Improvement on the AMM model for predicting wing-body juncture flows

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Motivation

- The k-ε model is widely used in engineering calculations, but not in aeronautical flows
- This reflects impaired predictions for TBLs with separation
- In particular, we see a smaller separation bubble for k-ε than for experiments and SA and SST
- This issue would also be associated with the pressure-gradient response of k-ε in separated flows
- Although SST (blended k-ε/k-ω) improves the prediction for separated flows significantly, the motivation is to have a single model (possibly using a QCR) instead of a blended one

AMM model (Abe-Mizobuchi-Matsuo 2019) (1/2)

Two-equation eddy viscosity model (low Re k-ε model)

Eddy viscosity approximation

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2\nu_t S_{ij} \qquad \left(S_{ij} \equiv \left(\bar{U}_{i,j} + \bar{U}_{j,i} \right) / 2 \right)$$

Representation of ν_t

$$\nu_t = C_\mu f_\mu k^2 / \varepsilon$$

k equation

$$\frac{\partial k}{\partial t} + \bar{U}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right\} + P_k - \varepsilon$$

ε equation

$$\frac{\partial \varepsilon}{\partial t} + \bar{U}_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right\} + \frac{\varepsilon}{k} \left(C_{\varepsilon 1} P_k - C_{\varepsilon 2} f_\varepsilon \varepsilon \right)$$

Model functions

$$f_\mu = \left\{ 1 - \exp\left(-\frac{R_y}{A} \right) \right\} \left[1 + \frac{5}{R_t^{3/4}} \exp\left\{ -\left(\frac{R_t}{200} \right)^2 \right\} \right] \quad (A = 120) \qquad f_\varepsilon = \left\{ 1 - \exp\left(-\frac{R_y}{B} \right) \right\} \left[1 - \frac{2}{9} \exp\left\{ -\left(\frac{R_t}{6} \right)^2 \right\} \right] \quad (B = 12)$$

Model coef.	C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
AMM	0.09	1.4	1.4	1.5	1.9

AMM model (Abe-Mizobuchi-Matsuo 2019) (2/2)

Two-equation eddy viscosity model (low Re k-ε model)

Limiter for ν_t

$$f_\mu = min\left(f_\mu, 10\right)$$

Realizability limiter

$$\nu_t = C_\mu f_\mu k T_\mu$$

$$T_\mu = \min\left(\frac{k}{\varepsilon}, \frac{1}{\sqrt{6}C_\mu S}\right) \qquad \left(S = \sqrt{S_{ij}S_{ij}},\ S_{ij} \equiv \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right) \right)$$

Quadratic constitutive relation (QCR, improved from Spalart's)

$$-\overline{u_i u_j} = -\frac{2}{3}k\delta_{ij} + 2\nu_t S_{ij} - C_1 \frac{k}{\varepsilon}\nu_t \left[\Omega_{ik}S_{jk} + \Omega_{jk}S_{ik}\right] - C_2 \frac{k}{\varepsilon}\nu_t \left[S_{ik}S_{kj} - \frac{1}{3}S_{mn}S_{mn}\delta_{ij}\right]$$

$$\left(S_{ij} \equiv \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right),\ \Omega_{ij} \equiv \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial \overline{U}_j}{\partial x_i}\right) \qquad C_1 = C_2 = 0.6 \right)$$

Improvement of the outer edge behavior in a TBL

- Cazalbou-Spalart-Bradshaw (1994)'s mathematical analysis
- CSB noted that a constraint

$$2\sigma_k -1 \le \sigma_\varepsilon$$

is required for representing the outer edge of a TBL layer properly in a two-equation k-epsilon model.

- Except for the below model 3) and the standard high Re k-epsilon model, the condition "$2\sigma_k -1 \le \sigma_\varepsilon$" is not satisfied in a low Re k-epsilon model.
- It had yet to become clear if the constraint "$2\sigma_k -1 \le \sigma_\varepsilon$" affects the prediction of low Re k-epsilon model significantly. This was examined for AMM.

- Diffusion coefficients for low Re k-epsilon

1) $\sigma_k = 1.4$, $\sigma_e = 1.4$ (Abe-Konhon-Nagano1994)
2) $\sigma_k = 1.2/f_t$, $\sigma_e = 1.3/f_\varepsilon$ (Nagano-Shimada 1995)
3) $\sigma_k = 1.2/f_t$, $\sigma_e = 1.4/f_\varepsilon$ (Abe-Jang-Leschziner 2003)
4) $\sigma_k = 1.4$, $\sigma_e = 1.4$ (AMM)

Note that f_t and f_ε denote model functions so that σ_k and σ_ε in the works of 2) and 3) are not constant but depend on distance from the wall.

