Washington University in St.Louis
SCHOOL OF ENGINEERING & APPLIED SCIENCE

# Review & Potential of Wray-Agarwal Family of Turbulence & Transition Models for RANS Simulations

Ramesh Agarwal

Mechanical Engineering and Material Science Department

Washington University in St. Louis

NASA 2022 Symposium for Turbulence Modeling

(in honor of Phillipe R. Spalart)

# Wray-Agarwal (2017) Model

Washington University in St. Louis
School of Engineering & Applied Science

**https://turbmodels.larc.nasa.gov/wray_agarwal.html**

- One equation model derived from $k$-$\omega$ closure
- Switching function allows behavior like $k$-$\varepsilon$ model in the far field and like $k$-$\omega$ model in the wall region
- WA 2018 wall-distance free version
- Has been tested on 70+, 2D and 3D cases by researchers in US, China, India, Japan, Italy, Finland, Singapore, Czech Republic etc.
- Has been extended to include QCR and to hybrid DES, DDES, and IDDES models
- Has been extended to compressible high speed flows, rough wall flows and to include the effects of rotation & curvature
- Has been implemented in OpenFOAM, FUN3D and in some individually developed codes.
- UDF and source code modules are available on Github.

# Wray-Agarwal (2017) Model

Washington University in St. Louis
School of Engineering & Applied Science

$$\frac{\partial R}{\partial t}+\frac{\partial u_j R}{\partial x_j}$$

$$=\frac{\partial}{\partial x_j}\left[(\sigma_R R+\nu)\frac{\partial R}{\partial x_j}\right]+C_1 RS+f_1 C_{2k\omega}\frac{R}{S}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}-(1-f_1)min\left(C_{2k\varepsilon}R^2\left(\frac{\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}}{S^2}\right),\ C_m\frac{\partial R}{\partial x_j}\frac{\partial R}{\partial x_j}\right)$$

$\mu_t=\rho f_\mu R,\ R=k/\omega$, switching function $f_1=min(tanh(arg_1^4),0.9)$

$$arg_1=\frac{1+\frac{d\sqrt{RS}}{\nu}}{1+\left[\frac{max(d\sqrt{RS},1.5R)}{20\nu}\right]^2}$$

$$f_\mu=\frac{\chi^3}{\chi^3+C_w^3},\qquad \chi=\frac{R}{\nu}$$

$$S=\sqrt{2S_{ij}S_{ij}},\qquad S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

$$C_{1k\omega}=0.0833\quad C_{1k\varepsilon}=0.1127$$

$$C_1=f_1(C_{1k\omega}-C_{1k\varepsilon})+C_{1k\varepsilon}$$

$$\sigma_{k\omega}=0.72\quad \sigma_{k\varepsilon}=1.0$$

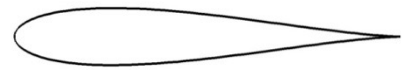
$$\sigma_R=f_1(\sigma_{k\omega}-\sigma_{k\varepsilon})+\sigma_{k\varepsilon}$$

$$\kappa=0.41$$

$$C_{2k\omega}=\frac{C_{1k\omega}}{\kappa^2}+\sigma_{k\omega}\quad C_{2k\varepsilon}=\frac{C_{1k\varepsilon}}{\kappa^2}+\sigma_{k\varepsilon}$$

$$C_w=8.54\quad C_\mu=0.09$$

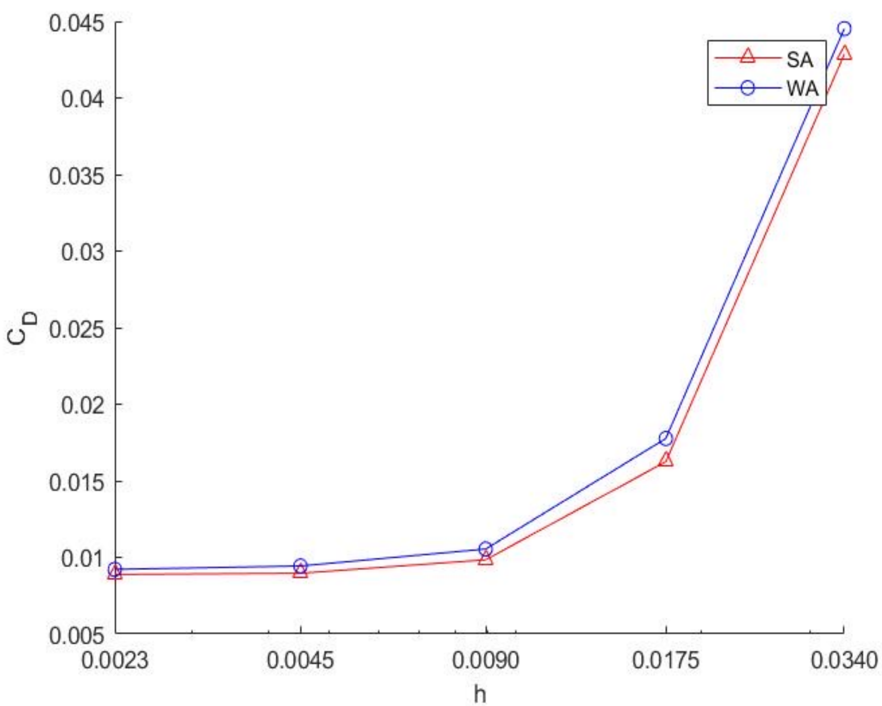
# HFW 2022
# Joukowski Airfoil

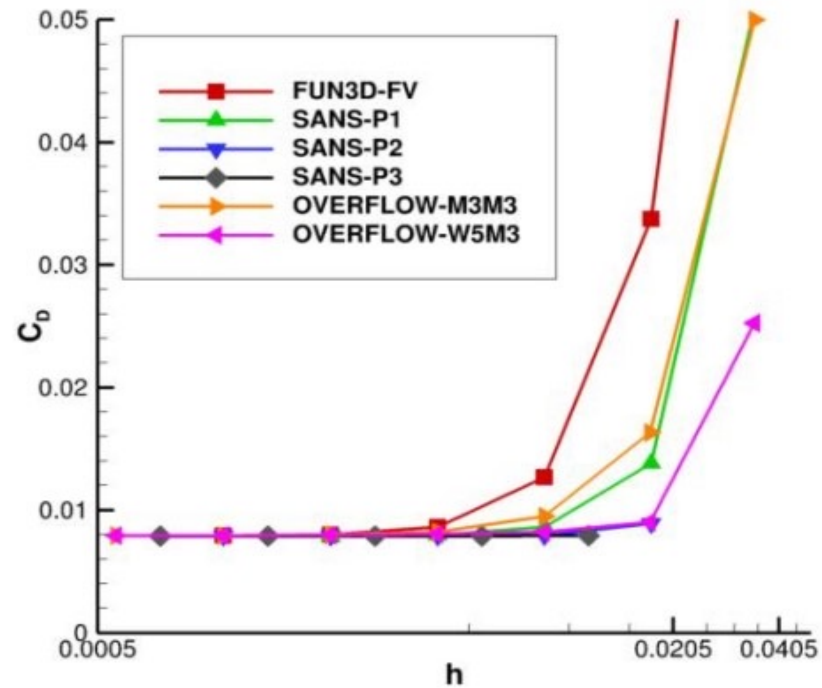


Washington University in St. Louis
School of Engineering & Applied Science

- M = 0.15, Re = $3 * 10^6$
- Incompressible RANS
- Green-Gauss Cell based second-order upwind
- Convergence of CD <10^-7
- Case based on guidelines for RANS SA-[neg]-QCR2000

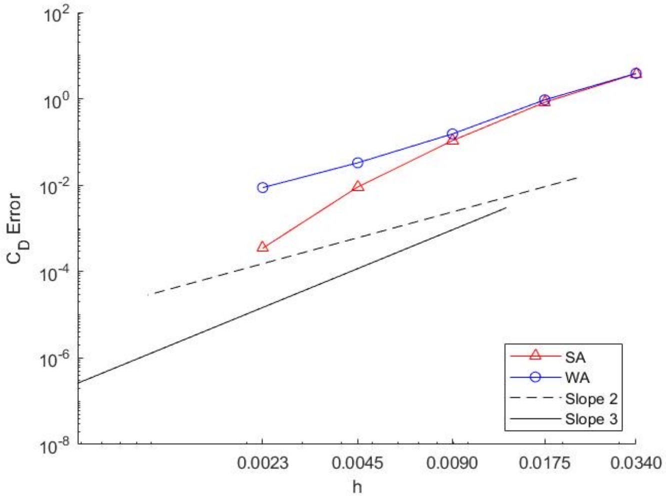
- M = 0.15, Re $6 * 10^6$
- Compressible RANS
- SA-QCR-2000 model verification results AIAA 2021-1552 (Diskin et al.)

