The Anchor Points of Turbulence Modeling

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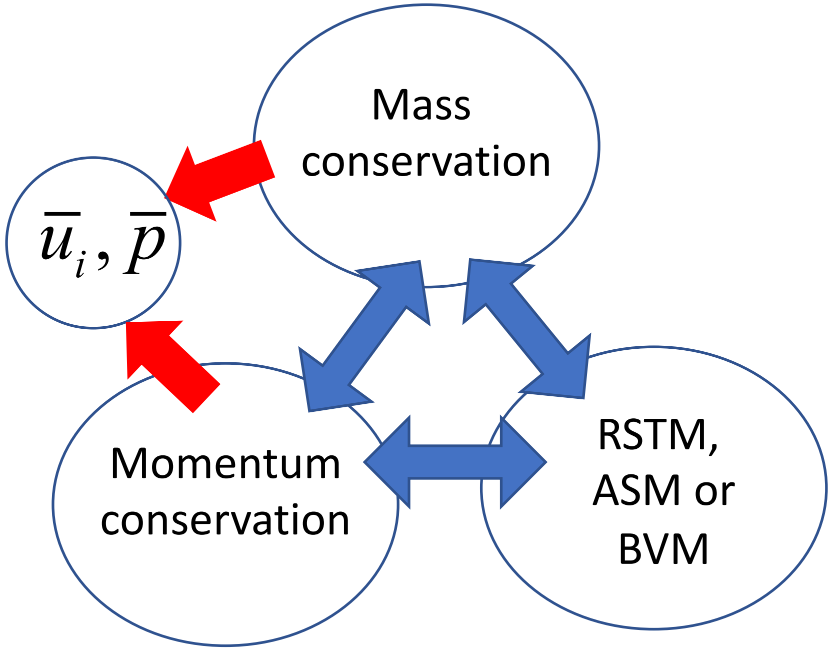
Closure Problem

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\rho \frac{D\bar{u}_j}{Dt} = -\frac{\partial \bar{p}}{\partial x_j} + \mu \frac{\partial^2 \bar{u}_j}{\partial x_i^2} + \frac{\partial \tau_{ij}^t}{\partial x_i}$$

where $\tau_{ij}^t = -\rho \overline{u_i' u_j'}$

$$= -\rho \begin{bmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{bmatrix}$$



RSTM

$$\rho \frac{D\overline{u_i' u_j'}}{Dt} = -\left(\rho \overline{u_i' u_k'} \frac{\partial \bar{u}_j}{\partial x_k} + \rho \overline{u_j' u_k'} \frac{\partial \bar{u}_i}{\partial x_k} \right) - 2\mu \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}} - \overline{\left(u_i' \frac{\partial p'}{\partial x_j} + u_j' \frac{\partial p'}{\partial x_i} \right)} - \frac{\partial}{\partial x_k} \left(\rho \overline{u_i' u_j' u_k'} - \mu \frac{\partial \overline{u_i' u_j'}}{\partial x_k} \right)$$

ASM

$$\rho \frac{D\overline{u_i' u_j'}}{Dt} = \rho \frac{\overline{u_i' u_j'}}{k} \frac{Dk}{Dt}$$

BVM

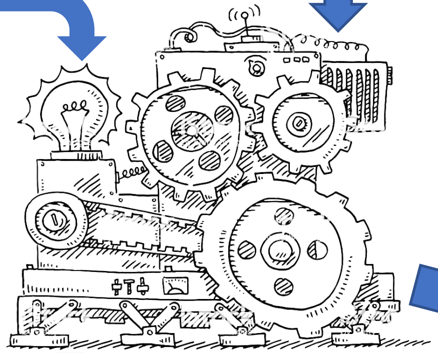
$$\tau_{ij}^t = -\rho \overline{u_i' u_j'} = \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{2}{3} \mu_t \delta_{ij} \frac{\partial \bar{u}_k}{\partial x_k} - \frac{2}{3} \rho \delta_{ij} k$$

Single-journey ML turbulence modeling

Invariant
Feature variables
S_{ij}, W_{ij}

Initiate with Solution of conventional turbulence model

Training cases

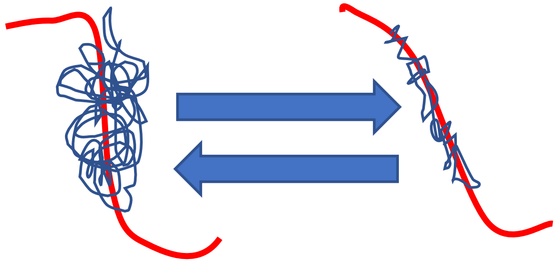


$-\overline{u_i'u_j'}$ or μ_t

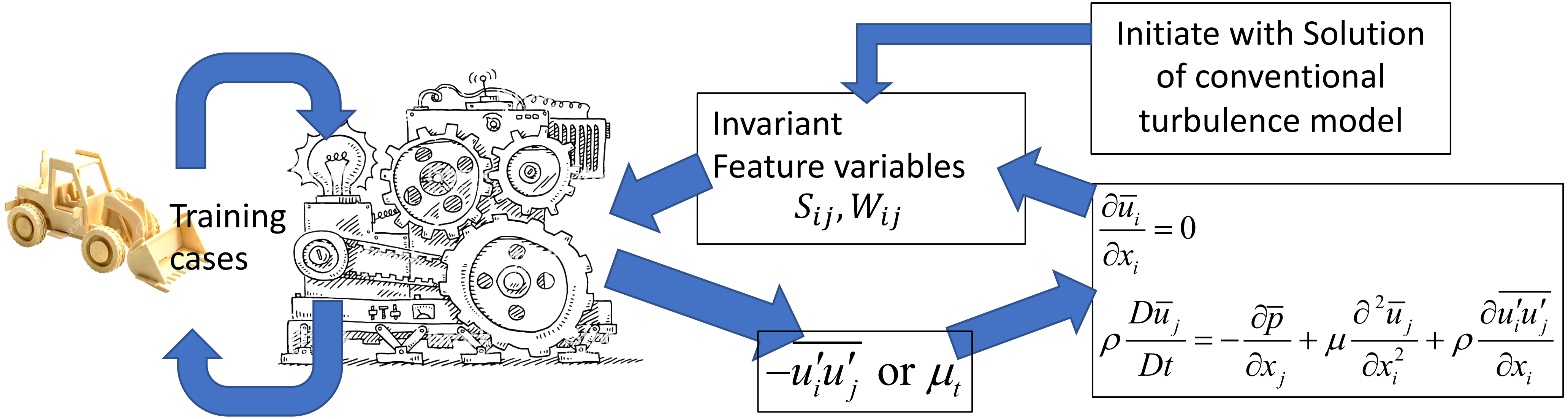
$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\rho \frac{D\bar{u}_j}{Dt} = -\frac{\partial \bar{p}}{\partial x_j} + \mu \frac{\partial^2 \bar{u}_j}{\partial x_i^2} + \rho \frac{\partial \overline{u_i'u_j'}}{\partial x_i}$$

- ❑ Solution will depend on initial solution.
- ❑ Lack of mean-flow feedback mechanism.
- ❑ May have physically unrealistic anti-diffusion.
- ❑ Numerically unstable.