1D inverted parabola analysis

➢ Initial profiles (t=0)

$$\nu_t = C_\mu \left(1 - x^2\right)$$

$$k = 1 - x^2$$

$$\varepsilon = 1 - x^2$$

➢ k and ε equations

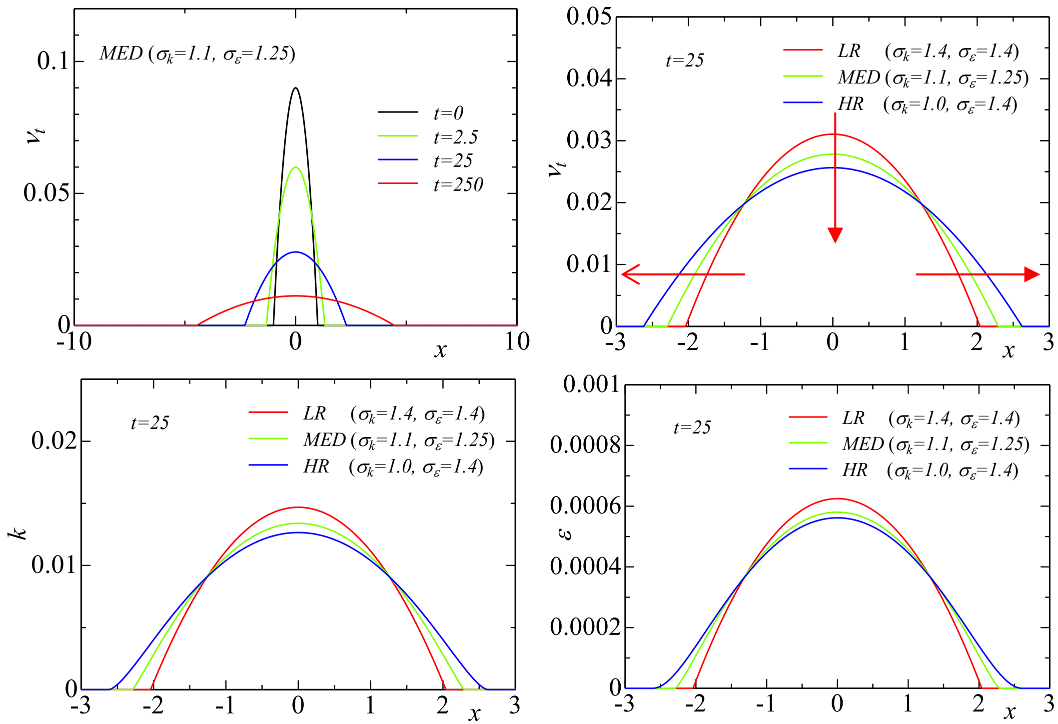
$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial x_j}\left\{\left(\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\} - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial x_j}\left\{\left(\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right\} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$\left(C_\mu = 0.09,\ C_{\varepsilon 2} = 1.5 \right)$$

	LR	MED	HR
$(\sigma_k, \sigma_\varepsilon)$	(1.4, 1,4)	(1.1, 1.25)	(1.0, 1.4)
$2\sigma_k - \sigma_\varepsilon$	1.4	0.95	0.6
CSB condition $2\sigma_k - \sigma_\varepsilon \leq 1$	Not satisfied	satisfied	Satisfied