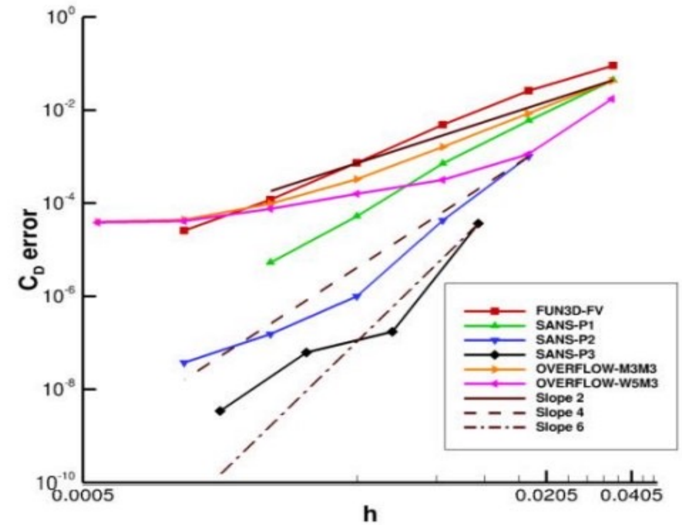


# Joukowski Airfoil Convergence History

Washington University in St.Louis
School of Engineering & Applied Science

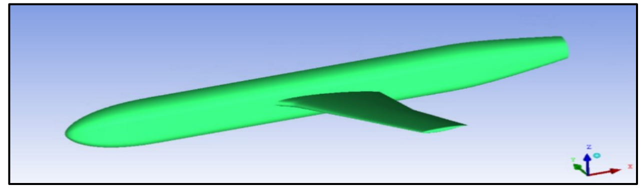



| Grid | Model | Cl | Cp | Cv | Cd | Error | N | h |
|---|---|---|---|---|---|---|---|---|
| 0 | SA | -7.64E-08 | 0.035976 | 0.00687 | 0.042846 | 3.823005621 | 864 | 0.034021 |
| 1 | SA | 3.12E-08 | 0.009151 | 0.007096 | 0.016248 | 0.828948721 | 3264 | 0.017504 |
| 2 | SA | -2.57E-08 | 0.002517 | 0.007325 | 0.009842 | 0.107887045 | 12480 | 0.008951 |
| 3 | SA | -4.52E-09 | 0.00155 | 0.007414 | 0.008965 | 0.009099917 | 49536 | 0.004493 |
| 4 | SA | -0.00032 | 0.00147 | 0.007417 | 0.008887 | 0.000347379 | 197376 | 0.002251 |
| 5 | SA | 4.66E-05 | 0.001438 | 0.007445 | 0.008884 | 0 | 787968 | 0.001127 |
| | | | | | | | | |
| 0 | WA | 4.31E-09 | 0.036392 | 0.008131 | 0.044522 | 3.872621559 | | 0.034021 |
| 1 | WA | -6.18E-08 | 0.009489 | 0.008276 | 0.017764 | 0.94417569 | | 0.017504 |
| 2 | WA | -5.39E-09 | 0.002671 | 0.007875 | 0.010546 | 0.154191605 | | 0.008951 |
| 3 | WA | 3.30E-10 | 0.001661 | 0.007777 | 0.009438 | 0.032969675 | | 0.004493 |
| 4 | WA | -1.96E-09 | 0.001509 | 0.007709 | 0.009218 | 0.00879792 | | 0.002251 |
| 5 | WA | 4.79E-08 | 0.001477 | 0.00766 | 0.009137 | 0 | | 0.001127 |

Meshes provided by Galbraith for Joukowski Airfoil (HFW 2022)

# HFW 2022
# Juncture Flow Model



Washington University in St. Louis
School of Engineering & Applied Science

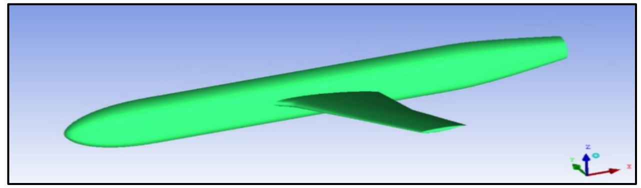
M = 0.189, Re = $2.4*10^6$ , α = -2.5°, 0°, 5° and 7.5°,19 million cells

Experiment: AIAA 2019-0077

Lift and Drag at α = 5°

| | Wray-Agarwal | Spalart-Allmaras | SA-QCR2000 | k-w SST | k-w SST-QCR2000 | High-Fidelity CFD Workshop |
|---|---|---|---|---|---|---|
| Drag Coefficient | 0.0713 | 0.0693 | 0.0690 | 0.0697 | 0.0688 | 0.07 |
| Lift Coefficient | 0.831 | 0.854 | 0.0849 | 0.857 | 0.851 | 0.85 |

# HFW 2022
# Juncture Flow Model



Washington University in St. Louis
School of Engineering & Applied Science

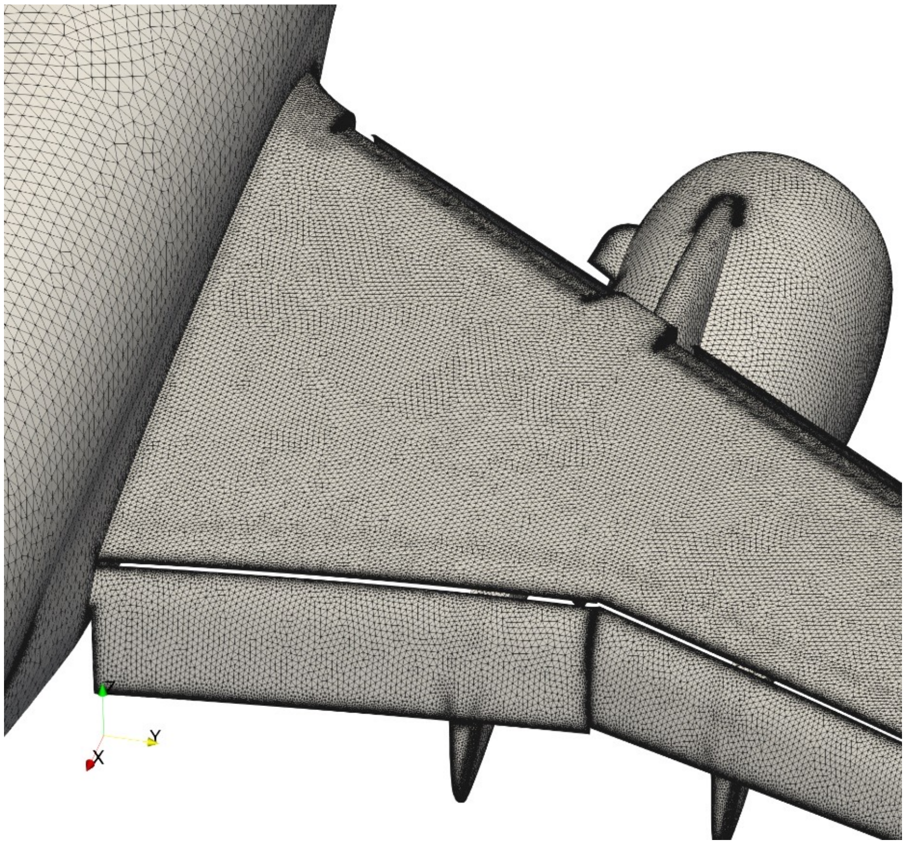
M = 0.189, Re = $2.4*10^6$ , $\alpha$ = -2.5°, 0°, 5° and 7.5°, 19 million cells

## Separation Size Prediction

| | Wray-Agarwal | Spalart-Allmaras | SA-QCR2000 | k-w SST | k-w SST-QCR2000 | Wind Tunnel |
|---|---|---|---|---|---|---|
| Length [mm] | 247.7 | 242.5 | 157.06 | 231.0 | 155.50 | 110.6 |
| Width [mm] | 55.4 | 49.0 | 42.319 | 43.0 | 41.39 | 40.3 |

# HLPW 4: NASA Common Research Model High-Lift (CRM-HL)

- $M = 0.2$, $Re_{MAC} = 5.49*10^6$, $\alpha = 19.47^o$, 232 million cells



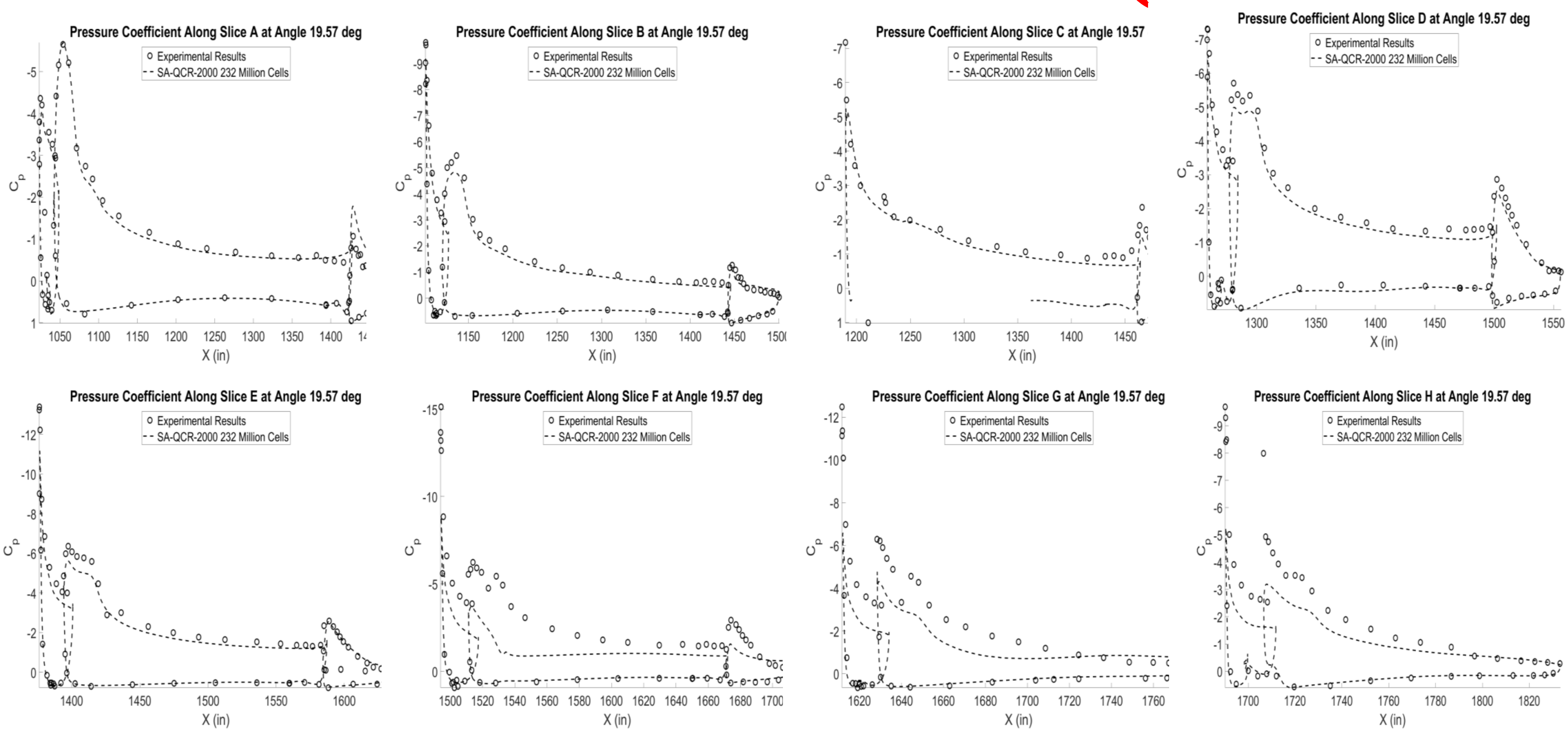

| 19.47 deg | CD | CL |
|---|---|---|
| Experiment | 0.362 | 2.515 |
| SA | 0.416 | 2.086 |
| SA-QCR-2000 | 0.397 | 2.235 |
| WA-QCR-2000 | 0.403 | 2.217 |