Two way coupling of ML turbulence modeling

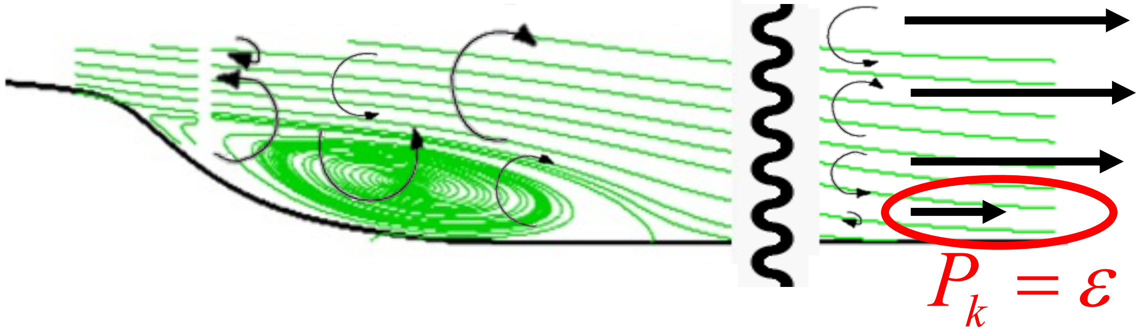


ML Machine may have several variations

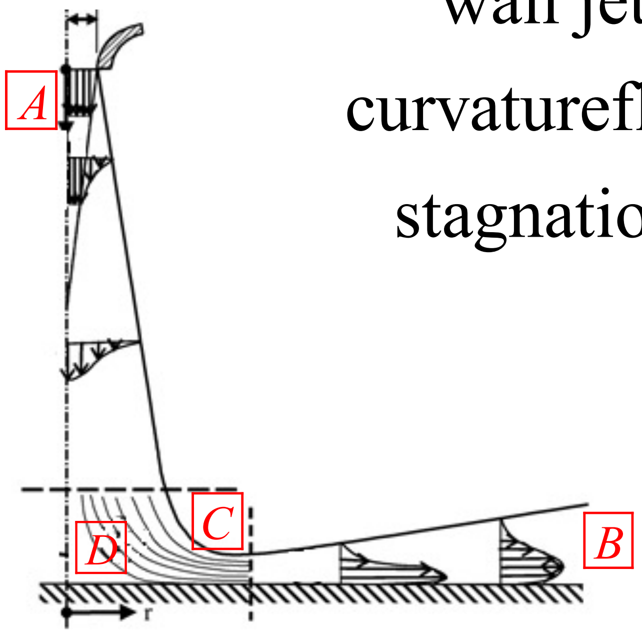
- ❑ Algebraic equations
  - Pope's Caley-Hamilton equations
  - Algebraic stress model equations
  - viscosity equation, ν_t *
  - $b = \nu_t S + \Delta \tau^{\perp}$
- ❑ Differential equations
  - One-equation model equations*
  - Two-equation model equations*
  - Reynolds stress model equations

* with Boussinesq eddy viscosity hypothesis

- ❑ Generalization of physical phenomena in different flow regimes may not be so obvious.



Benchmark building-block flows



free jet (A)
wall jet (B)
curvatureflow (C)
stagnation (D)

= Impinging jet

- Successes in predicting benchmark flows are necessary conditions, not sufficient conditions, for model generalization.
- The model (or machine) must know to obey asymptotic behaviors.

My personal observations

- ML is a good compromise of several training data (flows).
- ML may miss details of (unmonitored) flow physics.
- Galilean invariance is nice but:
  - Features invariance but *f:* may not
  - How to identify similarity-scaling invariance, for example?
- The accuracy of ML approach depends on the choice of the training data. ML models may be
  - Data dependent
  - Geometry dependent
  - Case dependent
- Extension of the ML models to high Re number is questionable.
  - Are the training data extrapolatable?
- Current ML approach is not flow-physics based.
- Current ML models do not reply on building-block (benchmark) experiments.

History of TMR

- 1968 – AFOSR-IFP Stanford Conference
- 1969 – Compressible TBL, NASA SP-216
- 1972 – Free shear flows, NASA SP-321
- 1980-81 – AFOSR HTTM-Stanford Conference of complex turbulent flows: Comparison of Computation and experiments
- 1996 – Bradshaw, Launder and Lumley's Olympics
- 2009 – Starting of Turbulence Modeling Resources Website

Good Benchmark flows

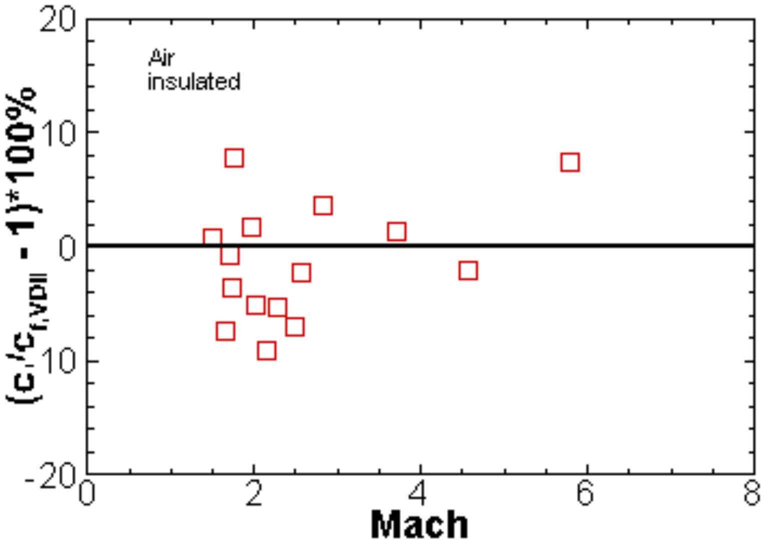
- With distinct physical feature(s)
- Cannot be a one-point data
- BC's well documented
- Insensitive to Re number
- High resilience to changes
  - Why it is important to look for resilient expression.

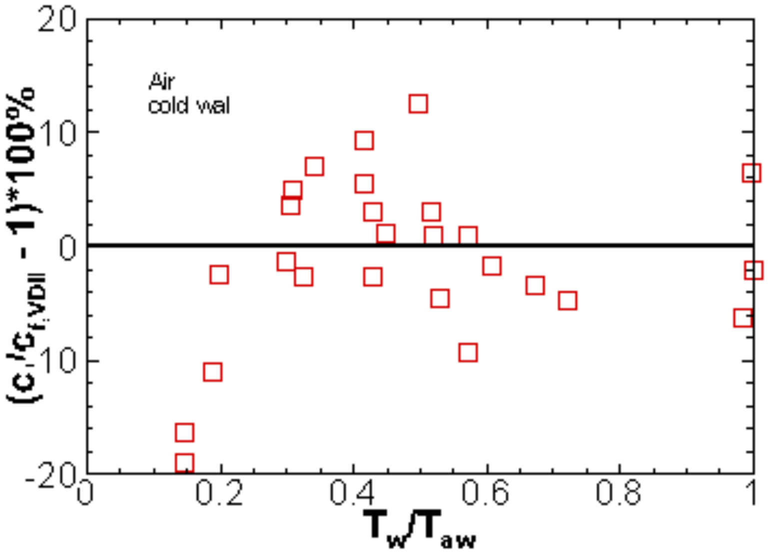
Cases Listing by Flow Physics

		Free shear flows			Wall flows		P-gradients	Curvature	Compressibility			Secondary flows	Turb Heat Flux	Higher Mach	Vortex flows	Shock	Separation
		Jet Anomaly	Mixing layer	Wakes	Law of wall	Law of wake			Mixing	Van Driest I	Van Driest II						
Boundary layers	2DZP*				Y	Y											
	2DZPH									Y	Y		Y	Y			
	ASBL*				Y		weak										weak
Mixing layer/ wakes	2DML*		Y														
	2DANW*			Y													
Jets	ASJ*	Y															
	ANSJ*	Y							Y					Y			
	AHSJ*	Y											Y				
	ACSSJ*	Y							Y					Y			
	AHSSJ*	Y							Y				Y	Y			
Airfoils	2DN00*						Y										weak
	2DN44						Y										Y
Bump flows	ATB*						Y							Y		Y	Y
	2DWMH						Y										Y
Shock/boundary layer interaction flows	ASWBLI						Y						Y	Y		Y	Y
Internal flows	2DCC						Y	Y									
	2DBFS						strong										Y
	3DSSD						Y					Y	Y	Y			

