{\left(y^+\right)^2}$$

Step 3: Spatial discrepency term

$\kappa y^+\sqrt{1+\alpha y^+}$ → ν_t^+; $\frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$ → $\frac{d\omega^+}{dy^+}$; $\frac{\sqrt{1+\alpha y^+}}{\kappa y^+}$ → $\frac{du^+}{dy^+}$; $\frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$ → ω^+

$$-\frac{d}{dy^+}\left(\sigma_\omega \nu_t^+ \frac{d\omega^+}{dy^+}\right) = \gamma \left(\frac{du^+}{dy^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \quad -D_{\omega,t}^+ = P_\omega^+ - \epsilon_\omega^+ + m^+(y^+,\Delta p_x^+)$$

$$\Leftrightarrow \quad -\frac{\sigma_\omega}{a_1}\frac{1}{(y^+)^2} = \frac{1}{\kappa^2}\left(\gamma - \frac{\beta_\omega}{a_1^2}\right)\frac{1+\alpha y^+}{(y^+)^2} + m^+(y^+,\Delta p_x^+)$$

This gives an analytical expression for *m* as a function of y^+ and the pressure gradient parameter $\alpha = \Delta p_x^+$

Spatial model discrepency term ***m***

Step 3: Spatial discrepency term

$\kappa y^+\sqrt{1+\alpha y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{\kappa y^+}$ $\qquad \frac{\sqrt{1+\alpha y^+}}{a_1\kappa y^+}$

$$-\frac{\mathrm{d}}{\mathrm{d}y^+}\left(\sigma_\omega \nu_t^+ \frac{\mathrm{d}\omega^+}{\mathrm{d}y^+}\right) = \gamma\left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 - \beta_\omega \left(\omega^+\right)^2$$

$$\Leftrightarrow \quad -D^+_{\omega,t} = P^+_\omega - \epsilon^+_\omega \quad + m^+(y^+,\Delta p_x^+)$$

Analytical solution of a BL problem
Is equivalent to field inversion by
numerical methods (see FI/ML
approach by Duraisamy et al.)

$$\Leftrightarrow \quad -\frac{\sigma_\omega}{a_1}\frac{1}{(y^+)^2} = \frac{1}{\kappa^2}\left(\gamma - \frac{\beta_\omega}{a_1^2}\right)\frac{1+\alpha y^+}{(y^+)^2} \quad + m^+(y^+,\Delta p_x^+)$$

Inverse modelling: If we add the model discrepency term ***m*** to the *ω*-equation, then the assumed wall-law at APG solves the modified *ω*-equation

(Cf. T. Knopp, AIAA-paper 2016-0588)

Step 4: Functional discrepency term

- **Step 4: Express the discrepency term as a function of admissible mean flow and turbulence quantities**

$$m^+(y^+) = P^+_{\omega,4,\mathrm{bl}} \equiv C_{\omega 4} \left(\frac{\mathrm{d}u^+}{\mathrm{d}y^+}\right)^2 = C_{\omega 4} \frac{1+\alpha^+ y^+}{(K y^+)^2} \approx C_{\omega 4} \frac{\alpha^+}{K^2 y^+}$$

➔ **Modification of the coefficient of the ω-production-term**

$$-D^{t,+}_{\omega,\mathrm{bl}} = P^+_{\omega,\mathrm{bl}} + \boxed{P^+_{\omega,4,\mathrm{bl}}} - \epsilon^+_{\omega,\mathrm{bl}}$$

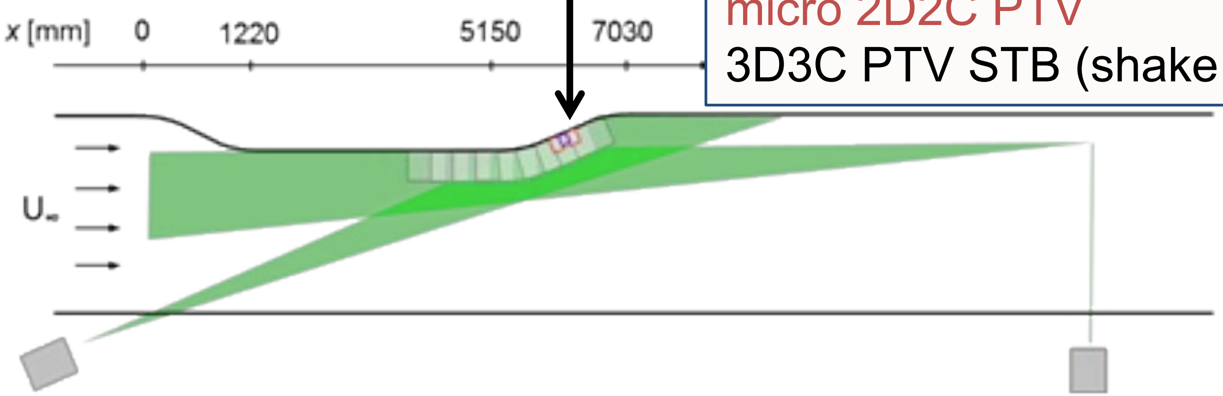
Activated in the half-power law region in an APG

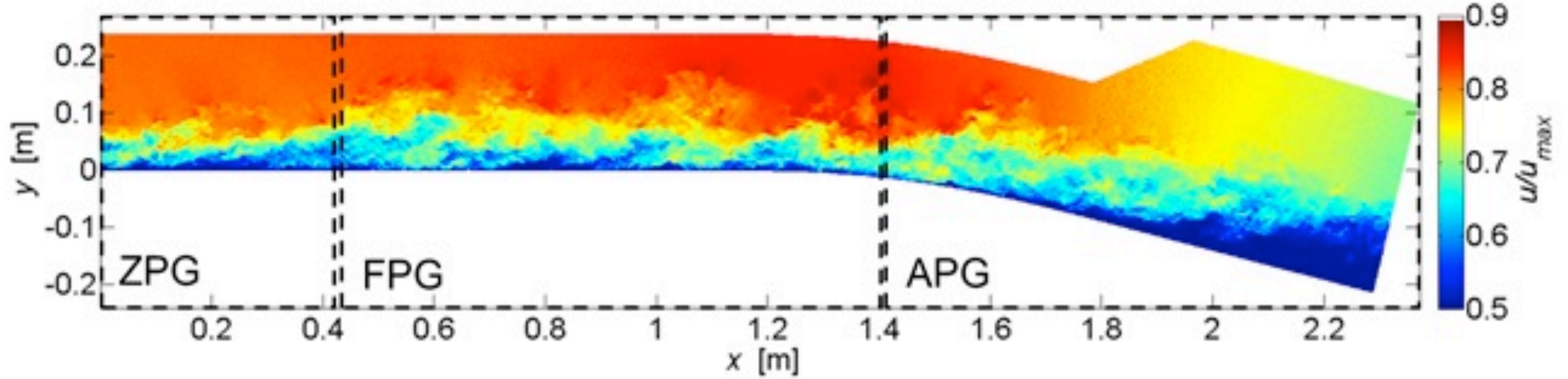
DLR/UniBw turbulent boundary layer flow (moderately strong APG)