# HLPW 4: NASA Common Research Model High-Lift (CRM-HL)

Washington University in St. Louis
School of Engineering & Applied Science

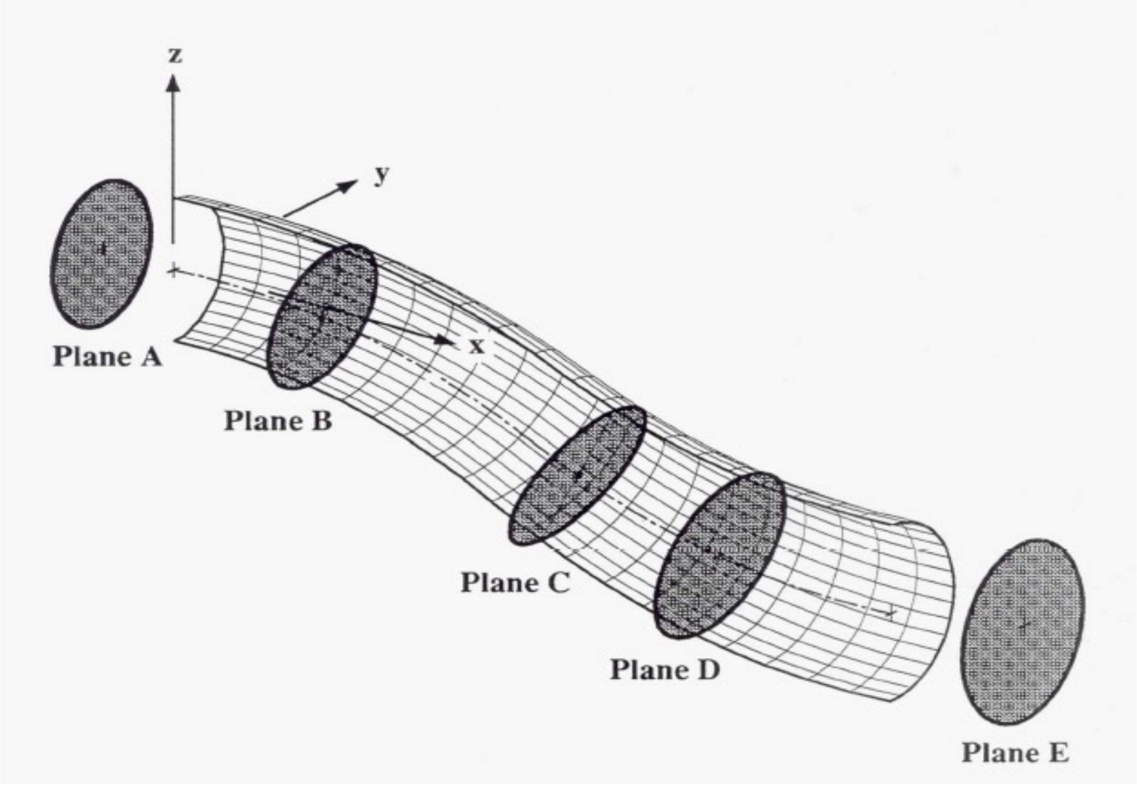
- M = 0.2, $Re_{MAC} = 5.49*10^6$, $\alpha = 19.47^o$

## Pressure Coefficient SA/WA-QCR



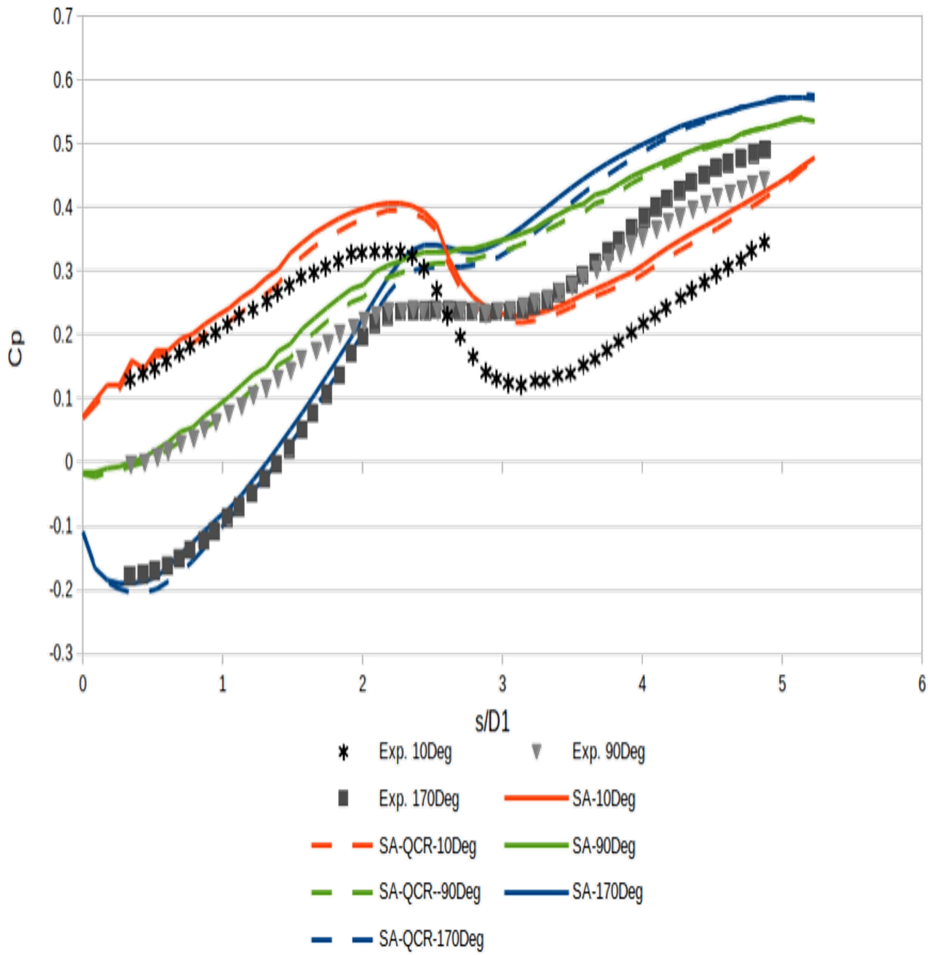
# NASA Glenn S-Duct

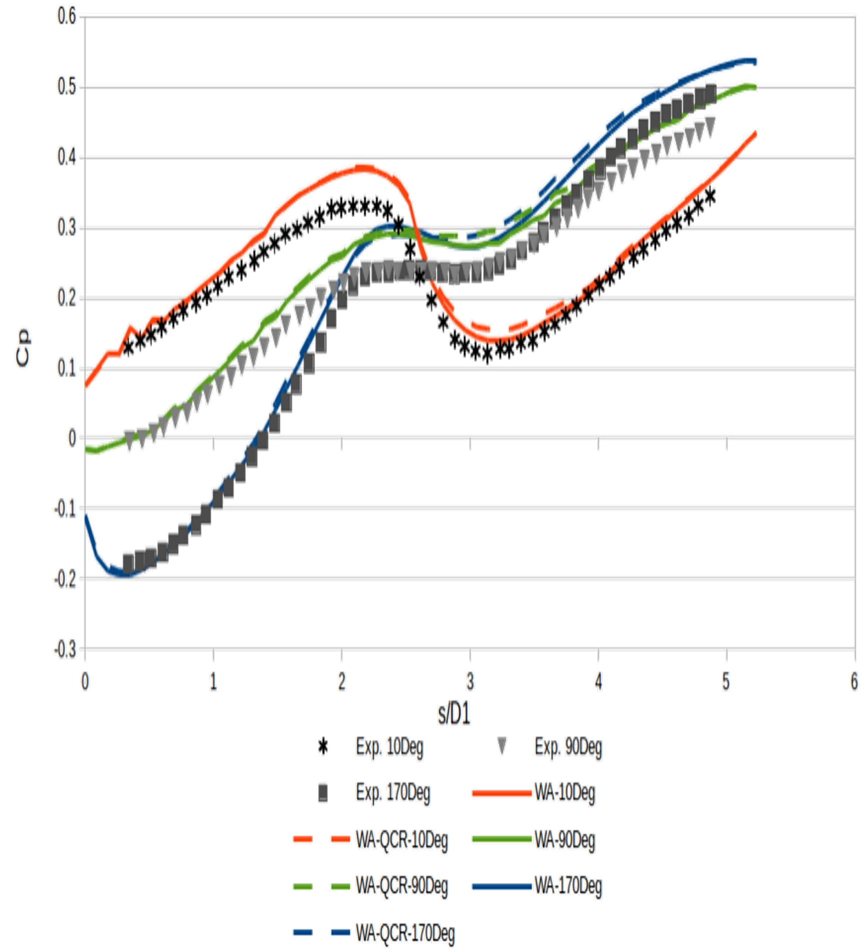
- M = 0.6, Re = 2,600,000 at $s/D_1$ = -0.5 (Plane A)
- The Aerodynamic Interface Plane (AIP), where the turbine face is located, is at $s/D_1$ = 5.73 (Plane E)