ZPG flat plate boundary layer

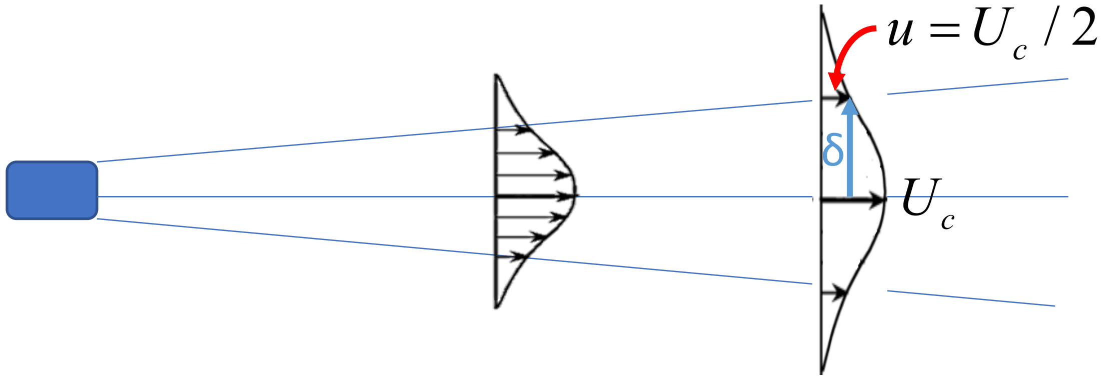
- Incompressible flows
  - c_f vs. Re_θ
  - u^+ vs. y^+
- Compressible flows
  - c_f, VD-II
  - Velocity and temperature transformation, VD-I, TL…
  - Pr_t
  - Use local properties

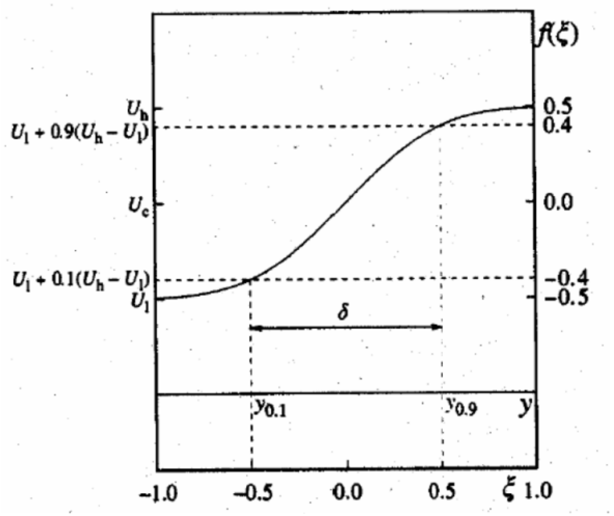




Free Shear Flows
(round/plane jet anomaly)

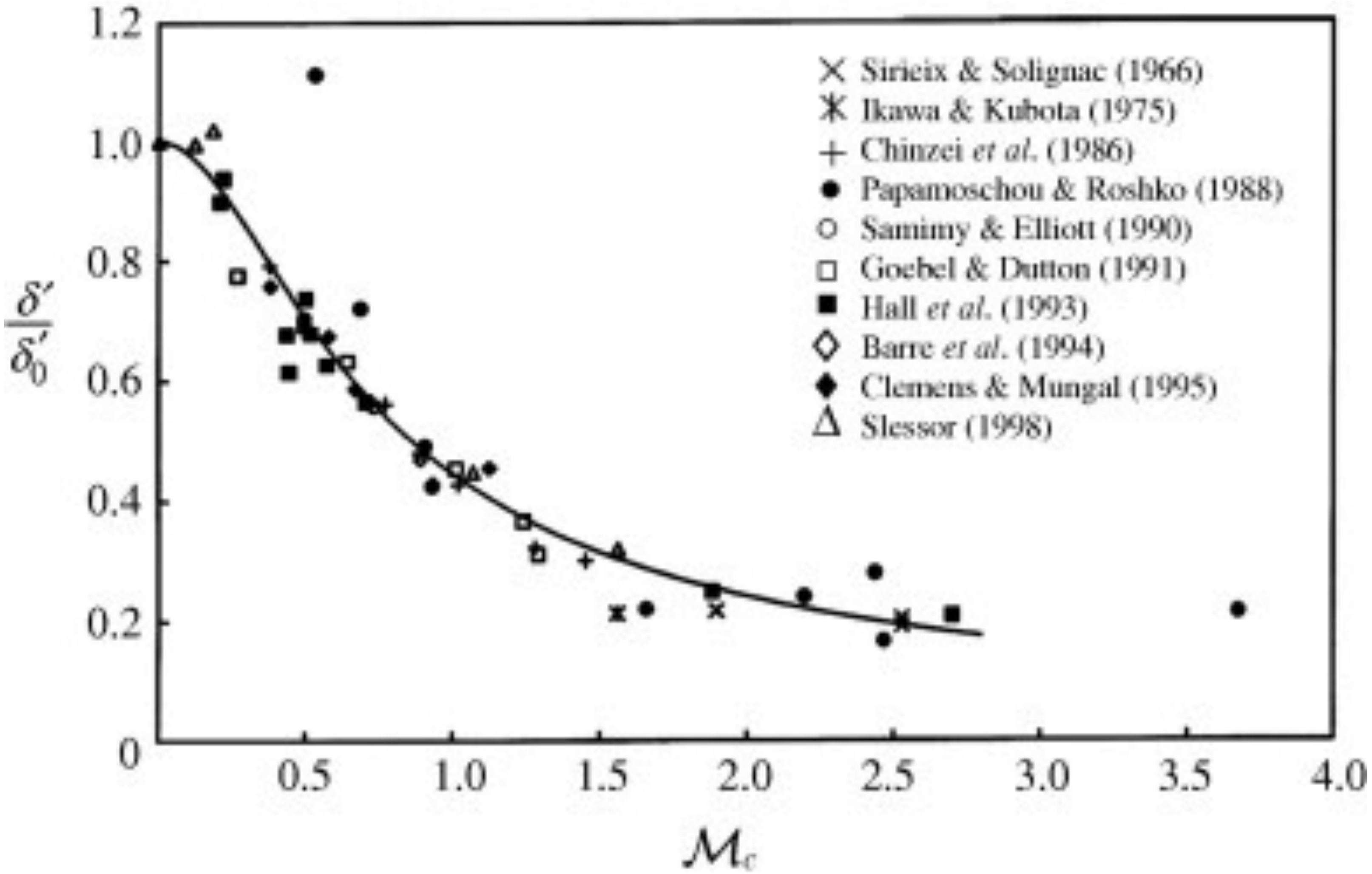
Flow types	Spread parameter	Experimental data
Round jet	$dr_{1/2}/dx$	0.086-0.095
Plane Jet	$dy_{1/2}/dx$	0.100-0.110
Mixing layer	$d(y_{0.9}-y_{0.1})/dx$	0.115