APG focus region

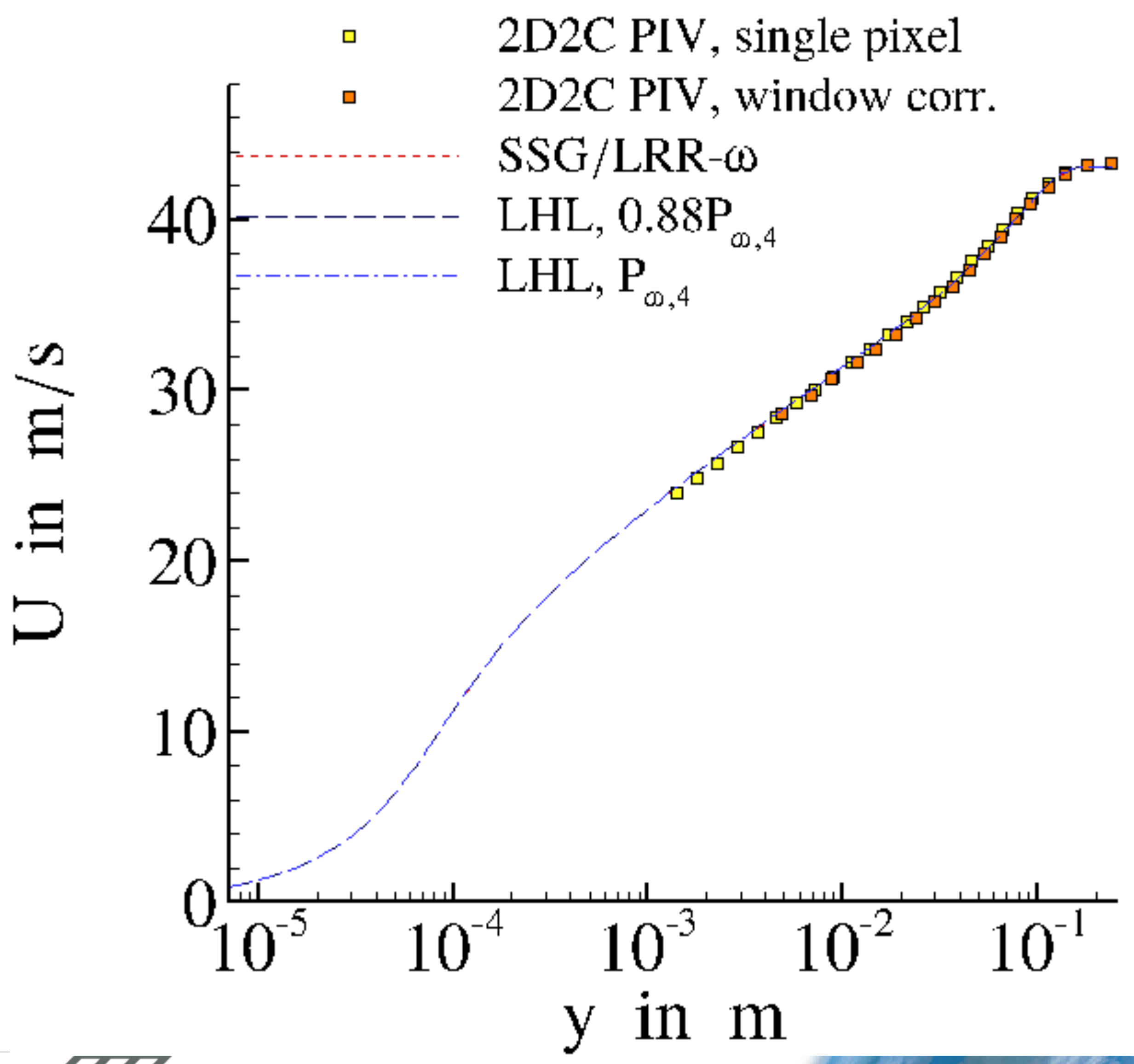
Large-scale 2D2C-PIV 9 cams

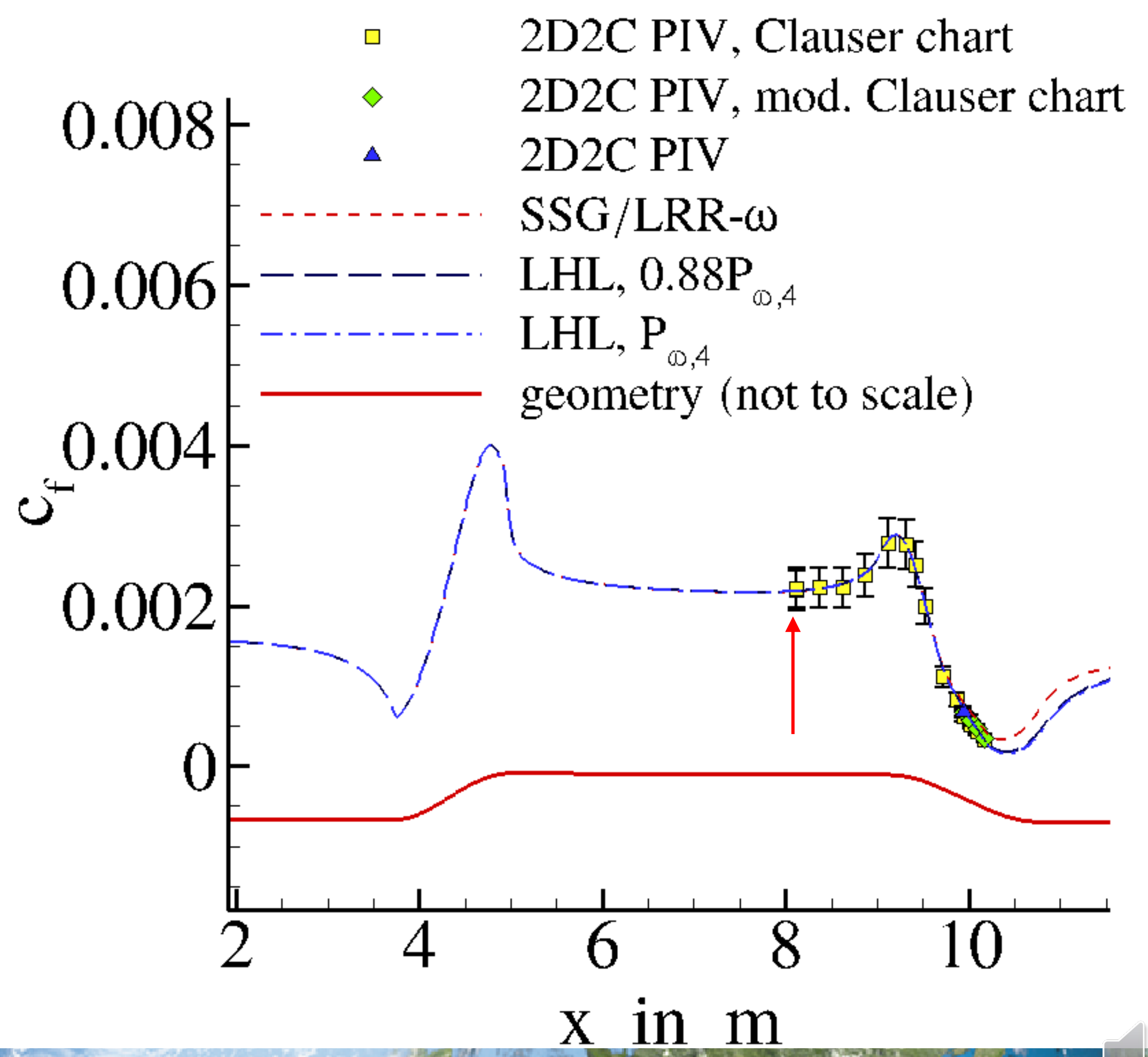
micro 2D2C PTV
3D3C PTV STB (shake the box)





RANS simulations for DLR/UniBw moderately strong APG flow exp.