# NASA Glenn S-Duct
# SA-QCR and WA-QCR

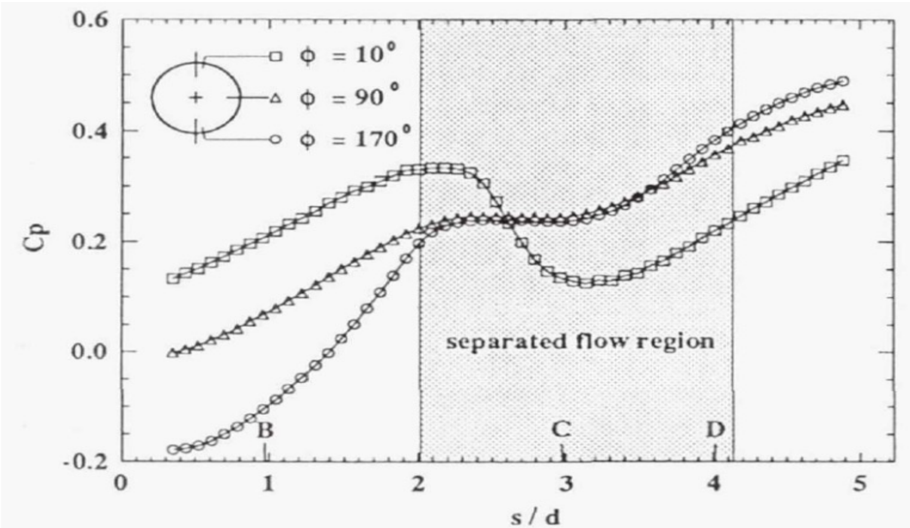
Washington University in St. Louis
School of Engineering & Applied Science