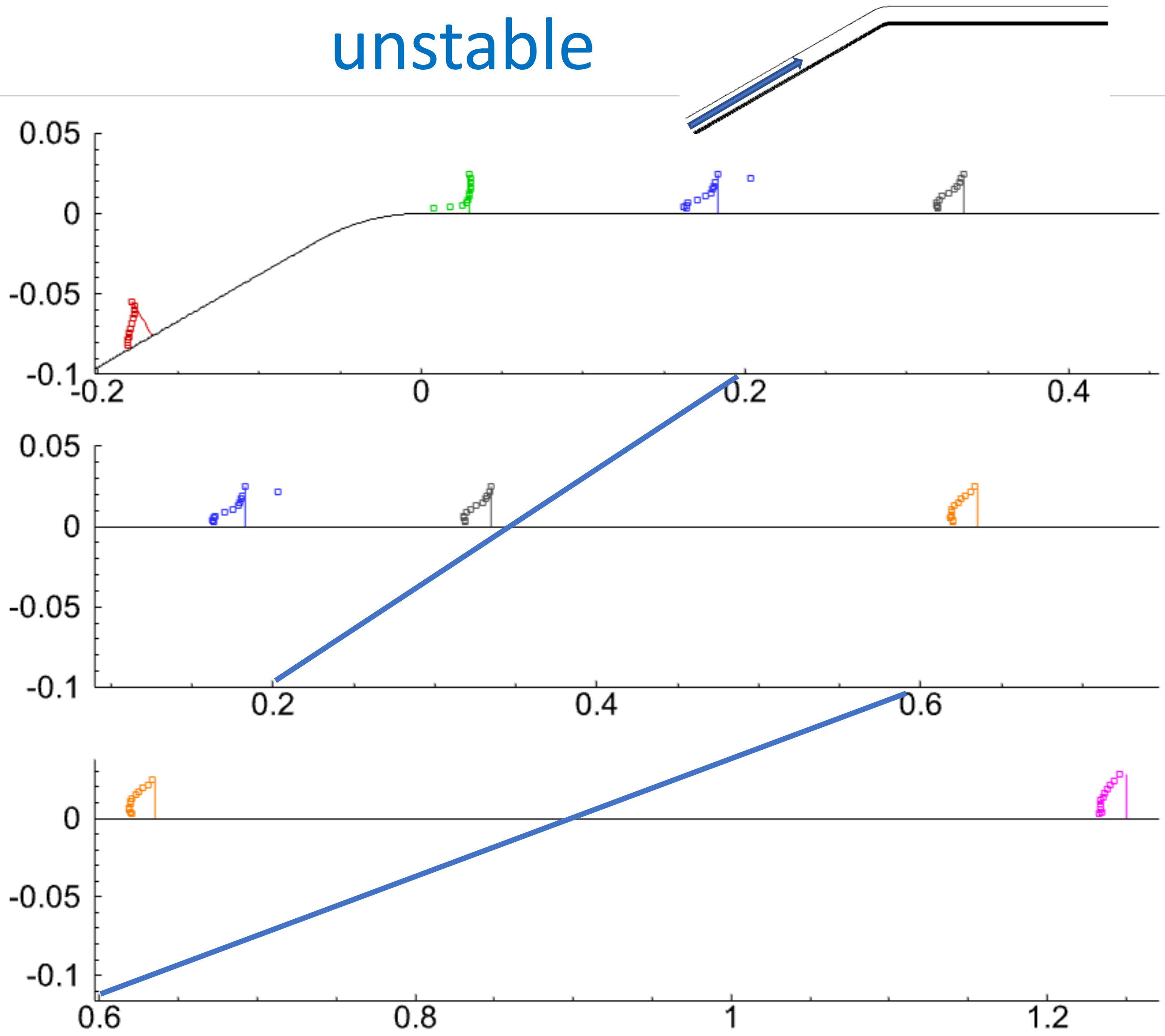
Mach number effects on mixing layer

compressibility effects

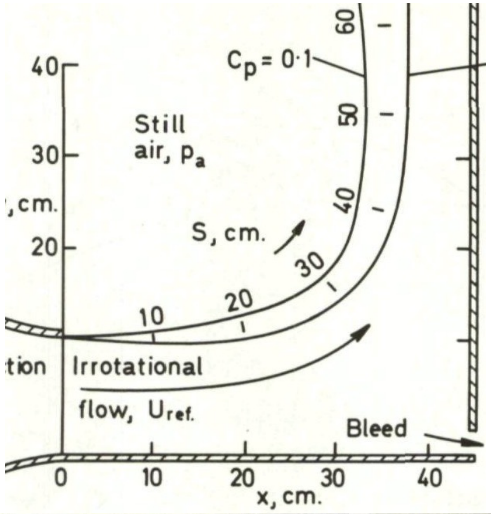


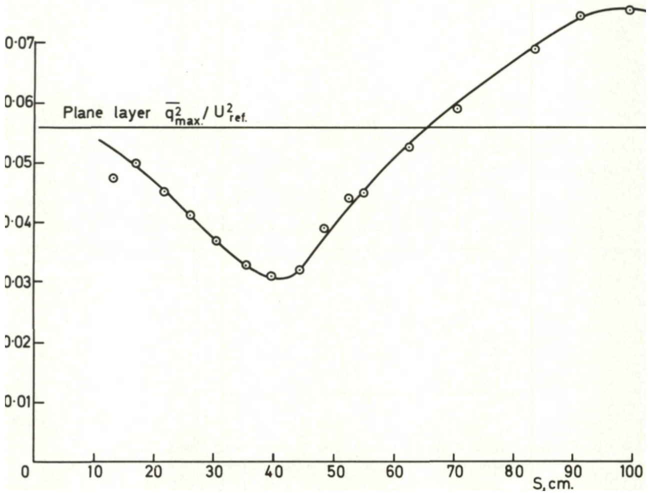
Curvature effects

unstable