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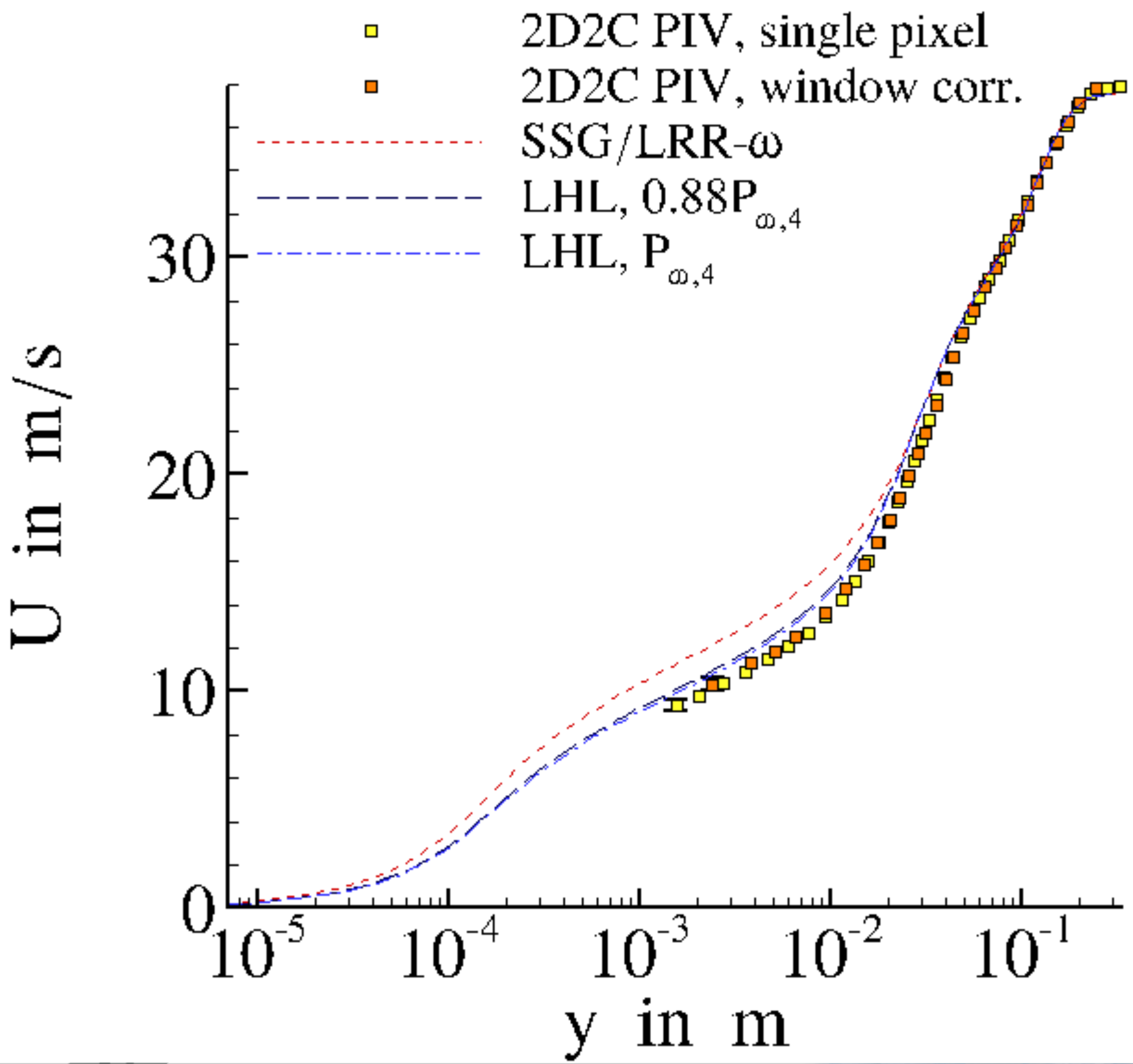
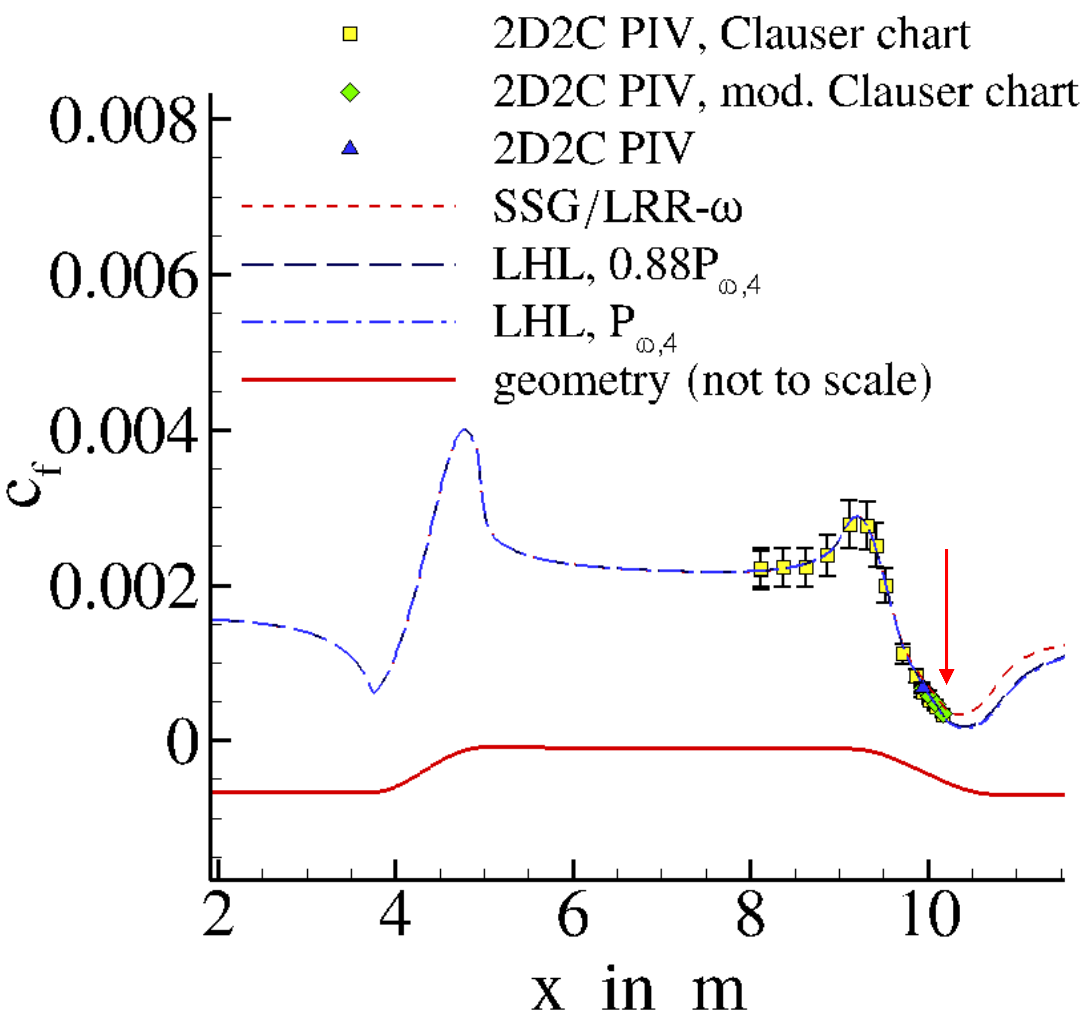
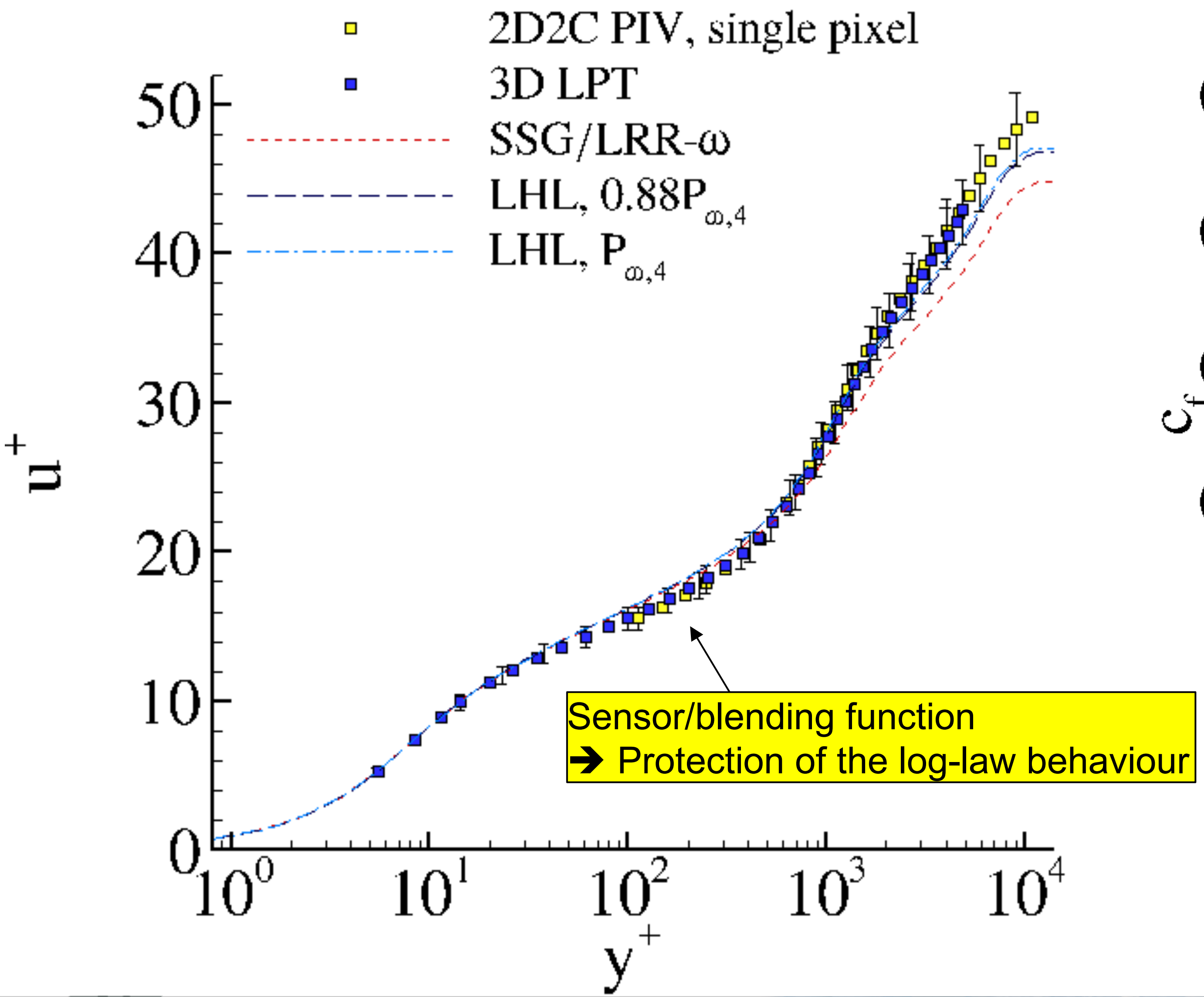
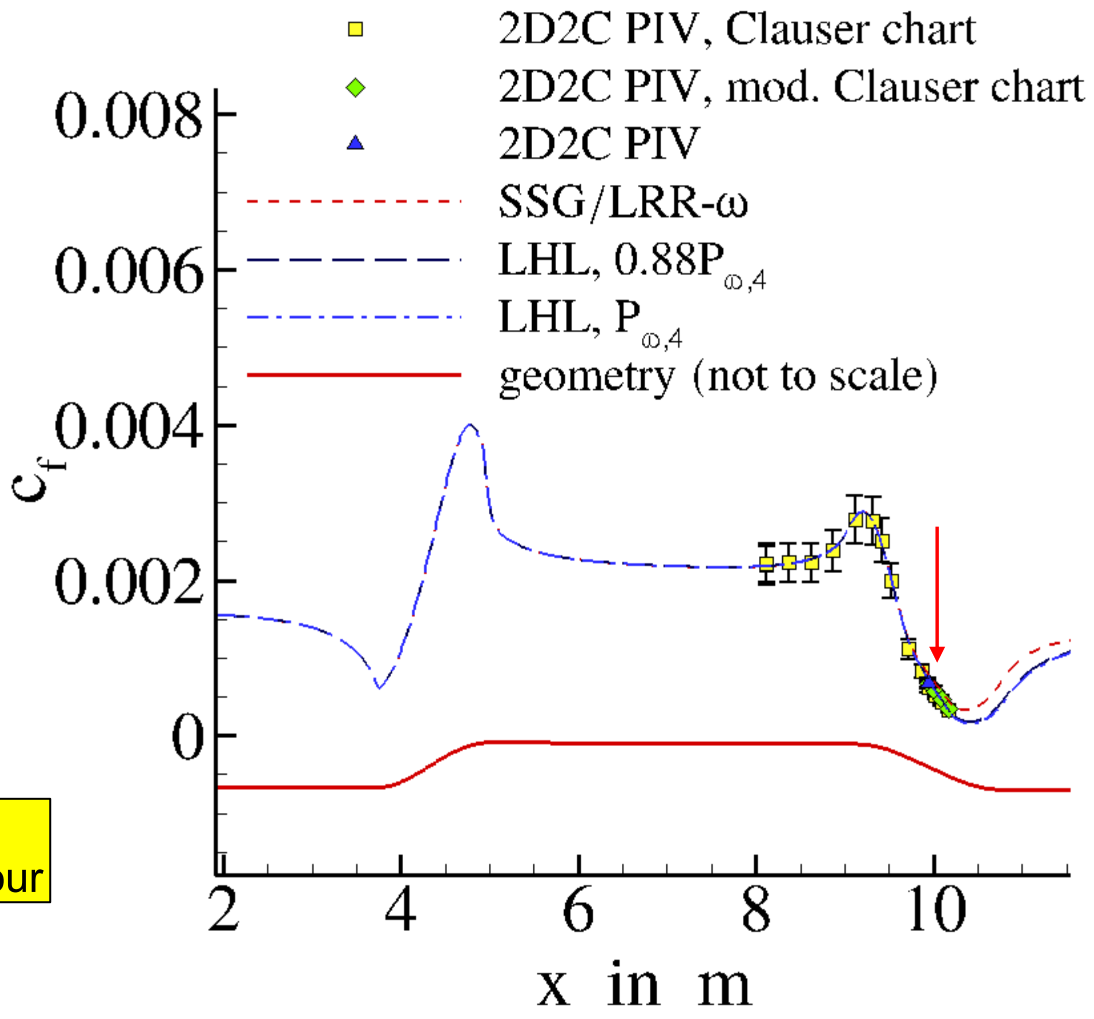