HR may not be applied to a low Re k-ε model

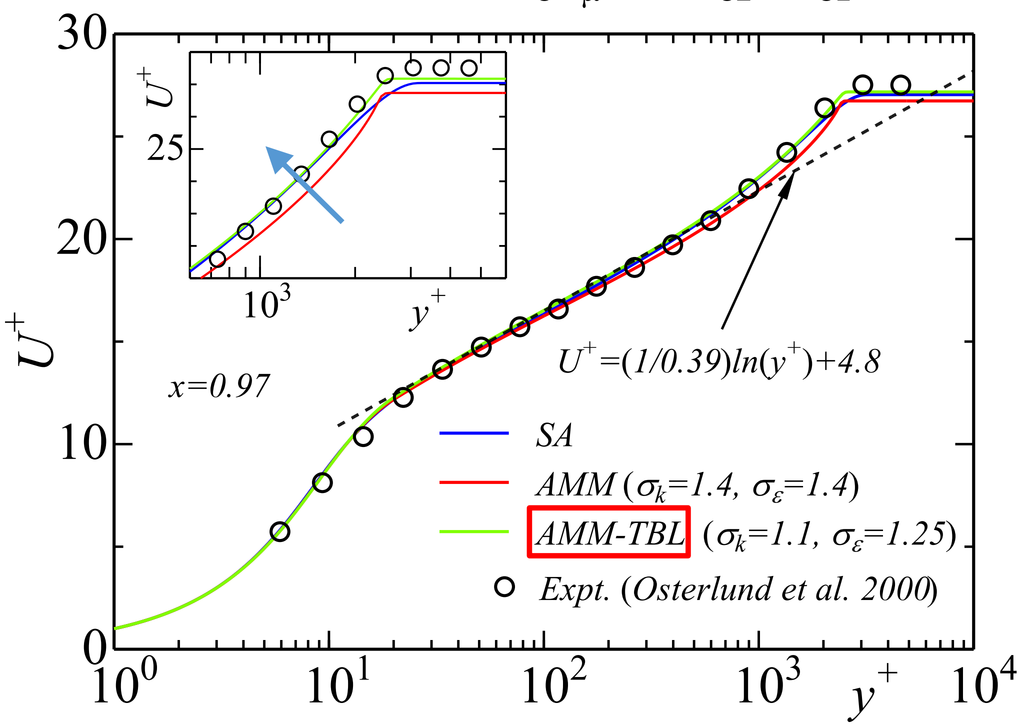


The propagation becomes faster with decreasing the magnitude of $2\sigma_k - \sigma_\varepsilon$.

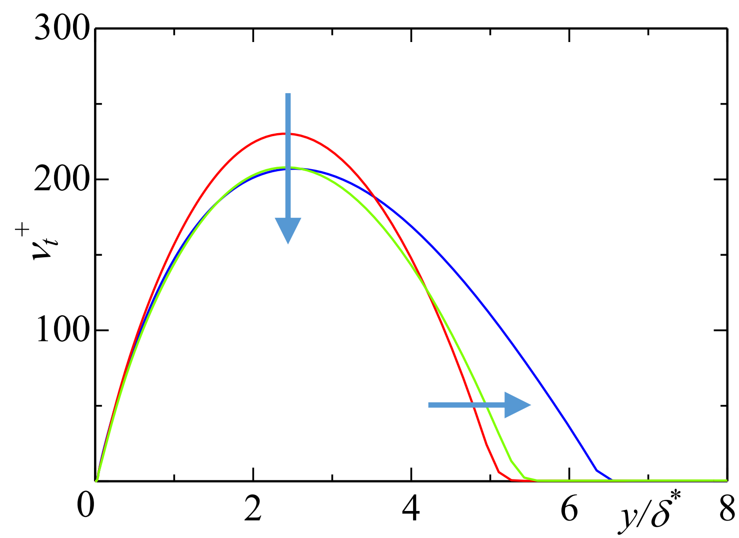
Modified AMM model

Model coef.	C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
AMM	0.09	1.4	1.4	1.5	1.9
AMM-TBL	0.09	1.1	1.25	1.5	1.9

- We have modified AMM model coefficients using the MED condition (denoted as AMM-TBL), which can be used for calculating both internal and external flows.
- The resulting von Karman constant for AMM-TBL is constraint is κ=0.39, which is estimated by $\kappa^2 = \sigma_\varepsilon C_\mu^{1/2} (C_{\varepsilon 2} - C_{\varepsilon 1})$ and is within the current κ value.

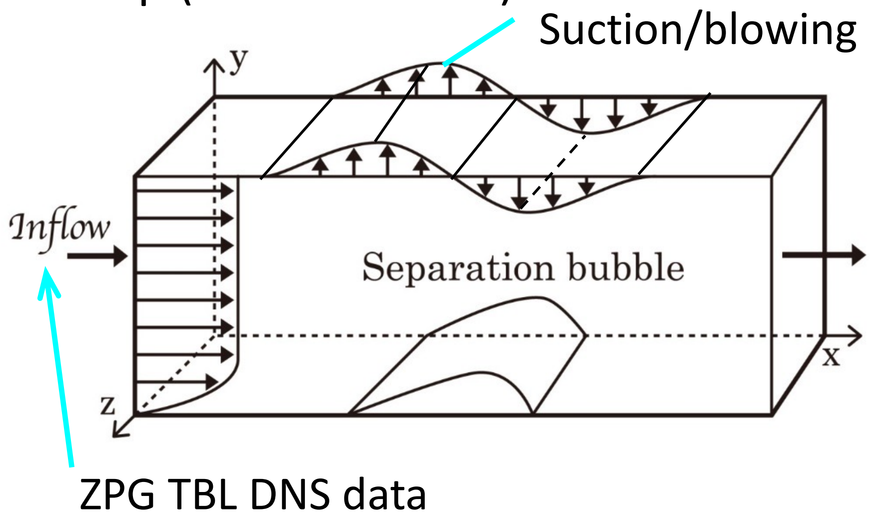


The outer edge behavior has been indeed improved!