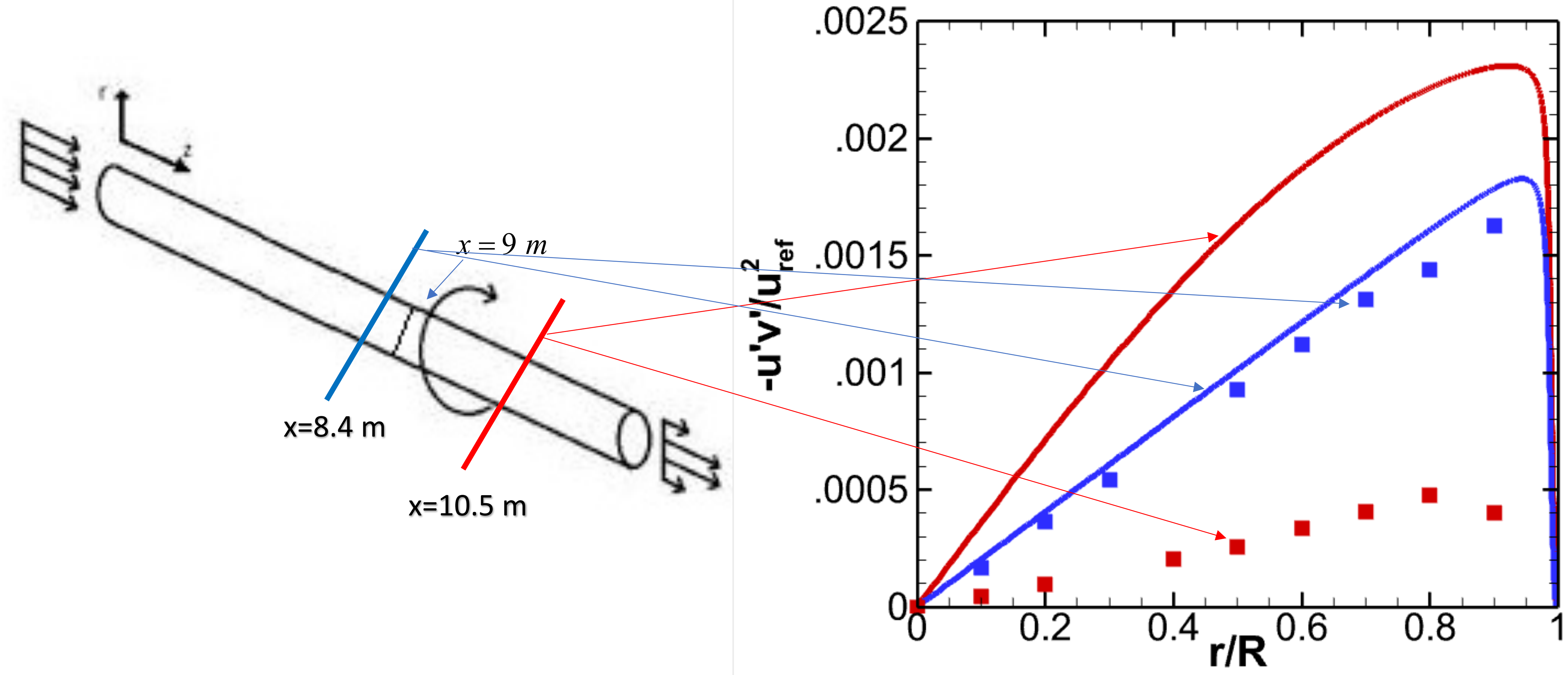


stable



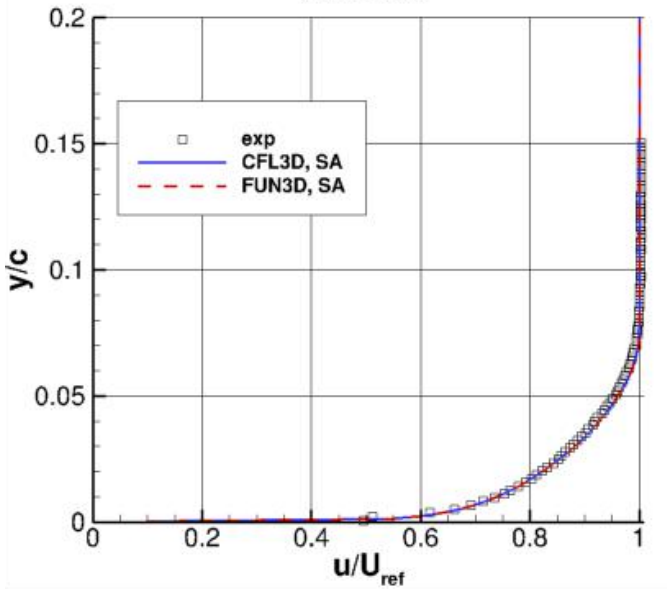


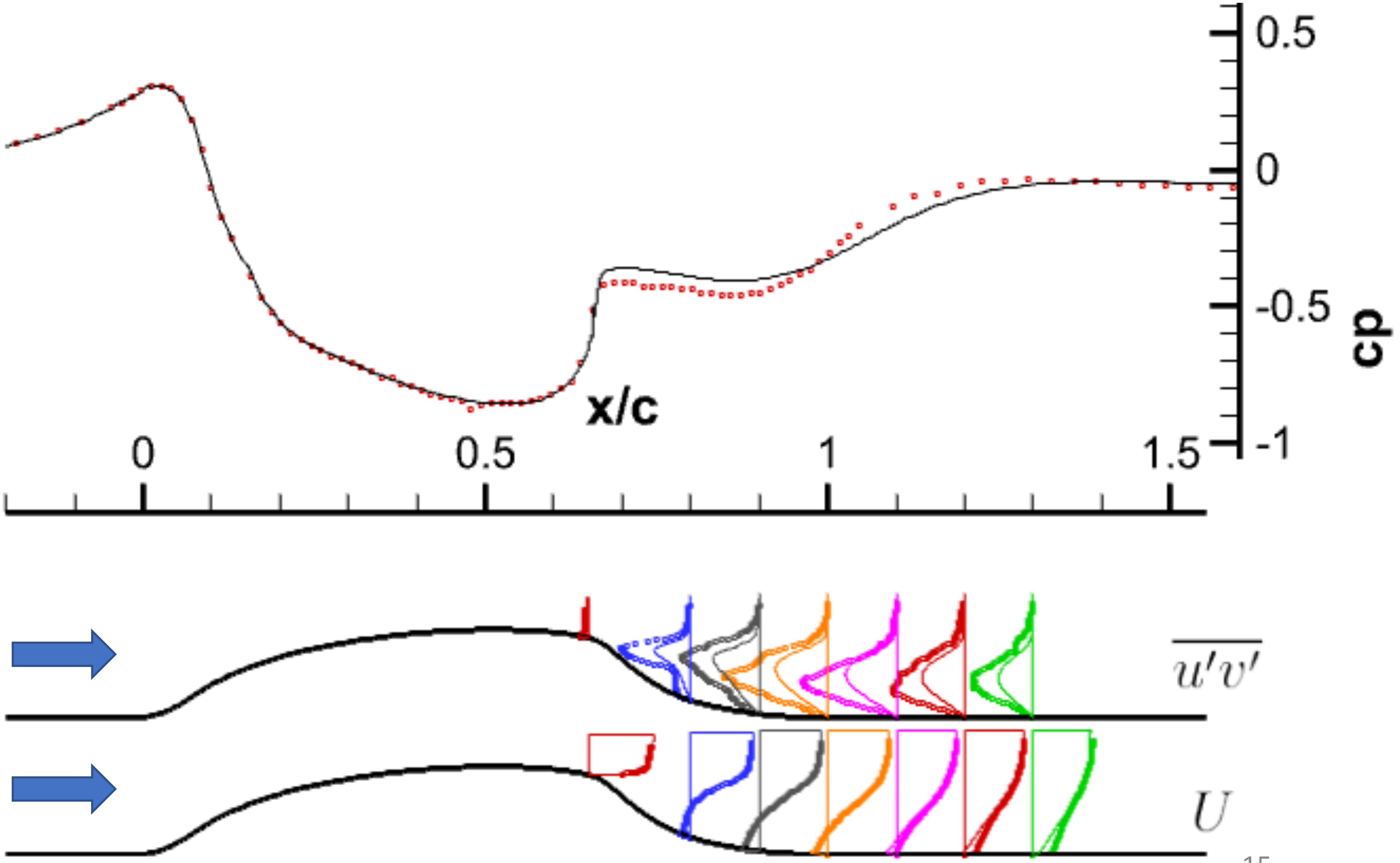
Vortex flows



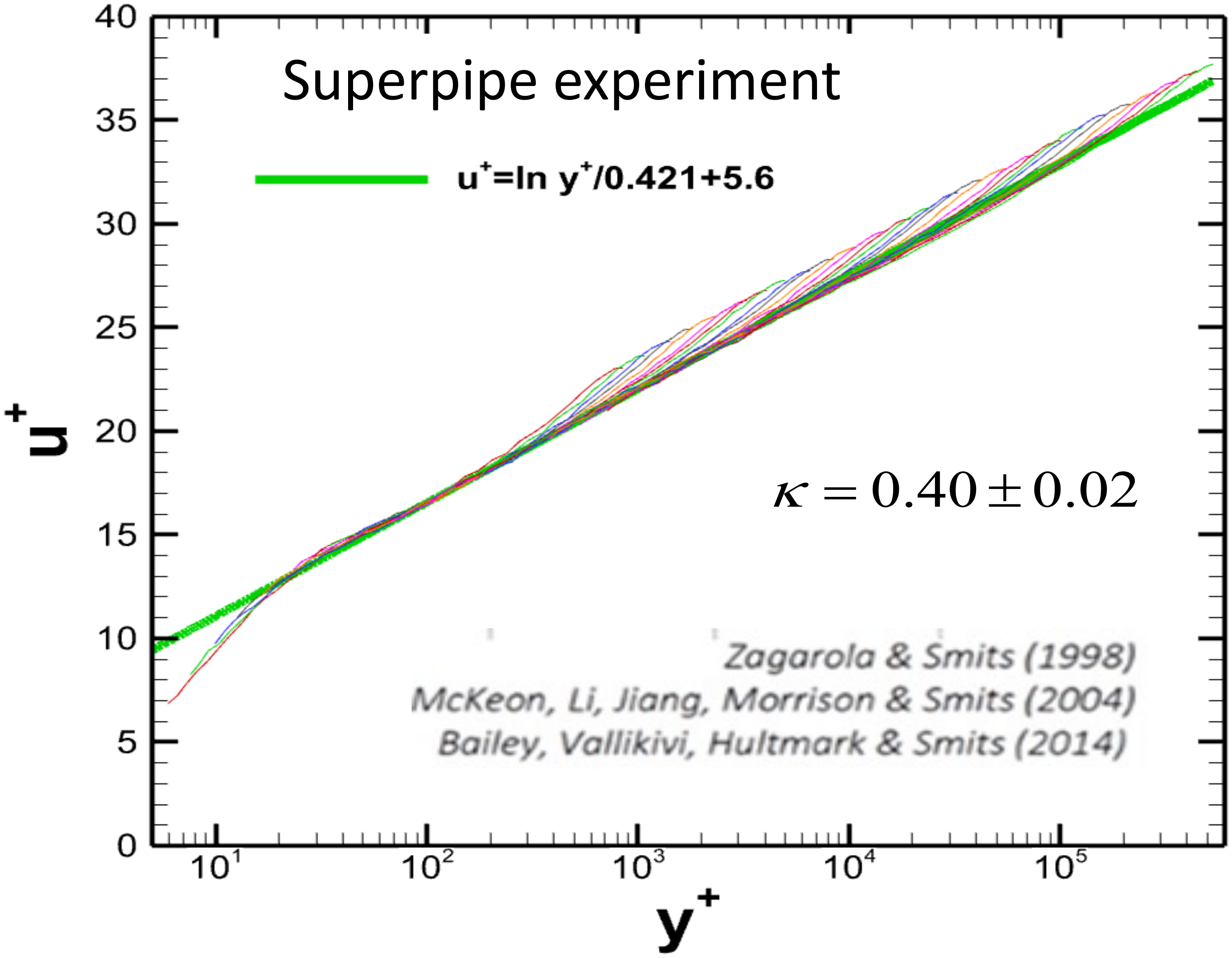
Well-documented BC's

$x/c = -2.14$

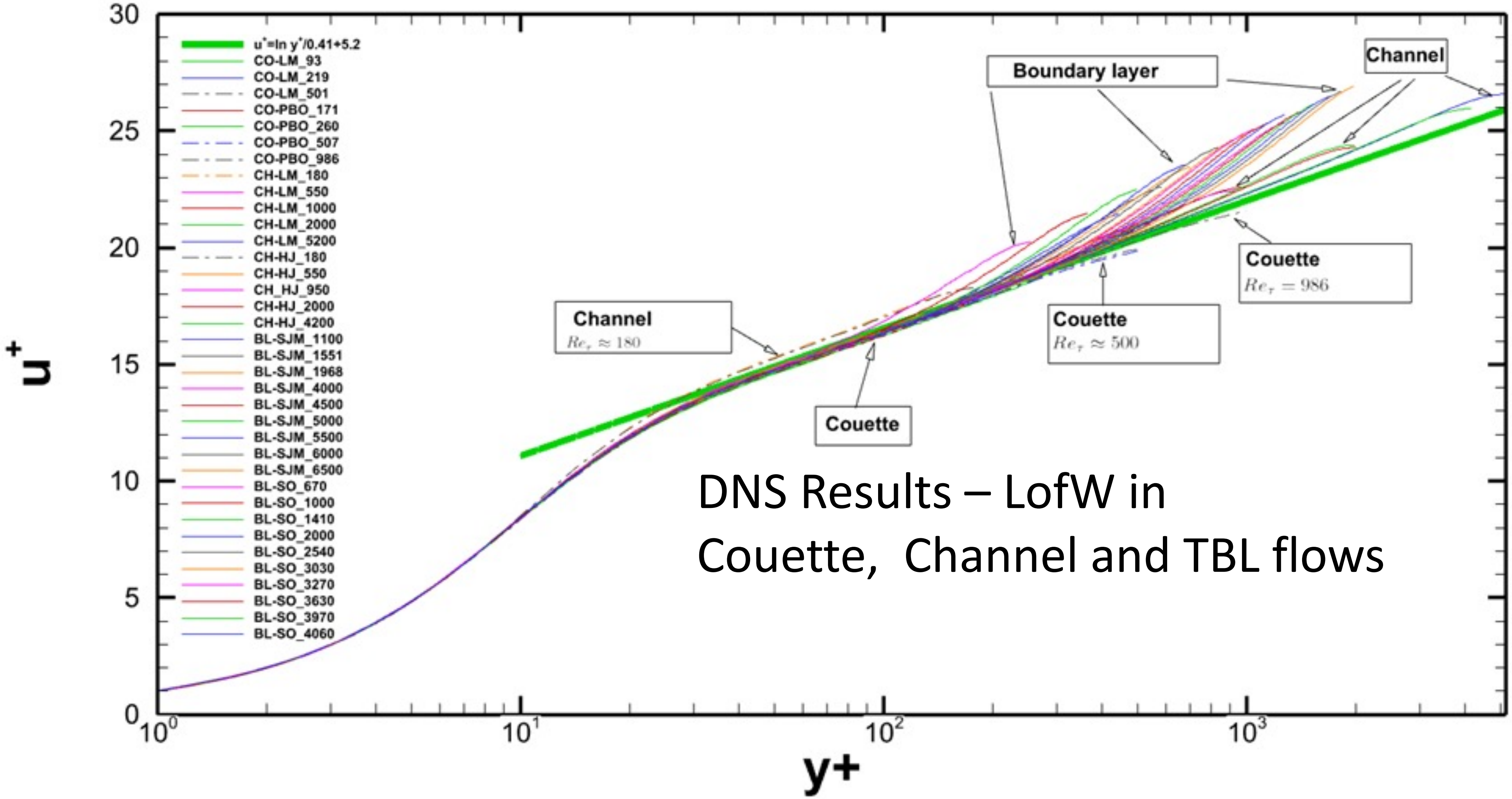