$\Delta p_x^+ = dp^+/dx^+ = 0.017$ at x=10.09m
(case U=36m/s)

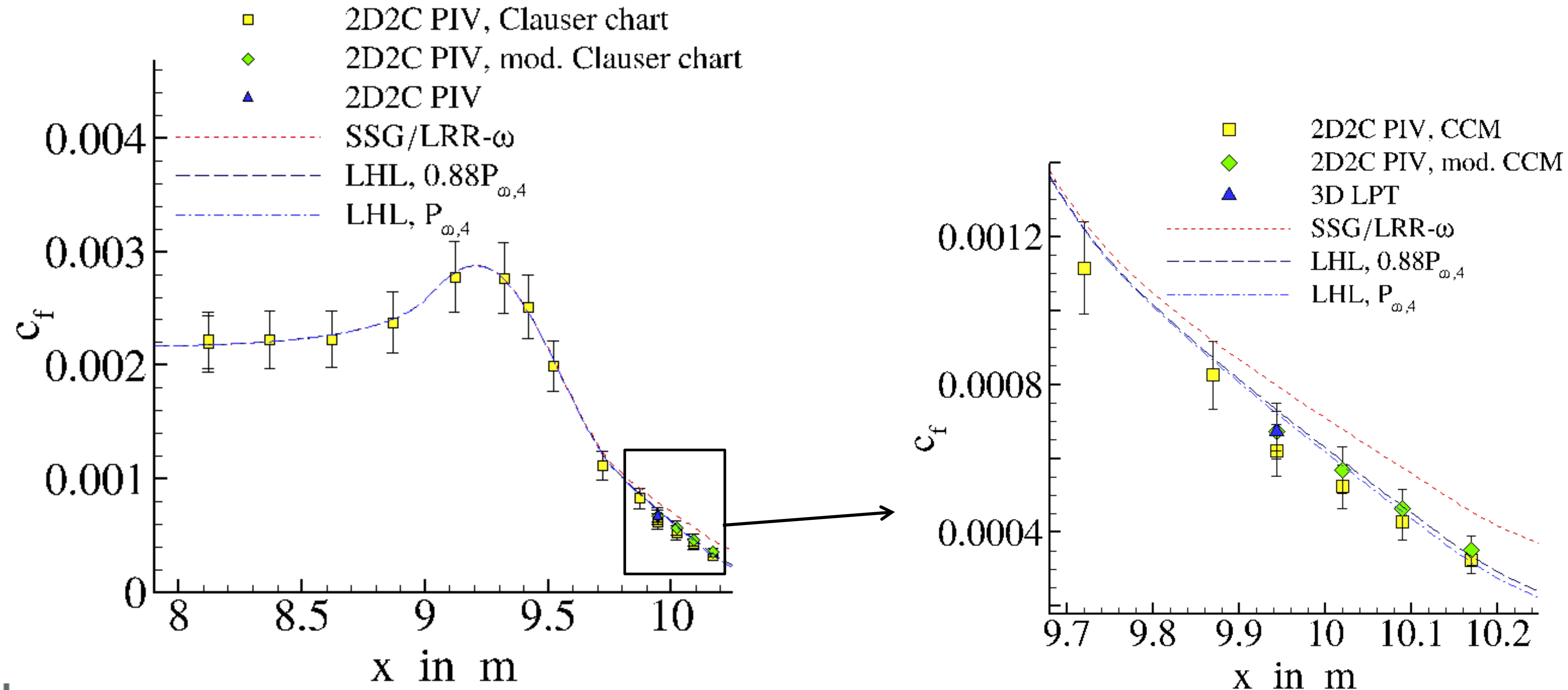
RANS simulations for DLR/UniBw moderately strong APG flow exp.





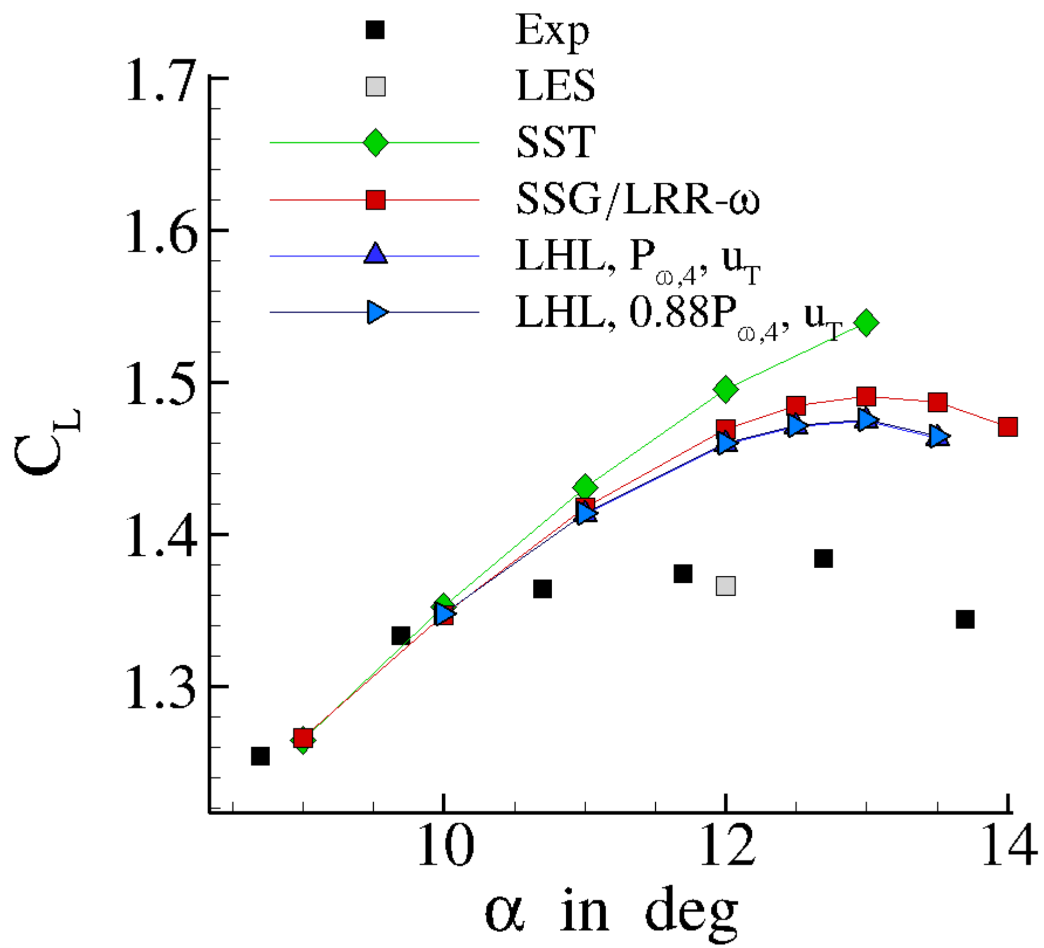
$\Delta p_x^+ = dp^+/dx^+ = 0.0114$ at x=9.944m (case U=36m/s)

RANS simulations for DLR/UniBw moderately strong APG flow exp.