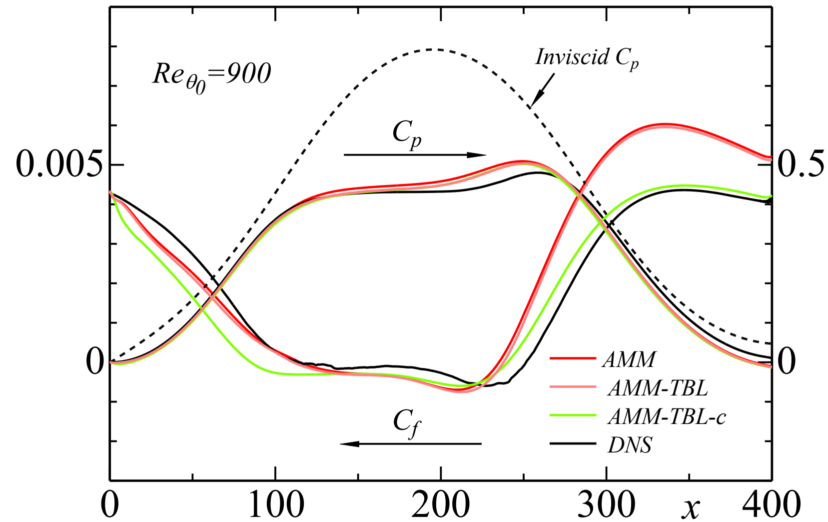


Modified AMM model and the prediction for a pressure-induced separation bubble

DNS setup (Abe 2017 JFM)

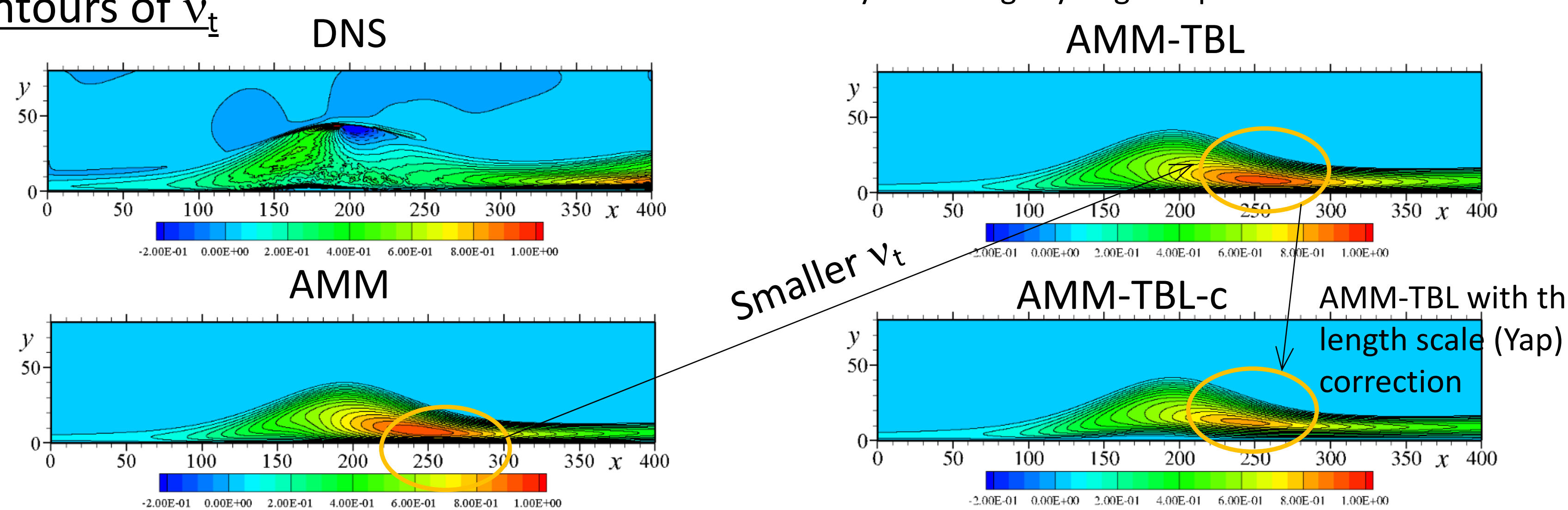


Cf and Cp profiles



AMM-TBL yields a slightly larger separation bubble than AMM

Contours of ν_t



AMM-TBL also improves the prediction for a separation bubble slightly

AMM-QCRcorner model

We consider the non-zero value of the mean streamwise vorticity in a corner flow where the Reynolds stress anisotropy plays a crucial role (Bradshaw 1987).