Effects of Re on the law of the wall



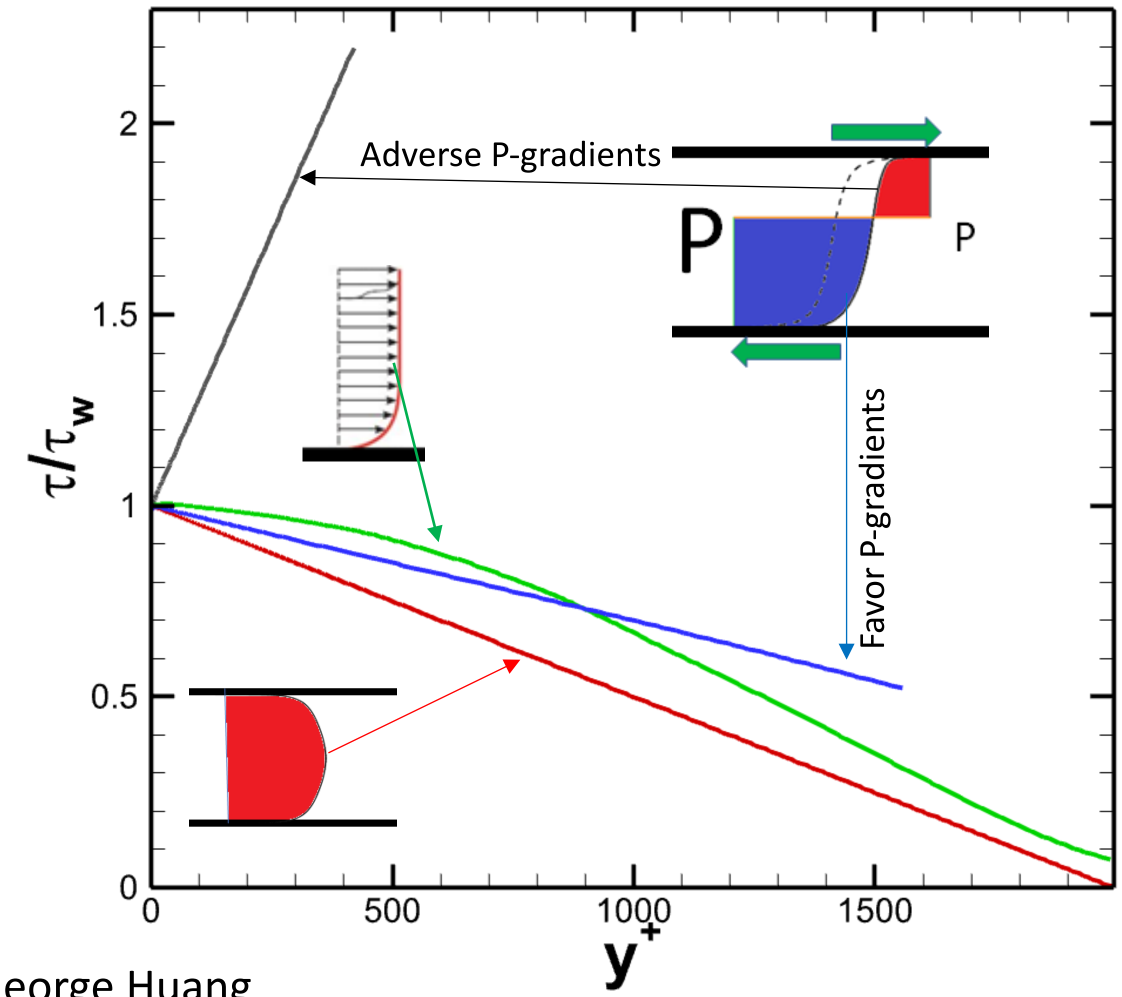
Resilience of the law of the wall

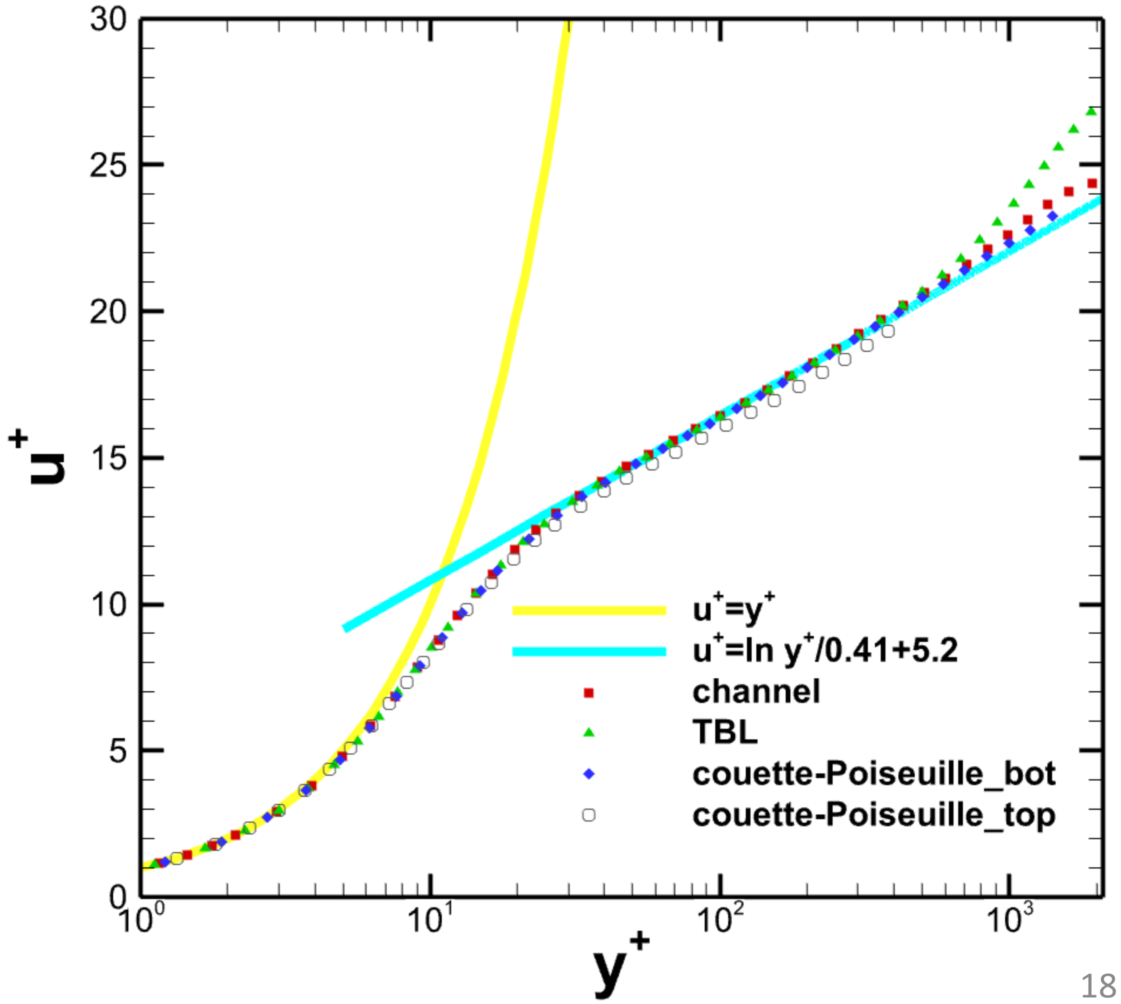


DNS Results – LofW in
Couette, Channel and TBL flows

Resilience of the law of the wall

$$\left(1+\mu_t^*\right)\frac{\partial u^+}{\partial y^+}=1+\int_0^{y^+}\left(u^+\frac{\partial u^+}{\partial x^+}+v^+\frac{\partial u^+}{\partial y^+}+\frac{dp^+}{dx^+}\right)dy^+=\frac{\tau}{\tau_w}$$