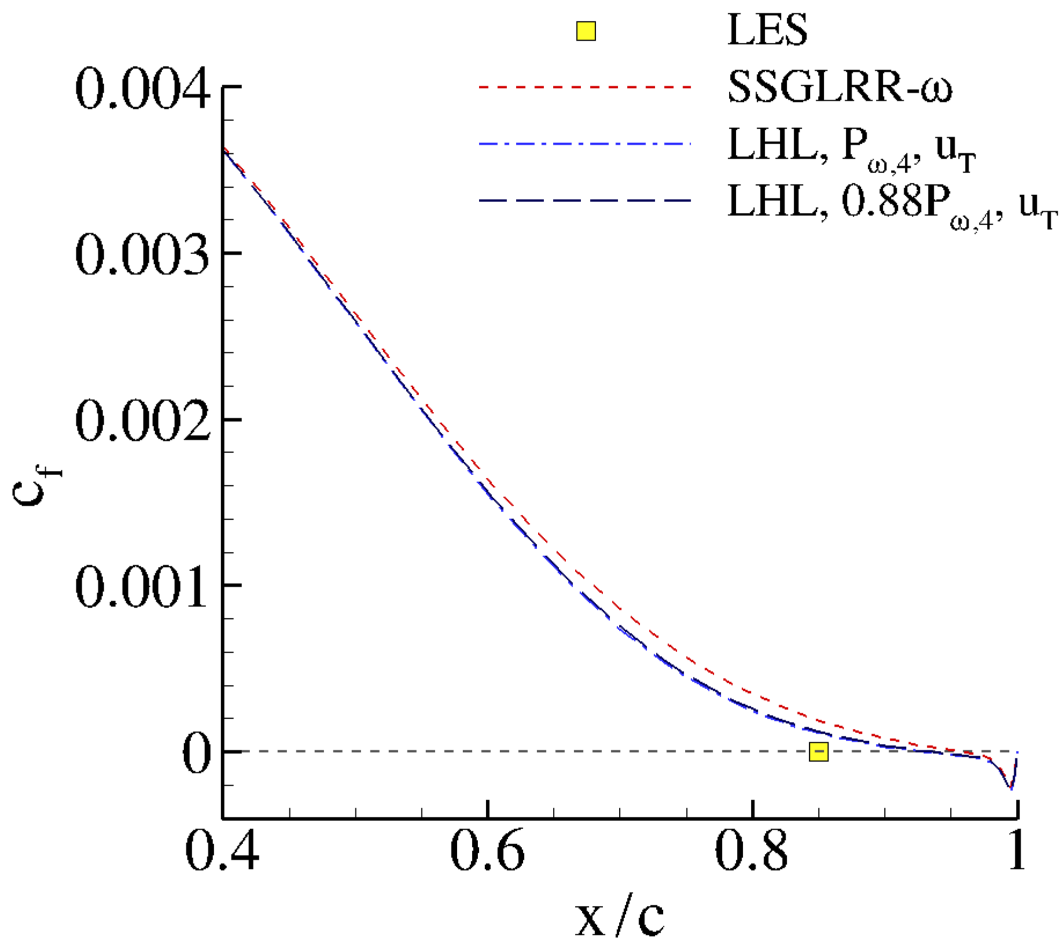


HGR01 airfoil. Re = 0.65 M, M=0.07

- Wind-tunnel measurement in MUB at ISM at TU Braunschweig
- Large-eddy simulation (LES) at AIA at RWTH Aachen