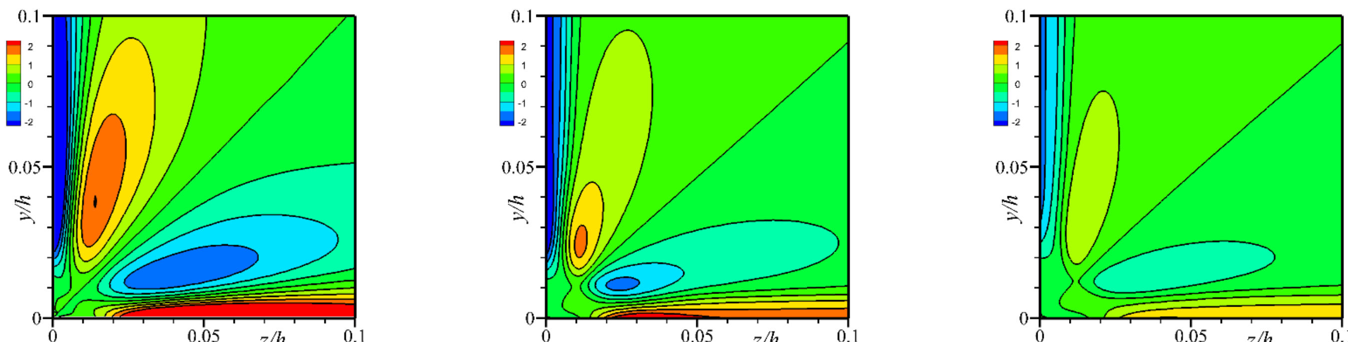
In AMM-QCRcorner, a $\Omega\Omega$ term is added to the original AMM-QCR, i.e.

$$-\overline{u_i u_j} = -\frac{2}{3}k\delta_{ij} + 2\nu_t S_{ij} - C_1 \frac{k}{\varepsilon}\nu_t \left[\Omega_{ik}S_{jk} + \Omega_{jk}S_{ik}\right] - C_2 \frac{k}{\varepsilon}\nu_t \left[S_{ik}S_{kj} - \frac{1}{3}S_{mn}S_{mn}\delta_{ij}\right]$$

$$- C_3 \frac{k}{\varepsilon}\nu_t \left[\Omega_{ik}\Omega_{kj} + \frac{1}{3}\Omega_{mn}\Omega_{mn}\delta_{ij}\right] \qquad C_1 = 0.6, C_2 = 0.2, C_3 = -0.3$$

C_1, C_2, C_3 have been determined using DNS data in the channel and square duct.

Distributions of the normalized mean streamwise vorticity in a square duct at Re_τ=1000

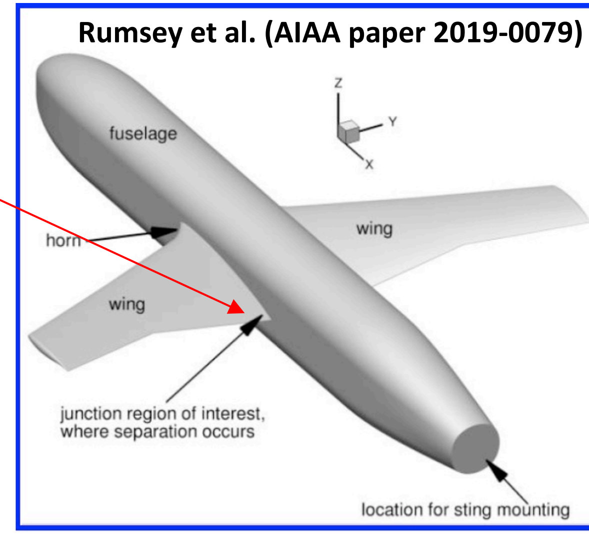


DNS (Pirozzoli et al. 2018 JFM) | AMM-QCRcorner | AMM-QCR

The prediction of AMM-QCRcorner is better than that of AMM-QCR, and agrees reasonably with the DNS data.

Prediction of AMM-QCRcorner for NASA Juncture Flow

The side-of-body separation occurs near the trailing edge of the wing near a wing-root junction.