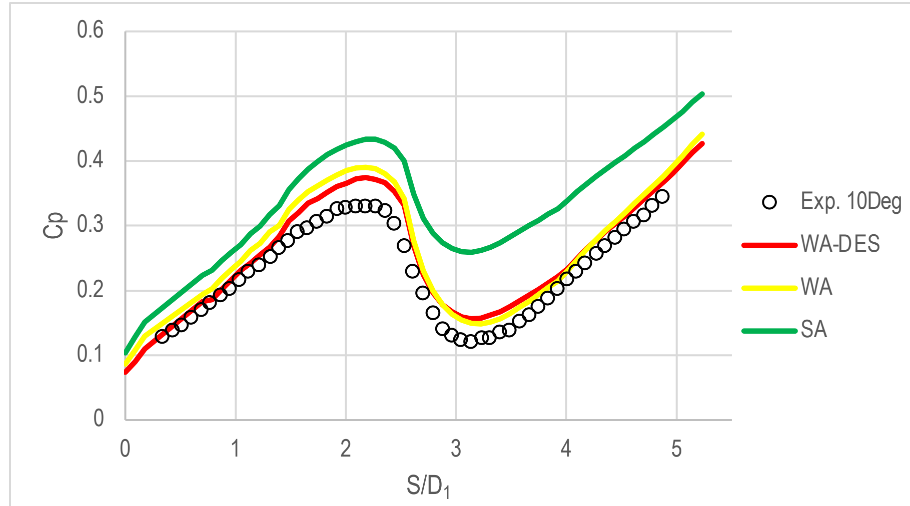


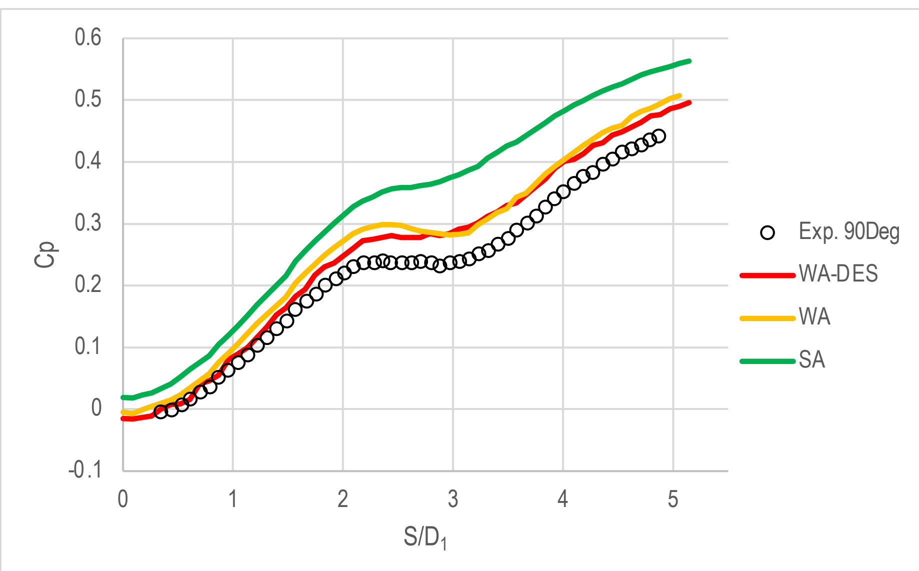



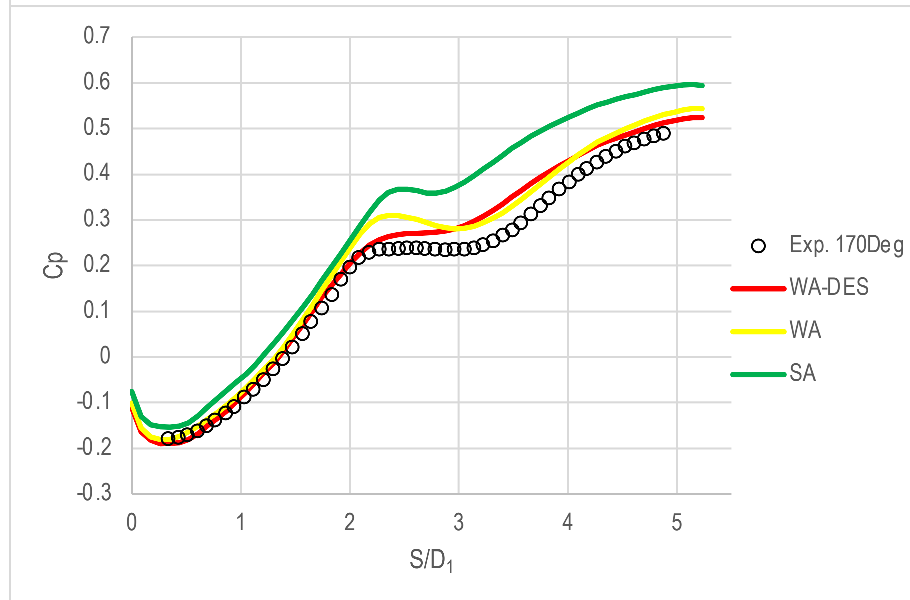
# NASA Glenn S-Duct
# SA, WA and WA-DES

Washington University in St.Louis
School of Engineering & Applied Science



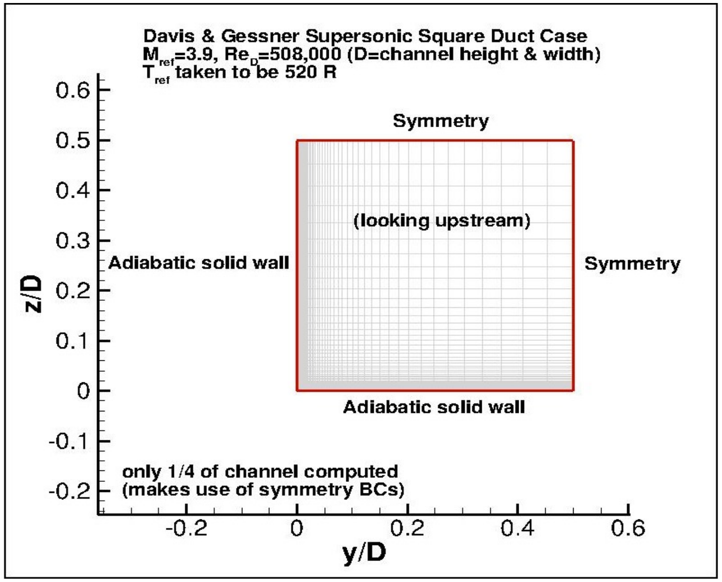



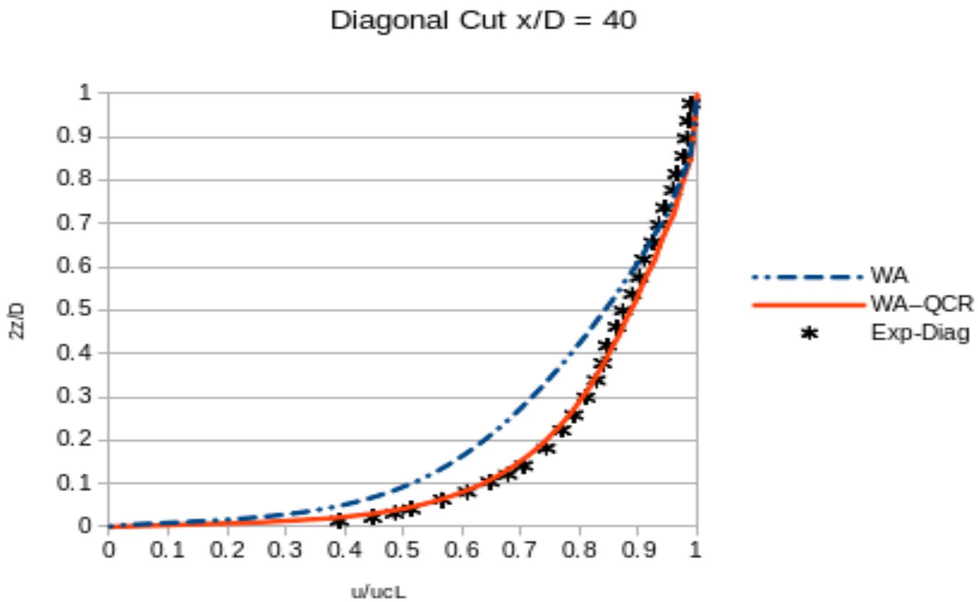



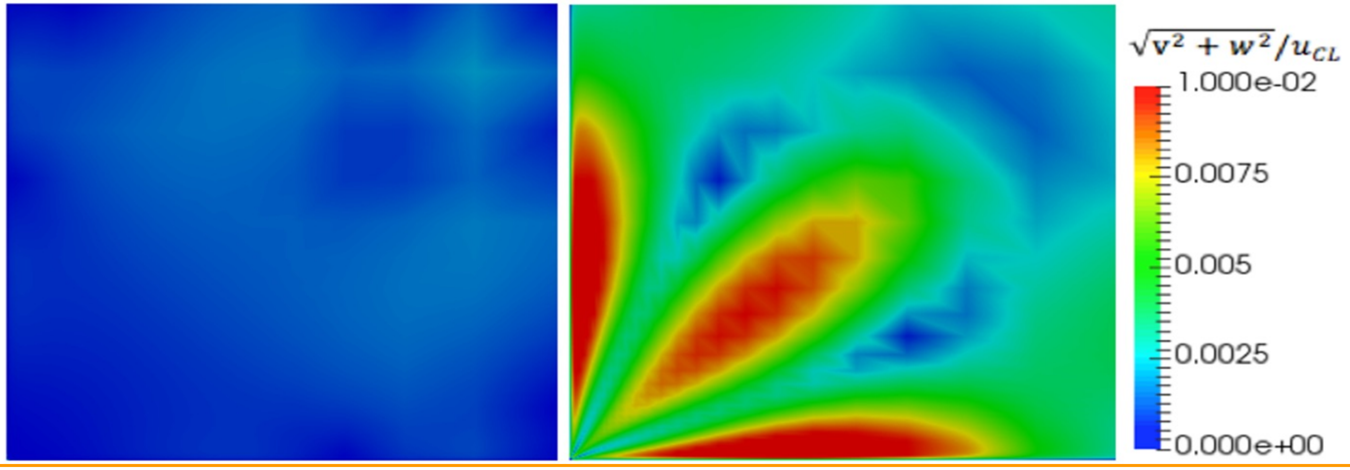
# 3D Supersonic Flow in a Square Duct
# WA and WA-QCR

Washington University in St.Louis
School of Engineering & Applied Science

Experiment of Davis and Gessner, M = 3.9, $Re_D$= 508,000, D = 25.4mm, x/D = 50



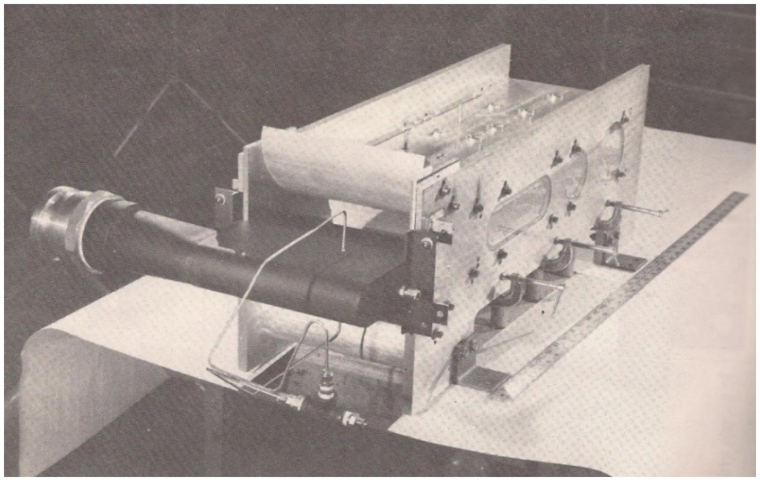



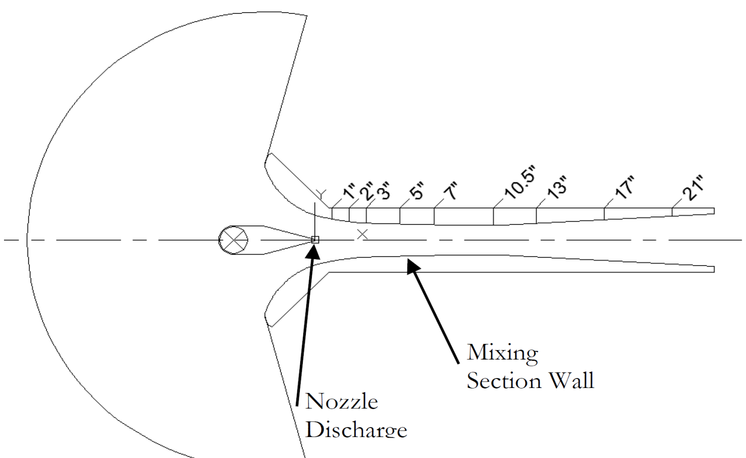



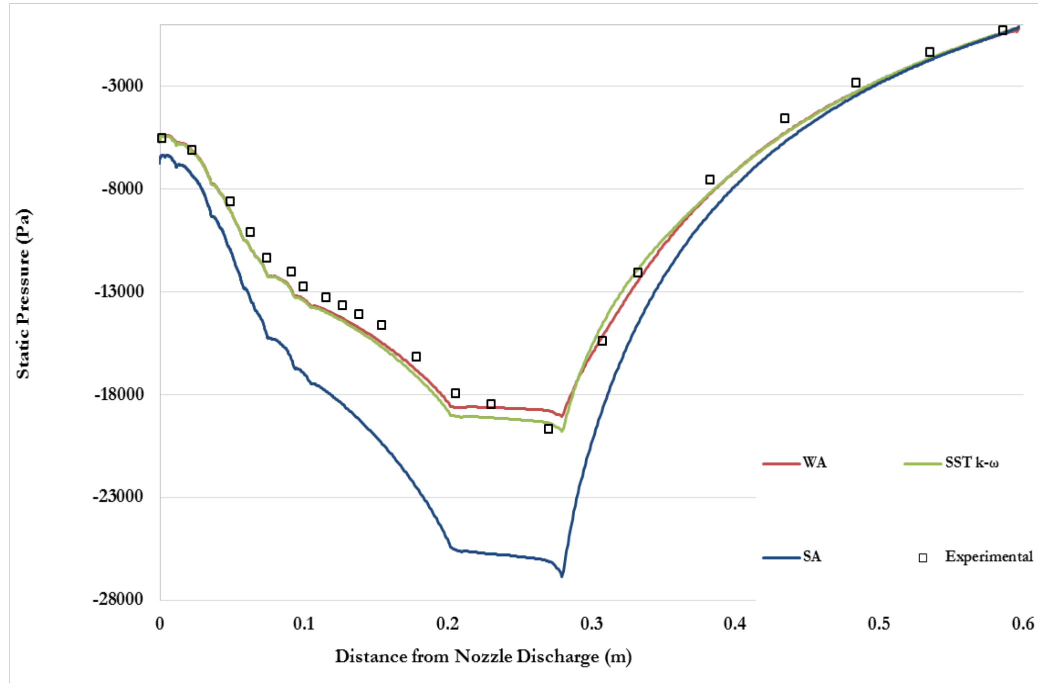
# 2D Slot Nozzle Ejector

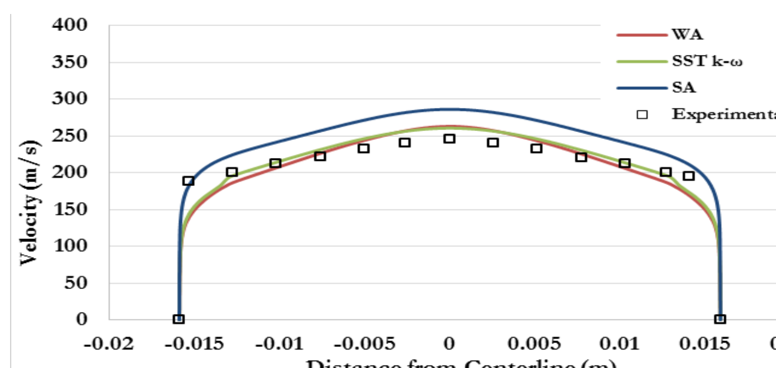
Washington University in St. Louis
School of Engineering & Applied Science

"Run5", $P_{nozzle}$ = 31.71 Psia, $T_{nozzle}$ = 648 R, Mixing Section Throat = 1.25", $\dot{m}_{nozzle}$ = 0.0787