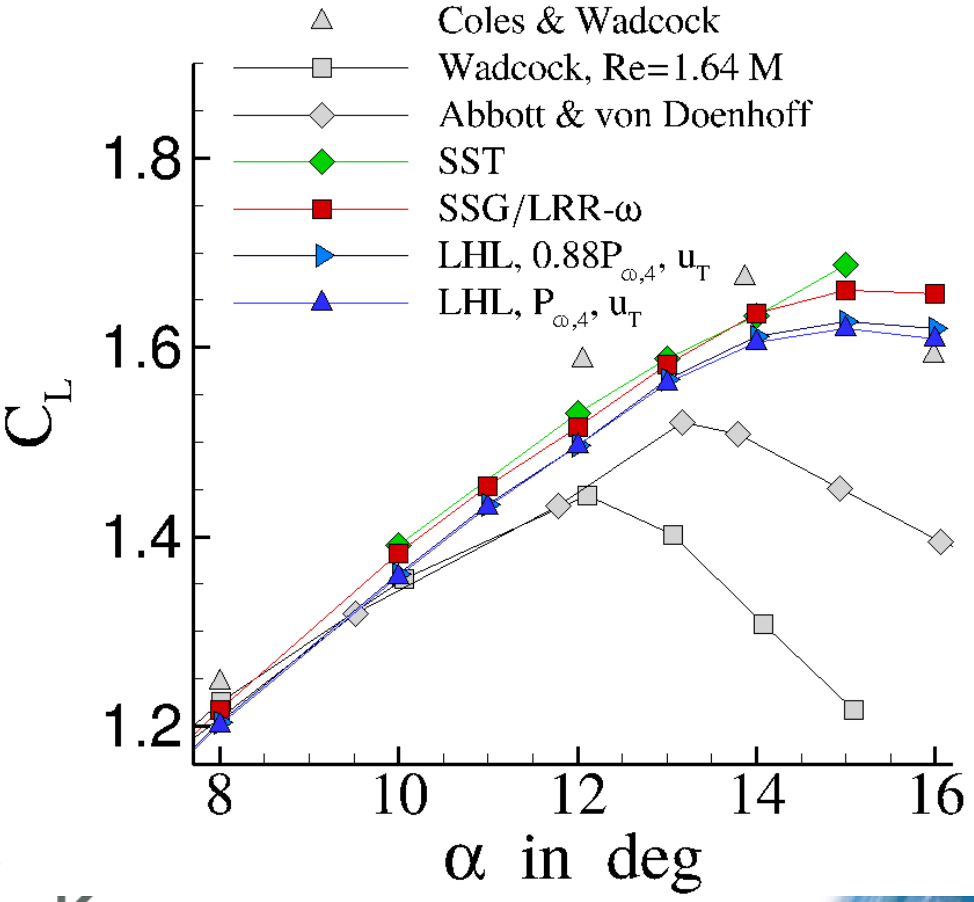


α = 12 deg

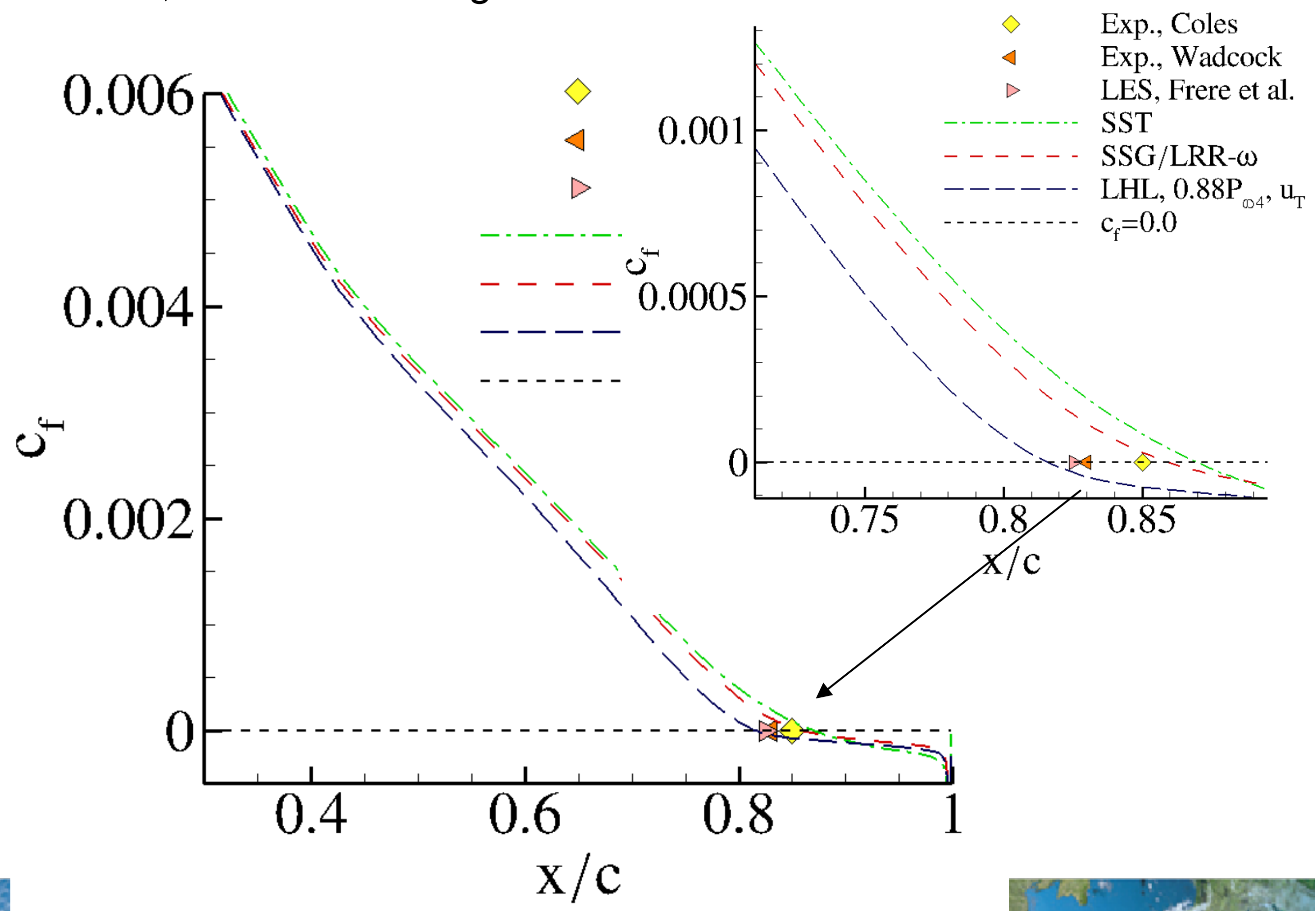


NACA 4412 airfoil. Re = 1.64 M, M=0.085

- Wind-tunnel experiments
  - Coles & Wadcock 1979 at Re=1.5Mio at CALCIT at CalTech
    - Test-section of length only 3.0m at an airfoil chord length 0.9m
  - Wadcock 1987 at Re=1.64Mio at NASA Ames
    - Test-section of length 4.6m and cross-section 2.1m x 3.0m, airfoil chord length 0.9m