- The experimental data :

  Kegerise and Neuhart (2019 NASA TM)

  NASA TMR website

- Re based on crank chord : 2.4million
- Mach number Ma : 0.189
- Attack of angle : α = 5 (-2.5 to 5 in the experiment)

  (to clarify to what extent AMM-QCRcorner predicts a separation bubble)

- Grids (NASA TMR website) : Coarse (12,312,544) and MED (39,121,991)
- Solver : FaSTAR (Unstructured grid solver developed by JAXA)

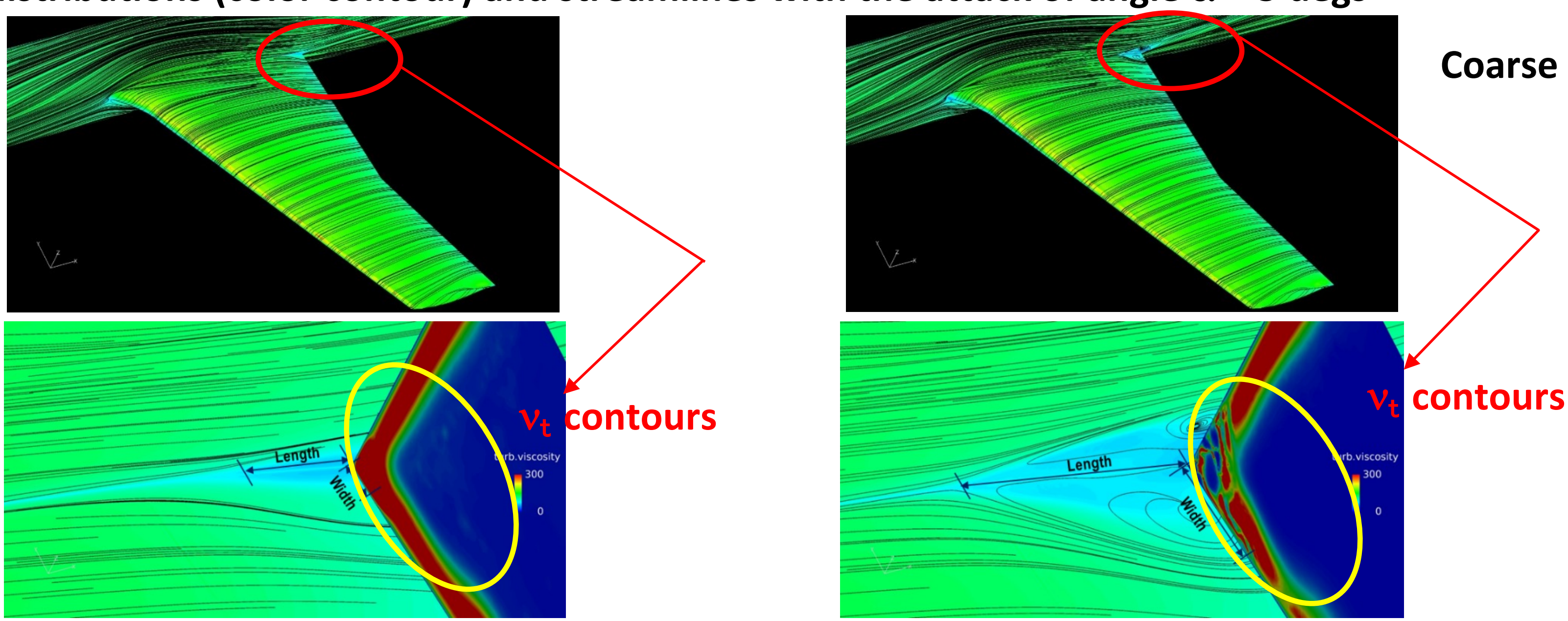
Modification for the eddy viscosity expression in the AMM-QCRcorner model

For airfoil calculations, we modify the expression for ν_t by incorporating a parameter $S^2 - \Omega^2$ (representing the acceleration and deceleration of the mean flow) into the turbulence time scale T_μ using the augmented time scale procedure by Yoshizawa et al. (2006 PoF).

$$\nu_t = \frac{C_\mu f_\mu k T_\mu}{\left(1 + C_{S\Omega}\left[\left(S_{ij}^2 - \Omega_{ij}^2\right)\left(k/\varepsilon\right)^2\right]^2\right)^{1/2}}, \quad T_\mu = \min\left(\frac{k}{\varepsilon}, \frac{0.6}{\sqrt{6}}\frac{1}{C_\mu S_{ij}^2}\right)$$

$$\left(S_{ij} \equiv \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right), \ \Omega_{ij} \equiv \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial \overline{U}_j}{\partial x_i}\right) \quad C_\mu = 0.09 \quad C_{S\Omega} = 1 \right)$$