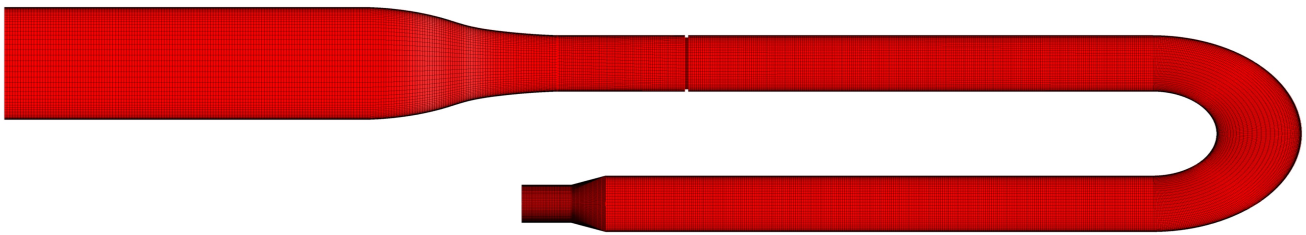



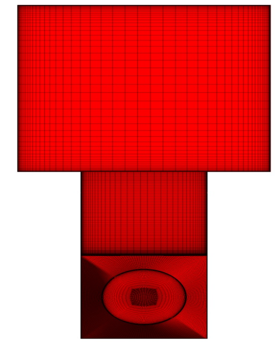
# RANS Simulations in a U-Bend

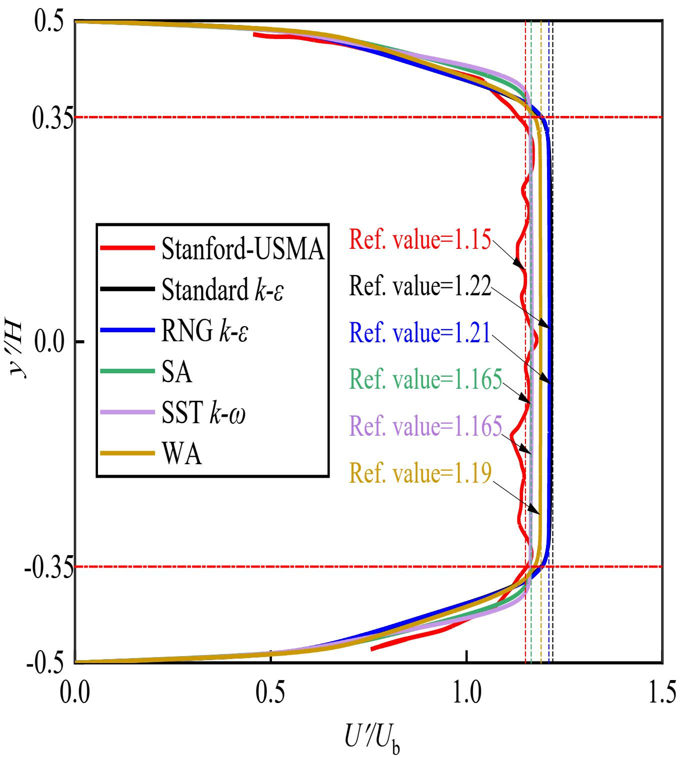
**Physics of Fluids, 2021, 33 (12) :125117**

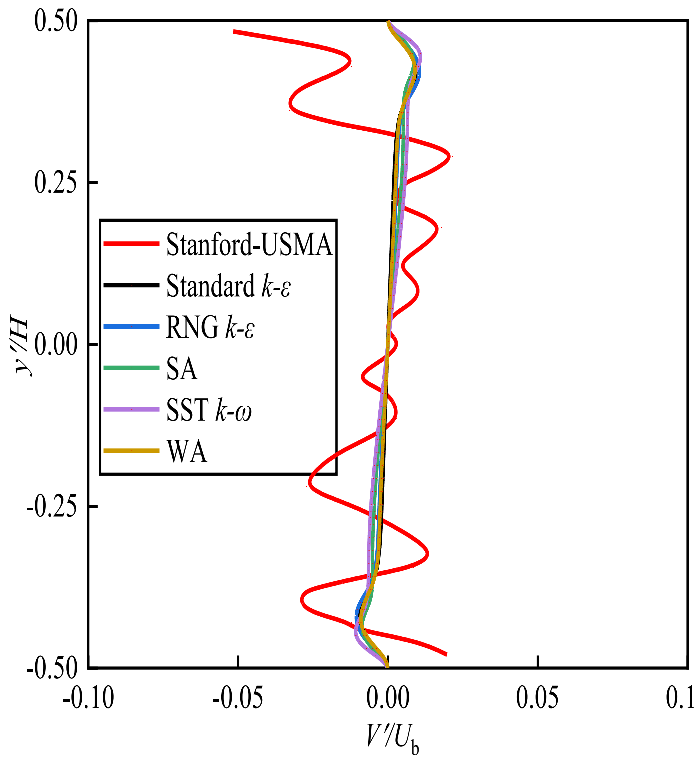
Washington University in St. Louis
School of Engineering & Applied Science

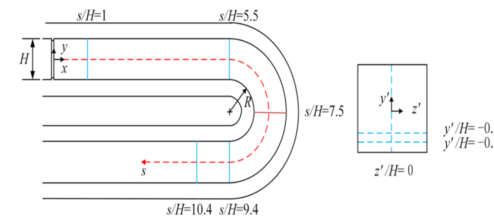
Velocity at inlet = 0.15m/s, Pressure at outlet = 1 atm.

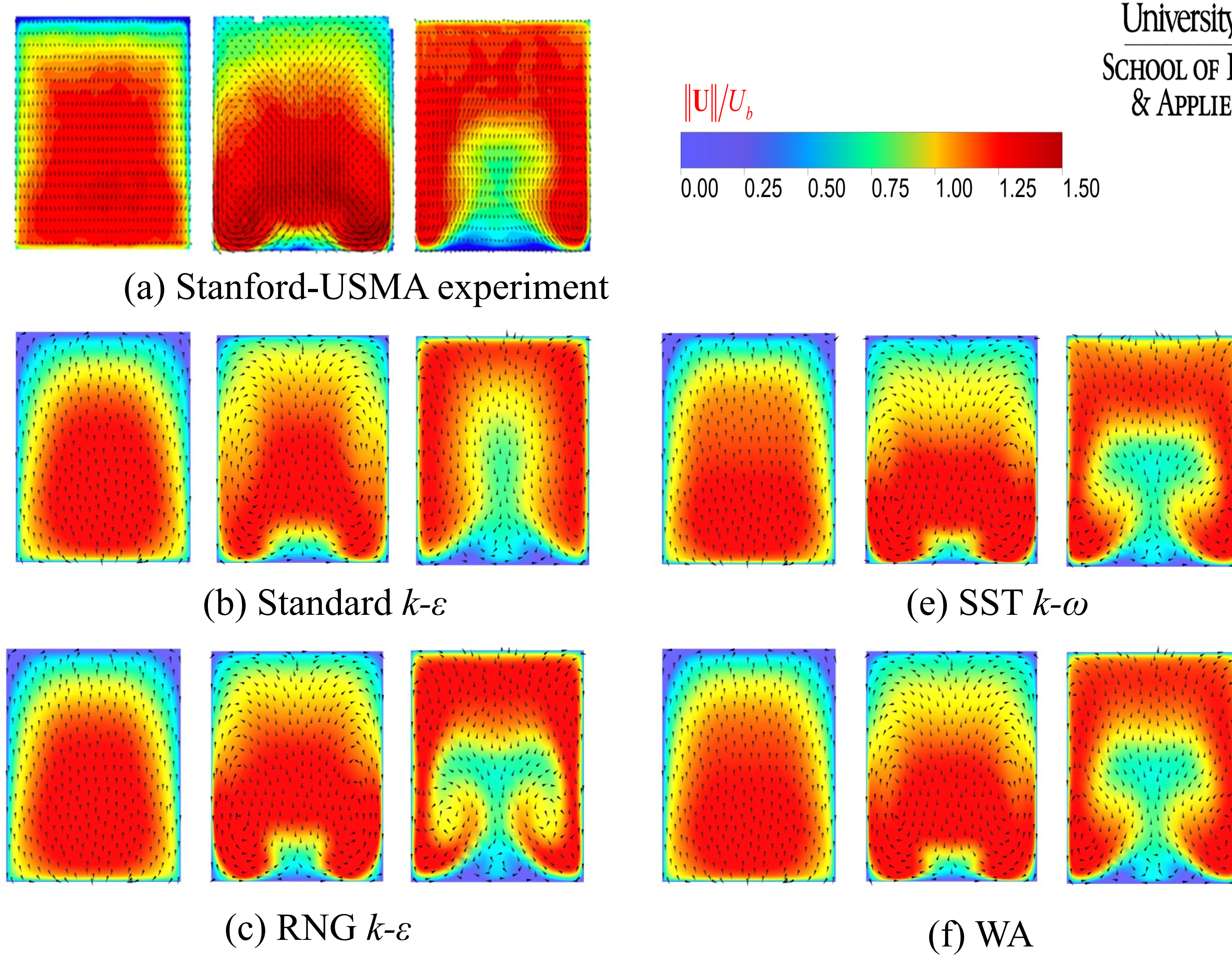









**Velocity contours and in-plane velocity vectors at *s*/*H* = 5.5 (left), *s*/*H* = 7.5 (middle), *s*/*H* = 9.4 (right)**

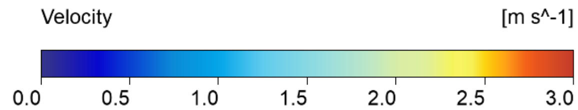
Washington University in St. Louis
School of Engineering & Applied Science

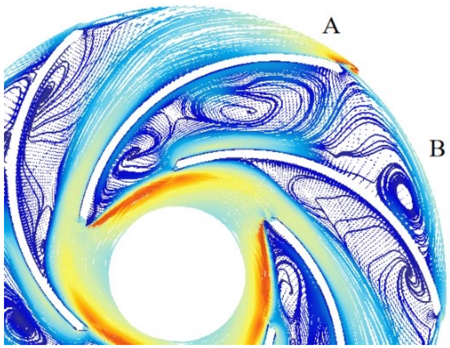
# Stall Prediction in a Centrifugal Pump

**Journal of Fluids Engineering, 2021, 143(3) : 031203**

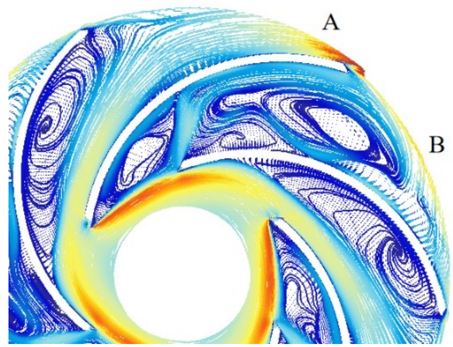
Washington University in St.Louis
School of Engineering & Applied Science