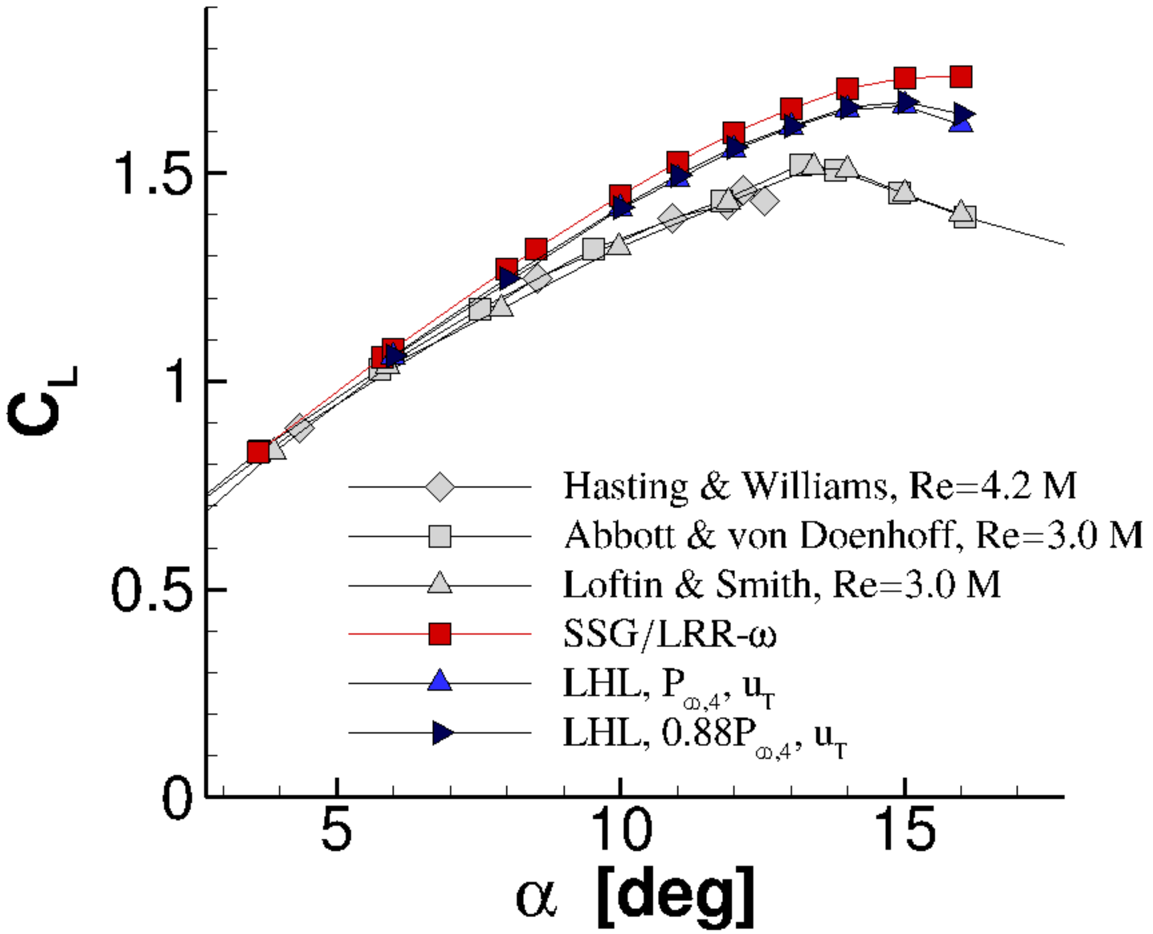


α = 12 deg

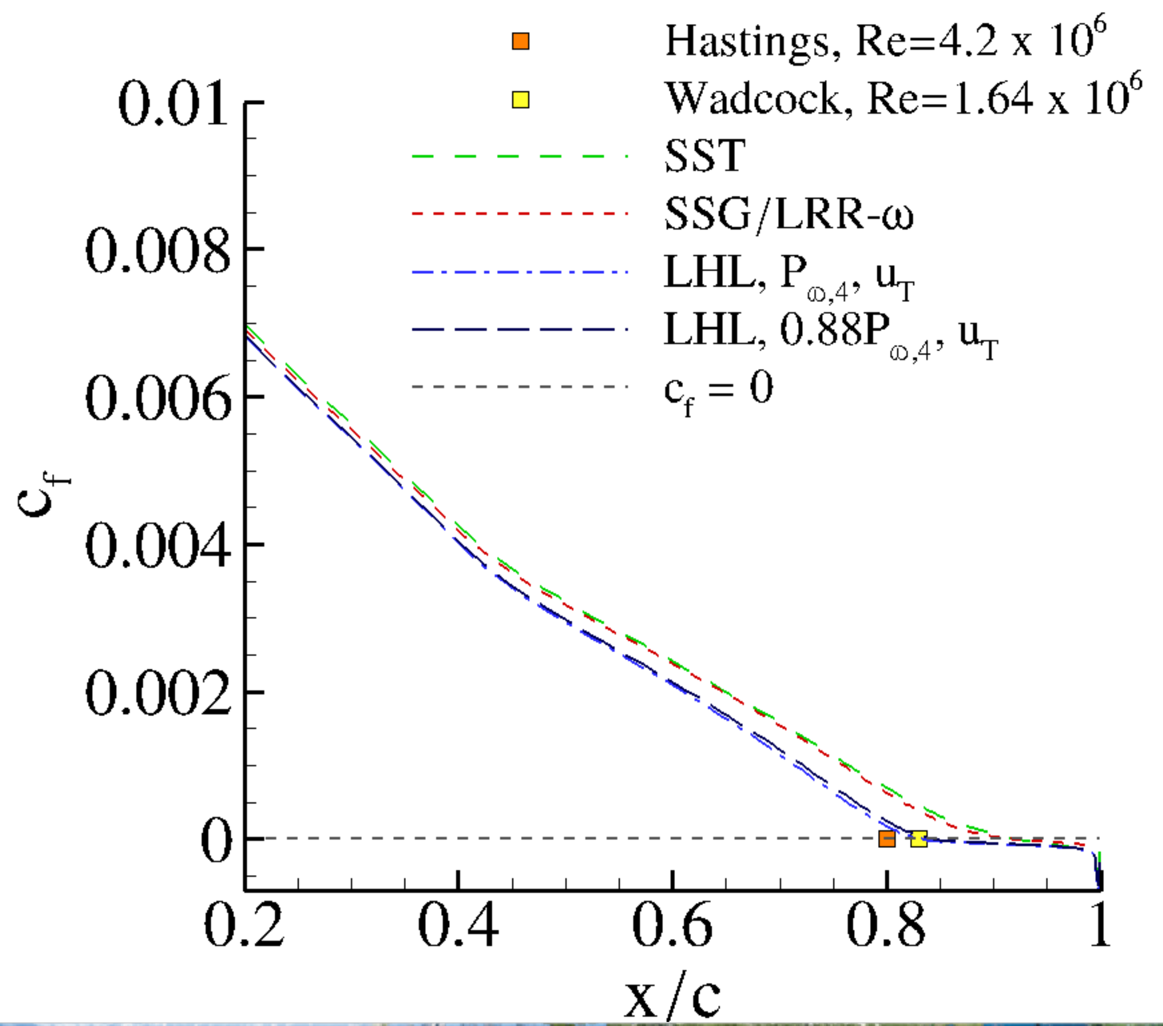


NACA 4412 airfoil. Re = 4.2 M, M=0.18

- Wind-tunnel experiments
  - Hasting and Williams (1987), at Bedford (too thick trip device?)



α = 12 deg

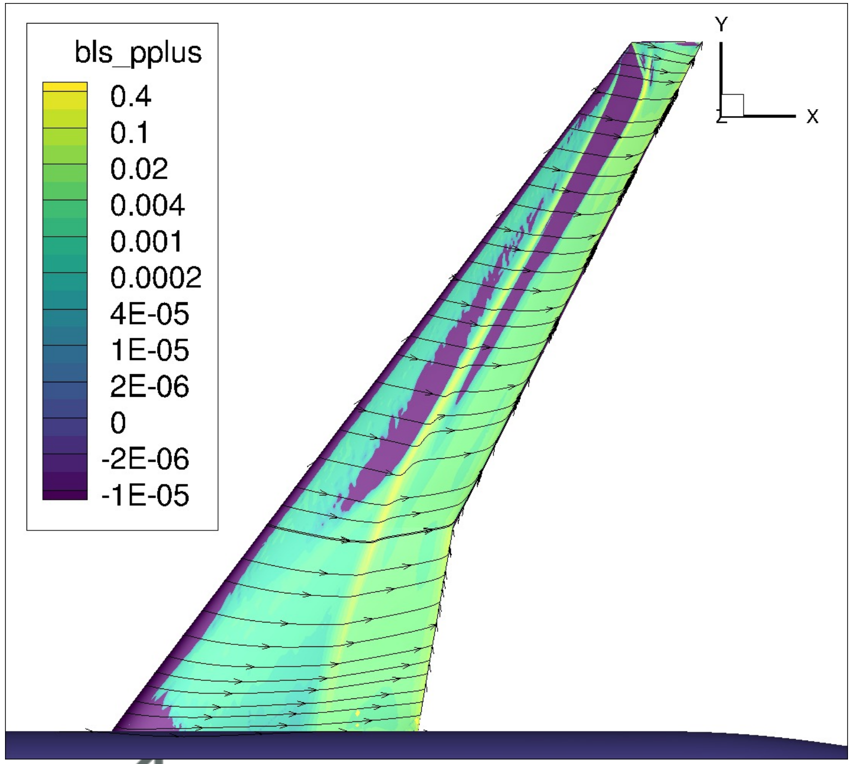


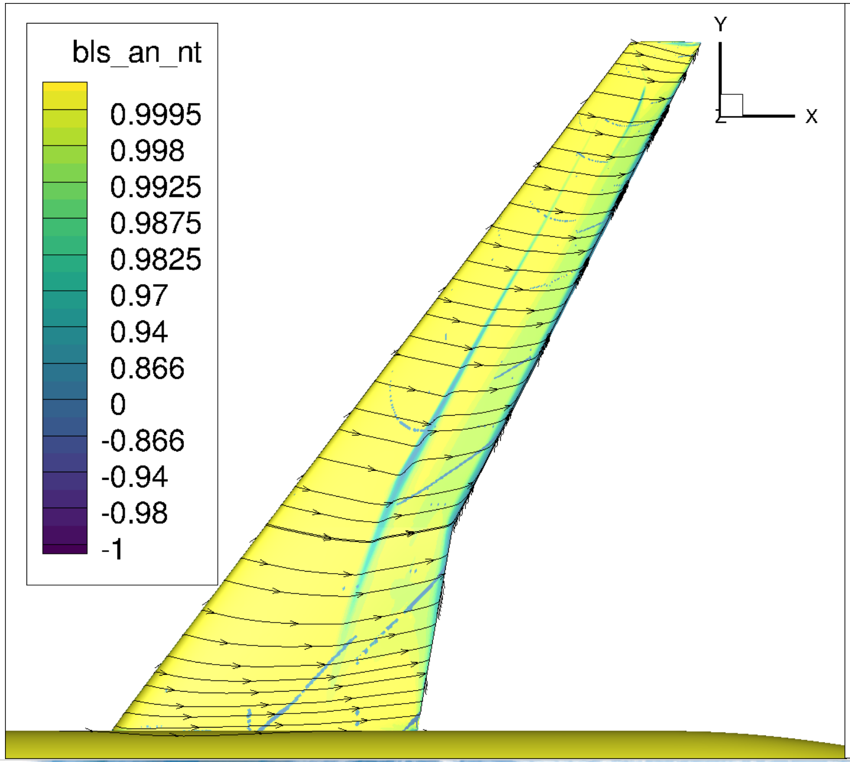
CRM DPW 6/7. M=0.85, Re=5M, α=2.5 deg

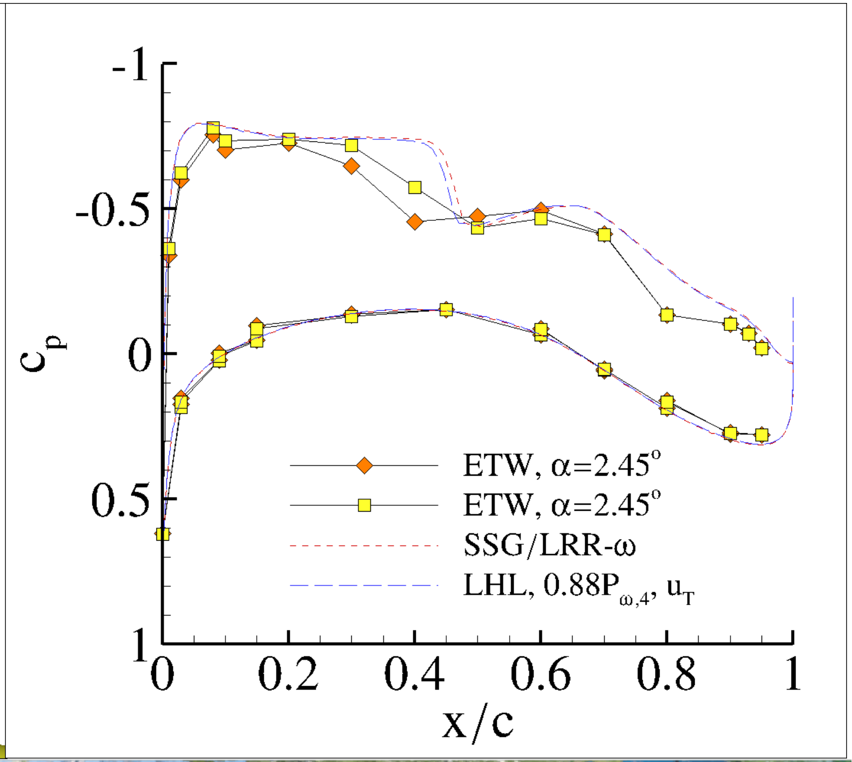
Surface distribution Δp_s^+

Definition of s used in Δp_s^+
Cosine angle between $U(y^+=1)$ and $U(y=0.1\delta)$

Cp at η=0.727







Summary & conclusion

Similarity of classical and DD/ML steps for the improvement of a RANS model

„Iteratively

Identification of a short-coming in the predictive accuracy (Data and validation test-cases)

↓

Identification of a term to be corrected in the turbulence equations

↓

Data = Test-cases for the improvement of a RANS model

Method to determine the discrepency term
- Numerically (FI/ML)
- Boundary layer approximation

Feature-Identification
- Statistical measures
- Theory

Parametrisation of the discrepency term
- Numerically (FI/ML)
- Boundary layer approximation

↓

Optionally: Sensor/blending function for local activation / deactivation of augmentation term

↓

Validation
Unit cases / Unit-interaction cases / Complex cases

Internat. cooperations of immeasurable value
DPW, HLPW, AVT, …

➔ Data driven methods and classical methods are very similar, DD/ML offers mighty (numerical) tools

Summary, Conclusion, and General Thoughts

- Modification of the SSG/LRR-ω in APGs
  - With APG modification: more susceptible to flow separation

- Classical approaches have always been data-driven, too.
  - Classical and DD methods share so many needs (need for good data, well-defined cases for validation)
  - They are/use complementary tools, which should work as friends
  - Human researcher's mind & experience and (ML) data-science tools are both needed for future progress

- Theory and data analysis are useful tools:
  - ➔ Reduction of a high-dimensional feature space
    - First order parameters : $(dP/ds)^+$, Re_τ ; higher order parameters : $(d^2P/ds^2)^+$, history effects
  - ➔ Avoids overfitting:
    - ➔Focus on first order effects
- Identify and filter out wind-tunnel effects (human's experience still needed) : This also avoids overfitting
  - ➔Using data for similar flows from different experiments
- Analytical inversion of boundary layer equations : remedies the problem of ill-posedness of FI if only using surface data

- Use of **blending functions** to activate a RANS augmentation term only in the target region (here: half-power law region)
  - ➔practical remedy (a single, composite model vs. a universal RANS turbulence model)
  - ➔Need to identify and to protect fundamental flow conditions (➔ Work by Bernhard Eisfeld on Friday morning)

Thank you for your attention.
Possibly time for a few questions…?