C_f distributions (color contour) and streamlines with the attack of angle α = 5 degs



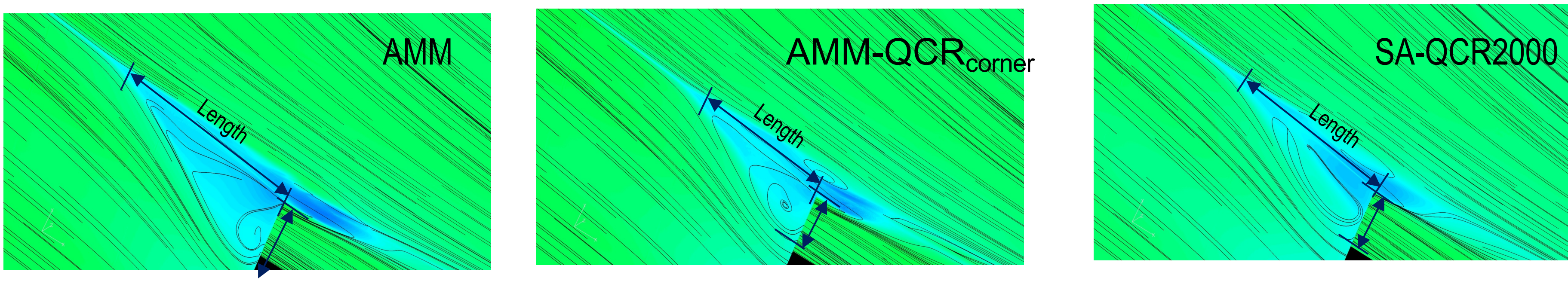
Coarse grid

In the trailing edge region where the APG is large, the modified ν_t expression improves the large magnitude of ν_t and hence the size of the separation bubble.

Prediction of AMM-QCRcorner for NASA Juncture Flow

C_f distributions (color contour) and streamlines with the attack of angle α = 5 degs

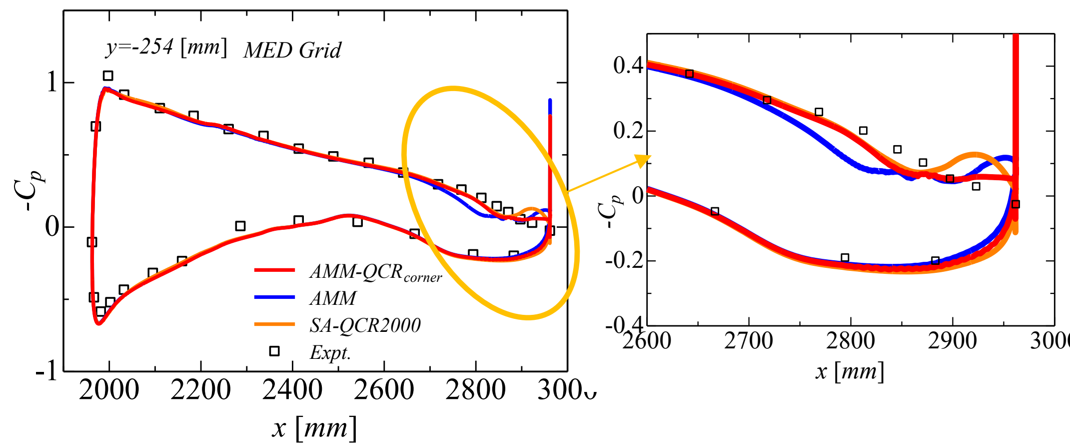
MED grid



Bubble size

	Length	Width
Expt	119mm	43mm
AMM-QCR$_{corner}$	127mm	50mm
AMM	173mm	71mm
SA-QCR2000	146mm	52mm

Cp distributions

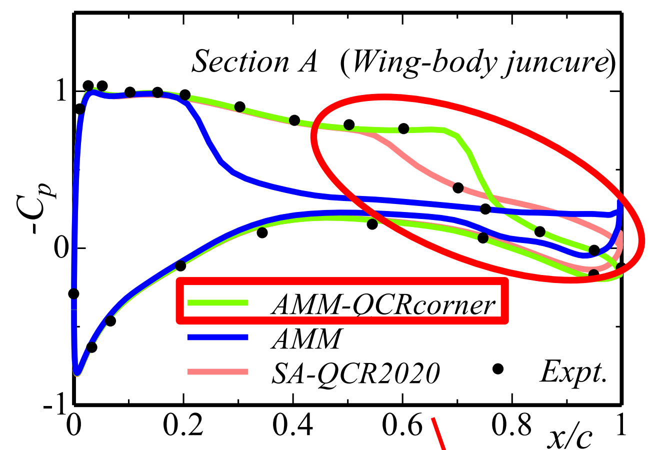


The size of the separation bubble predicted by the AMM-QCRcorner model agrees well with the experimental data.