N = 725rpm, $Q = 0.2\ Q_{des}$

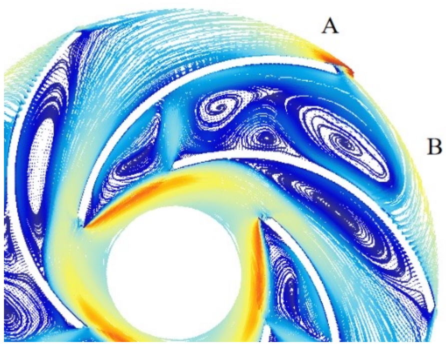


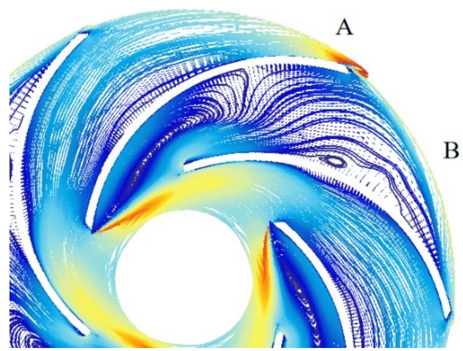

**Realizable *k-ε***



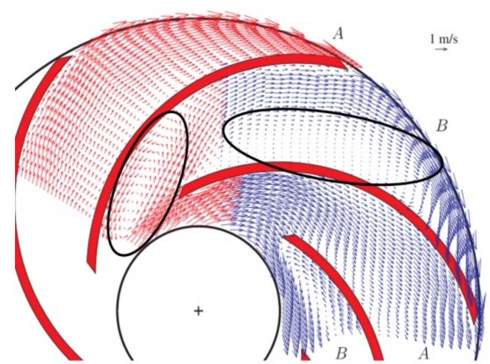

**RNG *k-ε***




**SST *k-ω***




**WA**



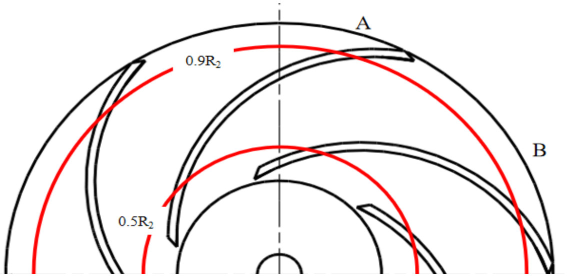

**PIV**

**Comparisons of the streamline distribution at impeller middle section**

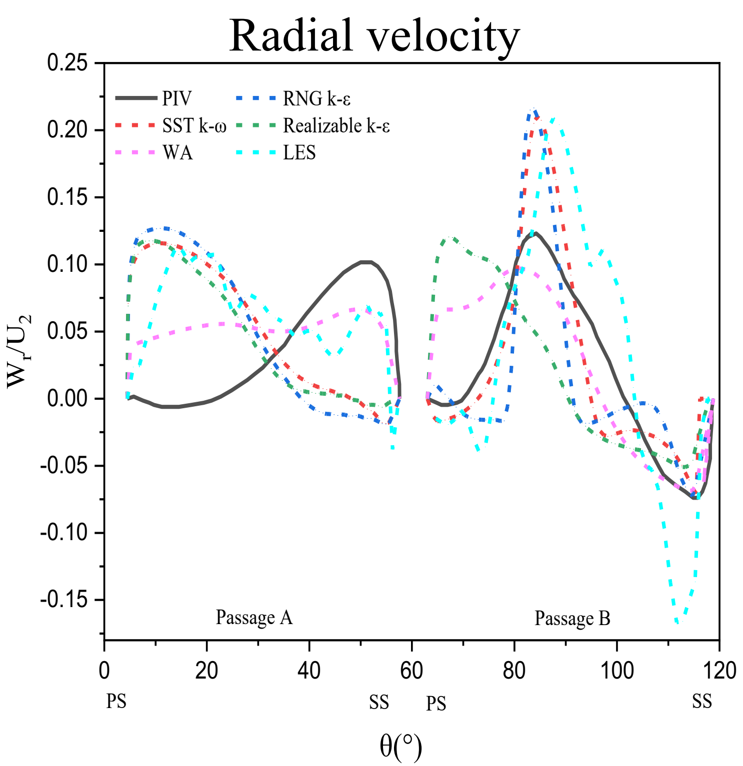
# Stall Prediction in a Centrifugal Pump

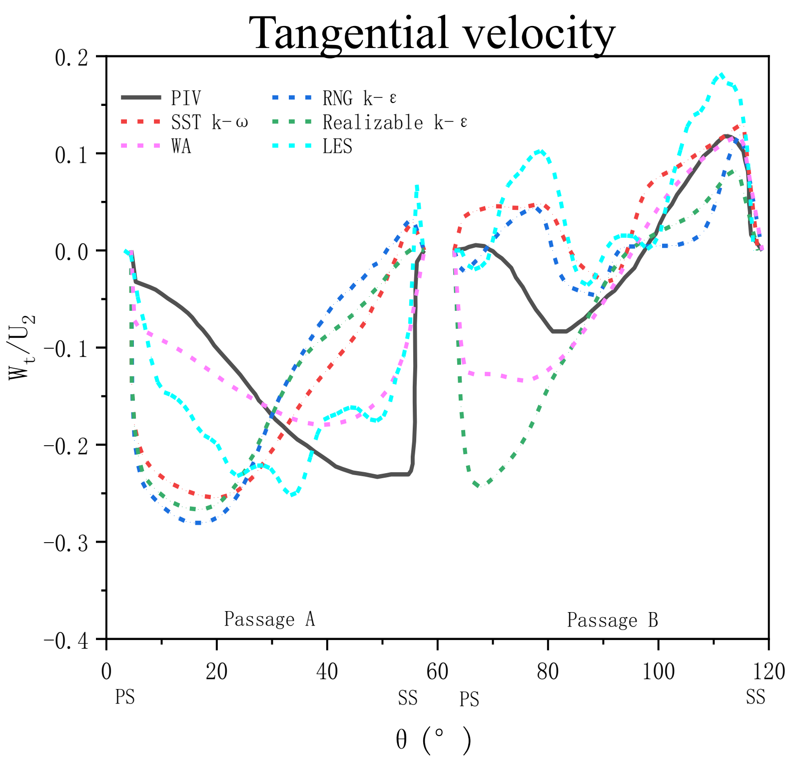
**Journal of Fluids Engineering. 2021, 143(3) : 031203**

Washington University in St.Louis
School of Engineering & Applied Science




## Comparisons of velocity distribution at $z/b_2$=0.5, $r/R_2$ = 0.5



# WA-γ Transition Models

Washington University in St. Louis
School of Engineering & Applied Science

- WA- γ (https://turbmodels.larc.nasa.gov/wa-gamma_transition_2eqn.html)

$$\frac{\partial \rho R}{\partial t} + \frac{\partial \rho u_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\mu + \sigma_R \mu_T)\frac{\partial R}{\partial x_j}\right] + \gamma\rho C_1 RS + \gamma\rho f_1 C_{2k\omega}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}\frac{R}{S} + P_R^{lim} - \max(\gamma, 0.1)(1 - f_1)\rho C_{2k\epsilon}\left(\frac{R\frac{\partial S}{\partial x_j}}{S}\right)^2$$

$$\frac{\partial \rho\gamma}{\partial t} + \frac{\partial \rho u_j \gamma}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_\gamma}\right)\frac{\partial \gamma}{\partial x_j}\right] + F_{length}\rho S\gamma(1-\gamma)F_{onset} - \rho c_{a2}\Omega\gamma F_{turb}(c_{e2}\gamma - 1)$$

- WA-AT (https://turbmodels.larc.nasa.gov/wa-at_transition_1eqn.html)

$$\frac{\partial R}{\partial t} + \frac{\partial u_j R}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\sigma_R R + \nu)\frac{\partial R}{\partial x_j}\right] + C_1 \boldsymbol{\gamma} RS + f_1 C_{2kw}\frac{R}{S}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j} - (1 - f_1) min\left[C_{2k\omega}R^2\left(\frac{\frac{\partial S}{\partial x_j}\frac{\partial S}{\partial x_j}}{S^2}\right), C_m\frac{\partial R}{\partial x_j}\frac{\partial R}{\partial x_j}\right]$$

$$\gamma = 1 - \exp\left(-\sqrt{Term_1} - \sqrt{Term_2}\right)$$

$$Term_1 = \frac{\max(1.2Re_\theta - Re_{\theta c}, 0.0)}{\chi_1 Re_{\theta c}},\ Term_2 = \max\left(\frac{\nu_t}{\nu}\chi_2, 0.0\right),\ \ \chi_1 = 0.02 \text{ and } \chi_2 = 50$$