Funding by the DLR program directory board within the DLR internal projects Rettina, VicToria, and Adamant, by the DLR Institute of Aerodynamics and Flow Technology, and by DFG (Grant KA 1808/14-1 & SCHR 1165/3-1) is gratefully acknowledged.

Thanks to my co-workers from DLR EXV Andreas Schröder, Matteo Novara, Daniel Schanz, and from UniBw München Christian Kähler, Nico Reuther, Martin Bitter

Special thanks to Philippe Spalart for years of friendship and for countless stimulating and fruitful discussions and emails, and very valuable comments to manuscripts

Backup material

Calibration of the wall law. Theoretical support from Self-Similarity Analysis

- Ansatz of a Self-Similar Solution

$$U(s,y) = U_e(s) f'(\eta)\,, \quad \overline{u'v'} = u_t^2(s)\, t(\eta)\,, \quad \eta(s,y) = \frac{y}{\delta(s)}$$

- Bounday layer eqution fur U

$$-\frac{\delta U_e}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}s}[f''f] + \frac{\delta^2}{\nu}\frac{\mathrm{d}U_e}{\mathrm{d}s}\left[(f')^2 - f''f - 1\right] = f''' + \frac{U_e\delta}{\nu}\left(\frac{u_t}{U_e}\right)^2 t'$$

- Self-similar solution if the following parameters are independent of streamwise position s

$$\beta_1 \equiv \frac{\delta U_e}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}s} = \mathrm{const}\,, \quad \beta_H \equiv \frac{\delta^2}{\nu}\frac{\mathrm{d}U_e}{\mathrm{d}s} = \mathrm{const}\,, \quad \beta_3 \equiv \frac{U_e\delta}{\nu}\left(\frac{u_t}{U_e}\right)^2 = \mathrm{const}$$

- From laminar case (Falkner-Skan equations):
  Hartree parameter

$$\beta_\mathrm{H} = \frac{\delta^2}{\nu}\frac{\mathrm{d}U_e}{\mathrm{d}s} = -\Delta p_s^+\, Re_\tau^2 \left(\frac{u_\tau}{U_e}\right)$$

$y^+_{log,max} \approx C(\Delta p_s^+)^-(\delta^+)^{1/2}$

Comparison with FI/ML

High-quality data base (exp., DNS/LES)

Empirical wall law for the mean velocity at APG

APG modification of RANS model

Comparison of approach with FI/ML

Consider a large database of „training data“
- Large parameter space of TBL in PG

- Idea: Reduce large-dimensional feature space
  - First-order and higher-order local effects
  - Equilibrium and non-equilibrium flows
  - Effects of the wind-tunnel and measurement uncertainties

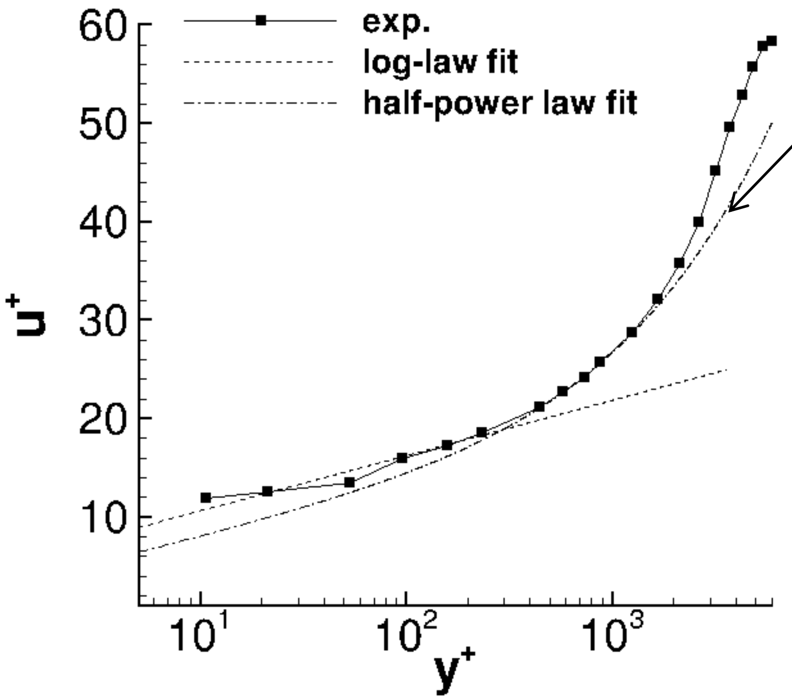
  ➔ Avoid overfitting (“Average/filter first, then fit”)
  ➔ Use data for similar flows from different experiments try to avoid fitting wind-tunnel effects

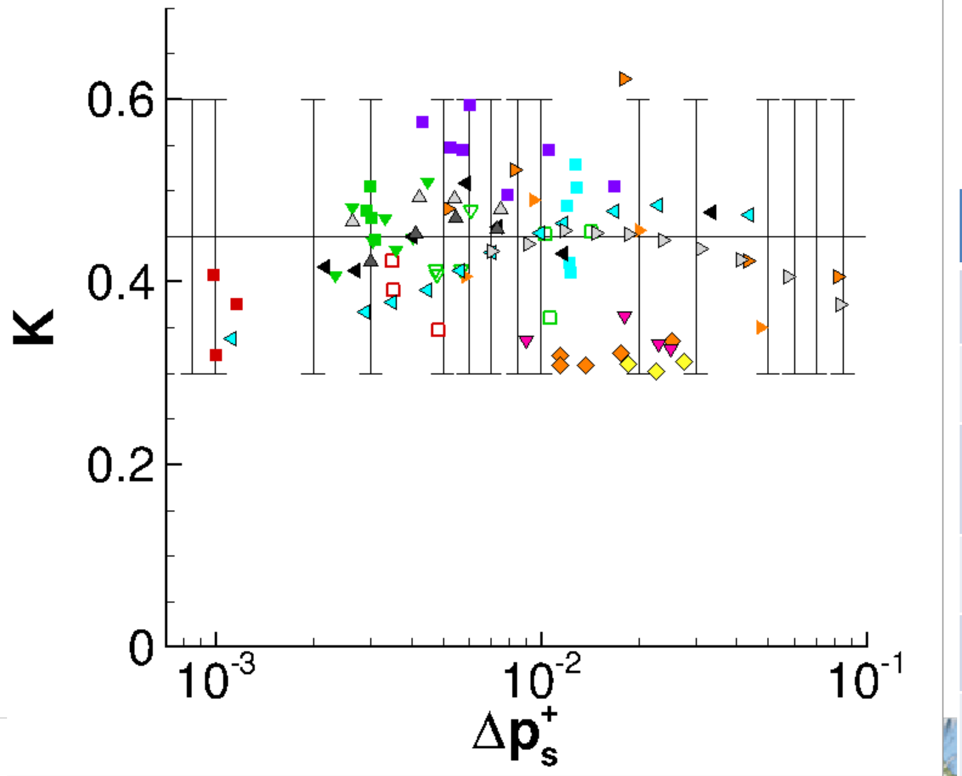
- Reduction to 1D boundary layer equations
  - Analytical field inversion possible (instead of numerical solution of an optimization problem)
  - Express the discrepency term as a function of admissible mean flow and turbulence quantities

Calibration of a wall-law at APG

Slope coefficient of half-power law

$$u^+ = \frac{1}{K}\left[\log(y^+) + 2\left(\sqrt{1+\Delta p_x^+ y^+} - 1\right)\right] + \tilde{B}$$





Author	K
Present (data base)	0.45±0.15
Townsend (1961)	0.48±0.03
Perry (1966)	0.48
Kader & Yaglom (1978)	**0.45**
Afzal (2008)	0.58
Mellor (1966) for data of Stratford cf=0 flow	0.44