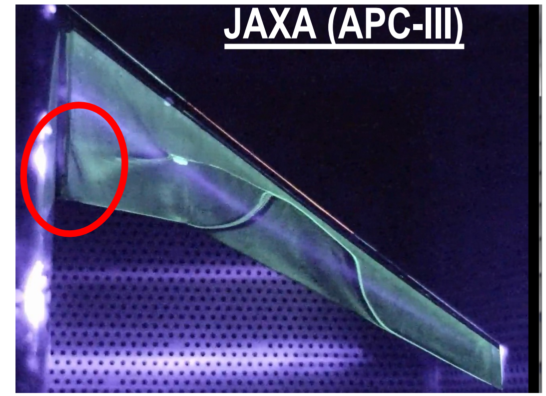
Prediction of AMM-QCRcorner for NASA CRM

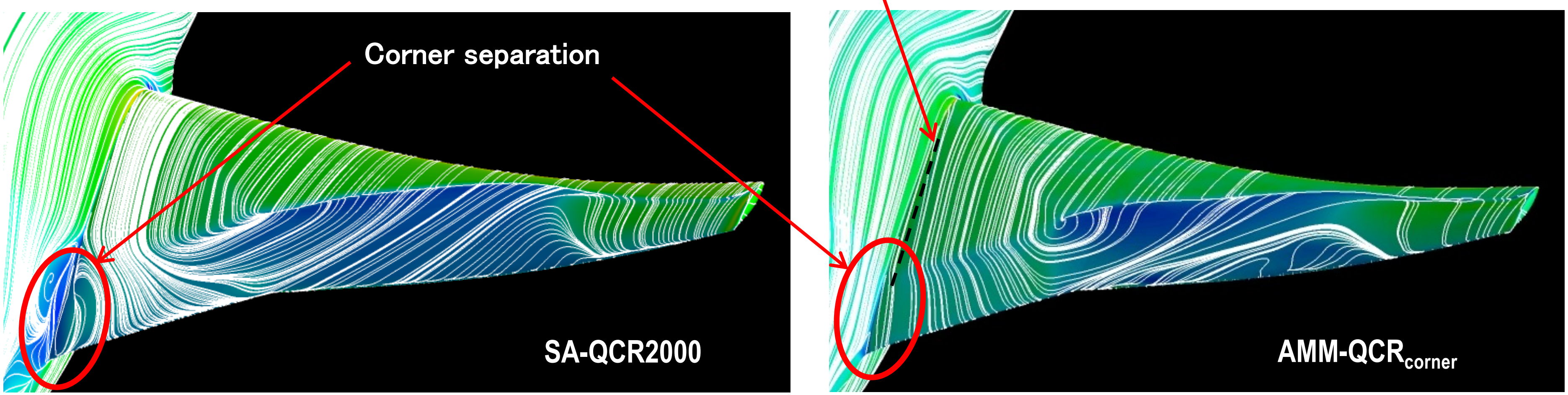
JAXA APC III
- Re=2.26 x 10^6
- Ma=0.847
- α=5.72 degs
- Number of grid points :
  N=9,006,808 (MED GRID)
- $T\mu$=0.08%
- ν_t/ν=0.10 (AMM)

Distribution of CP (wing body juncture)



Experiment (Oil flow)





For the NASA CRM, the AMM-QCRcorner model also predicts a corner separation bubble reasonably.

Summary

➢ AMM model modification

- The outer edge behavior in a TBL is repaired with the use of the CSB mathematical analysis
- The QCR for improving the prediction in a corner flow (AMM-QCRcorner)
- The eddy viscosity expression, by incorporating a parameter S^2-Ω^2 into the turbulence time scale, for avoiding the large ν_t in the APG region

➢ Improvement on the AMM model

- The outer edge behavior in a TBL and the prediction for a separation bubble are improved by the modified AMM model
- The AMM-QCRcorner model reproduces a strong secondary flow near a corner with large mean streamwise vorticity
- The corner separation predictions of AMM-QCRcorner for the NASA Juncture Flow and NASA CRM compare well with experimental data