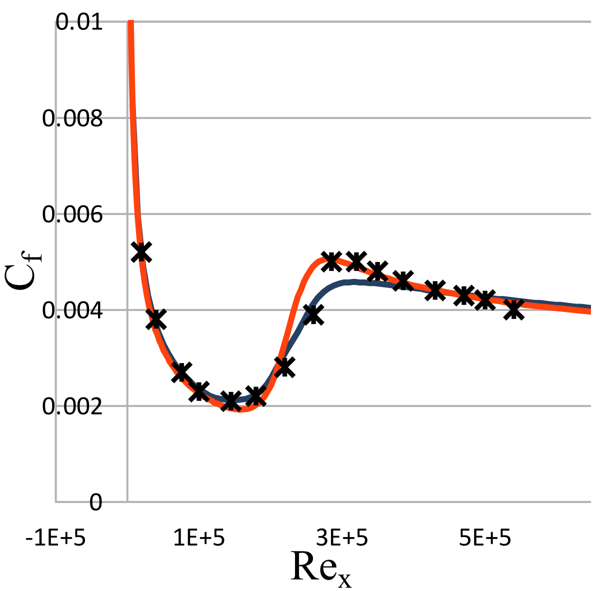
# WA-γ Transition Model

**Nagapetyan & Agarwal , AIAA 2018-3384**

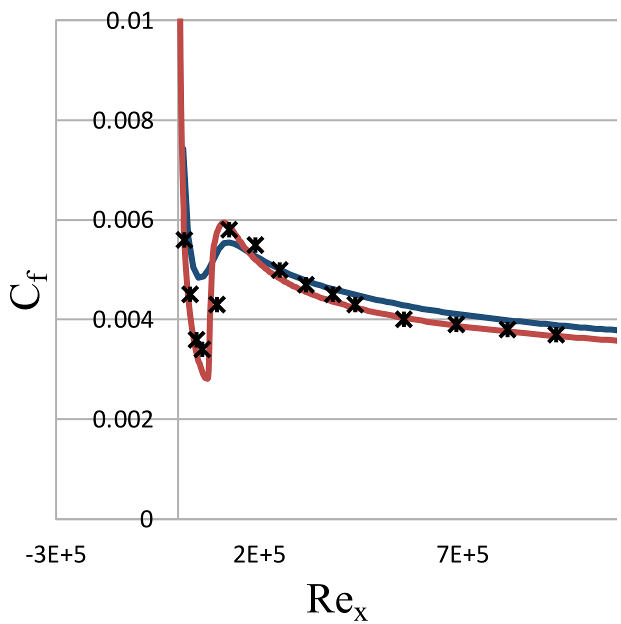


- Three zero pressure gradient flat plate cases : T3A, T3B, T3A-

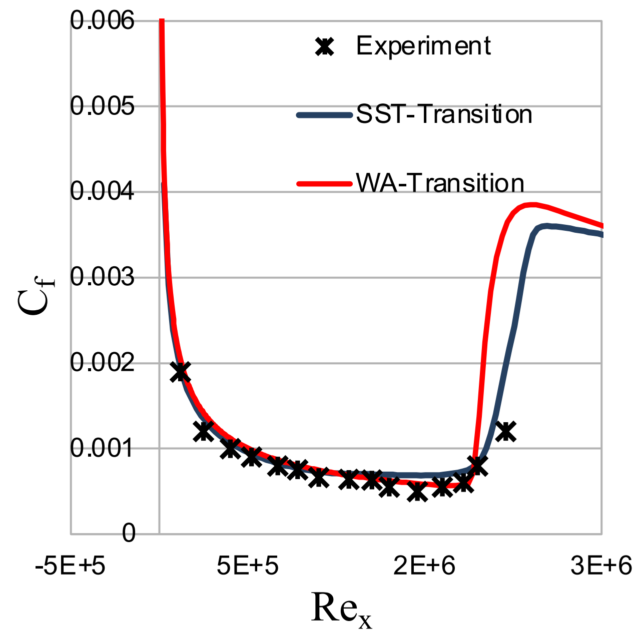
| | $U_\infty$ (m/s) | $Tu_\infty$(%) | $\mu_T/\mu$ | ρ (kg/m³) | μ (kg/ms) | Re |
|---|---|---|---|---|---|---|
| T3A | 5.4 | 3.5 | 13.3 | 1.2 | 1.8e-5 | 9e+5 |
| T3B | 9.4 | 6.5 | 100 | 1.2 | 1.8e-5 | 1.57e+6 |
| T3A- | 19.8 | 0.874 | 8.72 | 1.2 | 1.8e-5 | 3.3e+6 |




**T3A**




**T3B**

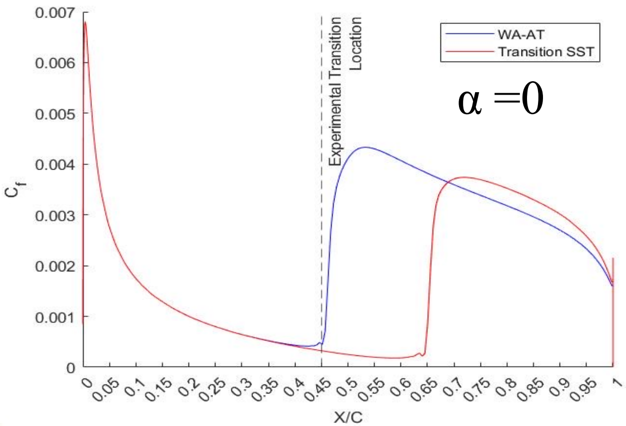


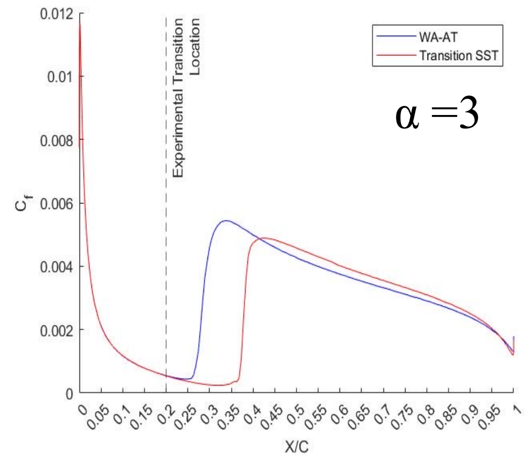

**T3A-**

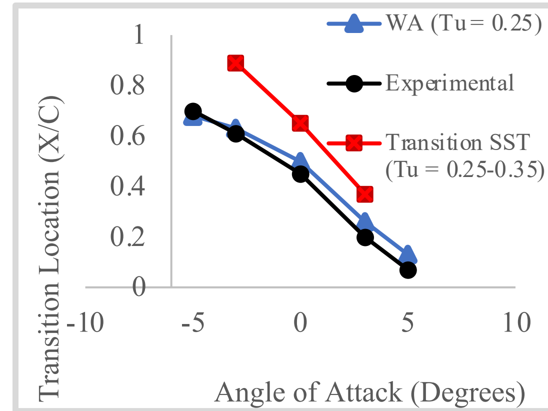
# WA-AT Transition Model

## Xue & Agarwal, AIAA 2021-2712

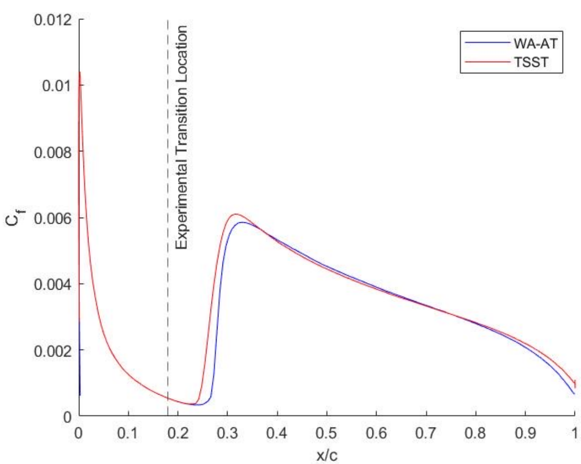
- NACA 0012, Re = $3*10^6$, Exp: Gregory & O'Reilly (1973), AIAA 2022-3411

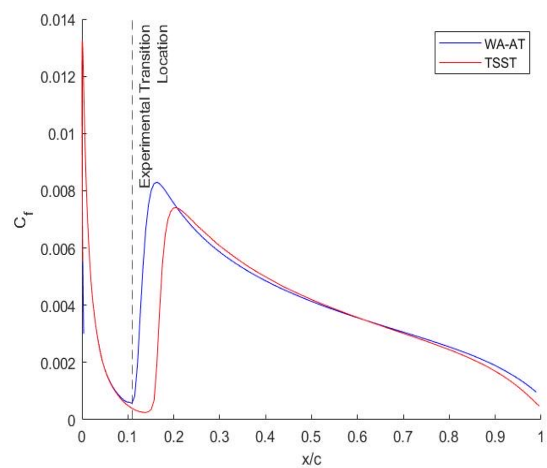


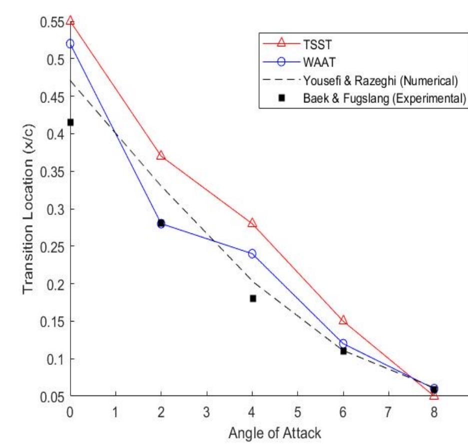




- NACA 0015, Re = $3*10^6$, Tu = 0.098%, $\mu_t/\mu$ = 10, Exp: Baek & Fuslang



# Summary

Washington University in St. Louis
School of Engineering & Applied Science

- A new one-equation turbulence model has been developed to have desirable characteristics of one-equation k-ω and one equation k-ε models.
- The new one-equation WA model has been used to simulate a number of wide-ranging canonical turbulent flow cases from NASA TMR and NPARC Alliance.
- The behavior of the WA model is very similar to the two-equation SST k-ω model.
- A clear advantage of the WA model's predictive capability over the SA model has been shown for a number of cases from subsonic to transonic to hypersonic wall bounded flows with small regions of separation and subsonic/supersonic free shear layer flows.
- Spalart-Shur R/C correction has been implemented and verified for WA model.
- Surface roughness corrections have been implemented and verified for WA model.
- Wall-Distance-Free WA model has been formulated and tested.
- Elliptic Blending has been included which showed improved predictions in few cases tested.
- The DES and IDDES versions of WA model have been developed which show improvement in accuracy over the WA model.
- The model has been extended to transitional flows (WA-γ and WA-AT).

Washington University in St. Louis
School of Engineering & Applied Science

# Acknowledgements


- Some of this research was partially supported by NASA EPSCoR Program.
- PI is grateful to Dr. Mujeeb Malik for his support and help.
- PI is also grateful to Dr. Chris Rumsey for including the WA , WA-γ and WA-AT models in NASA TMR website, pointing out several typos in the written version of the model and helping Aaron Erb of MS&T in putting the model in FUN3D.
- The presentation is based on the work of many graduate students: Tim Wray, Xu Han, Hakop Nagapetyan, Xiao Zhang, Bryce Thomas, Yan Xue, Karsten Hendrickson, Dean Ryan-Simmons, Colin Graham and Isaac Witte among others.
- The research has been presented at AIAA and ASME conferences.
- The conference papers and journal papers are available.
- Code modules for OpenFOAM and Fluent UDFs are available upon request.