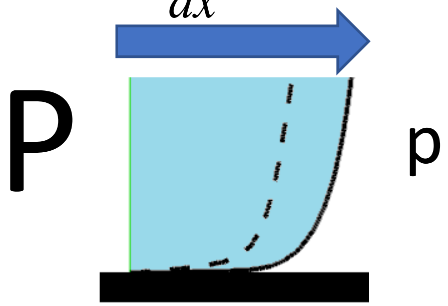


Resilience of the law of the wall

$$\frac{\partial u^+}{\partial y^+} = \frac{\tau / \tau_w}{1 + \mu_t / \mu} = \frac{1}{1 + \kappa y^+ D_\mu}$$

$$\frac{\tau}{\tau_w} = 1 + \frac{dp^+}{dx^+} y^+$$

$$\frac{dp^+}{dx^+} < 0$$



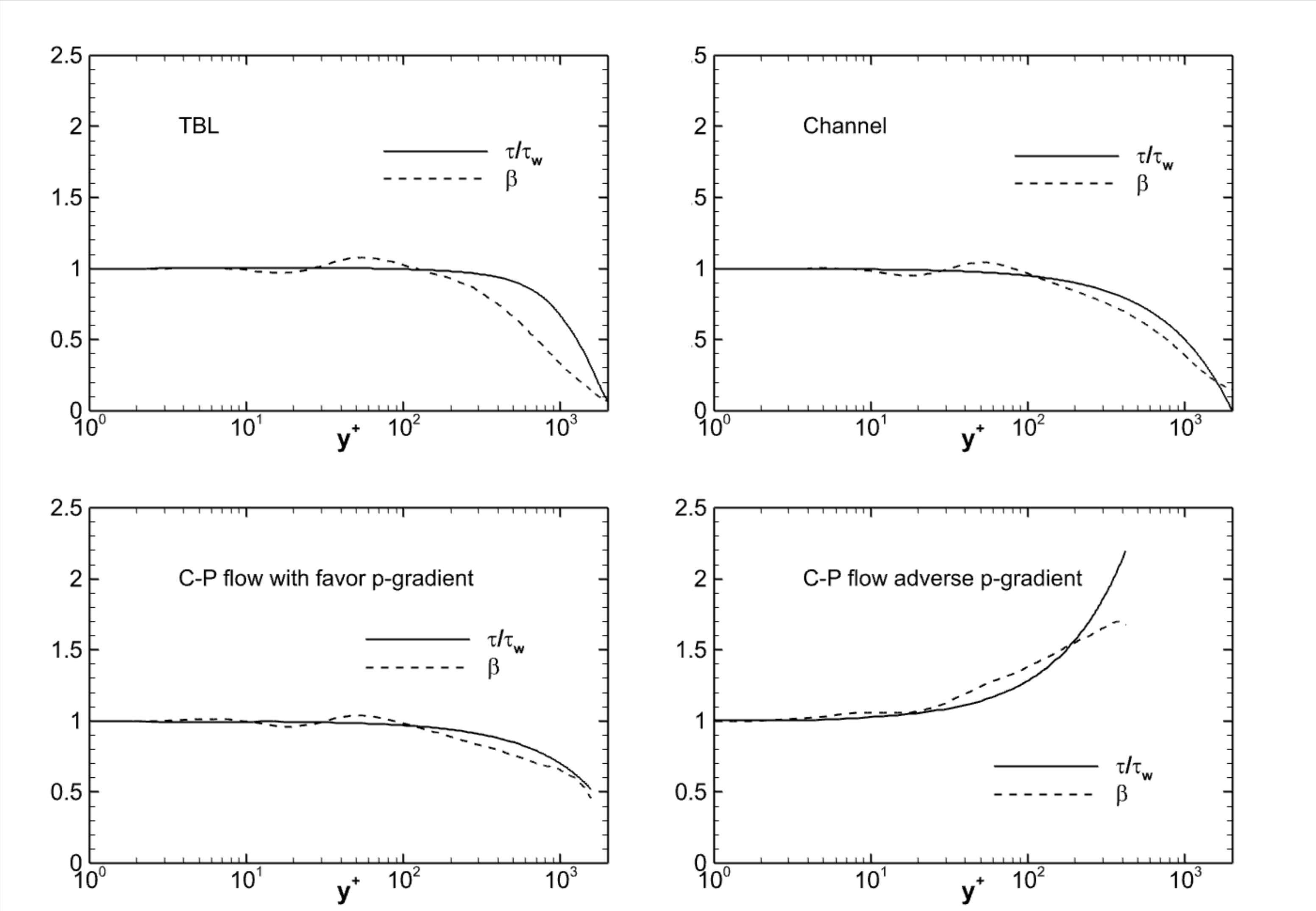
stable

$$\frac{dp^+}{dx^+} > 0$$

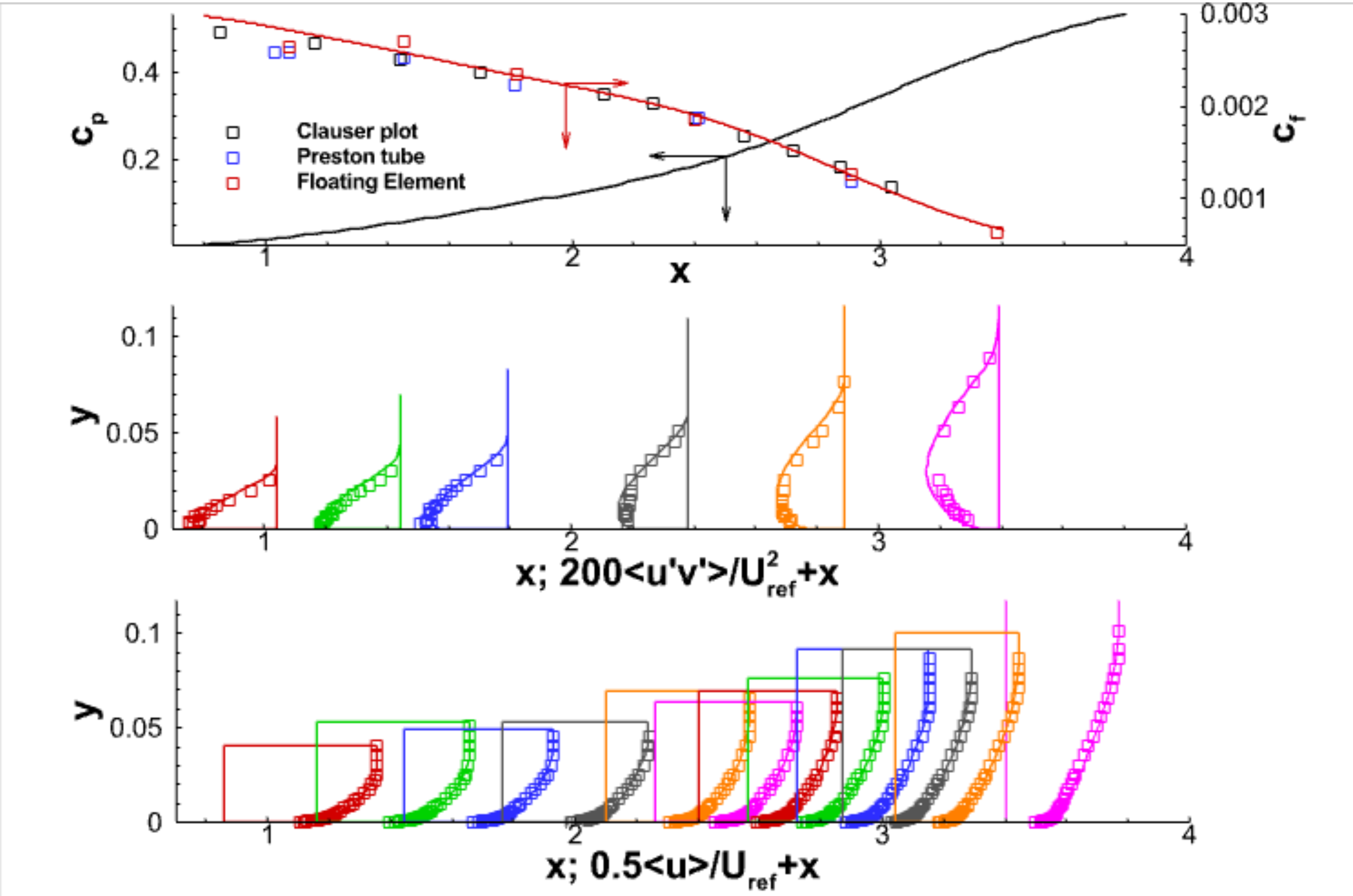
unstable

Resilience of the Law of the wall

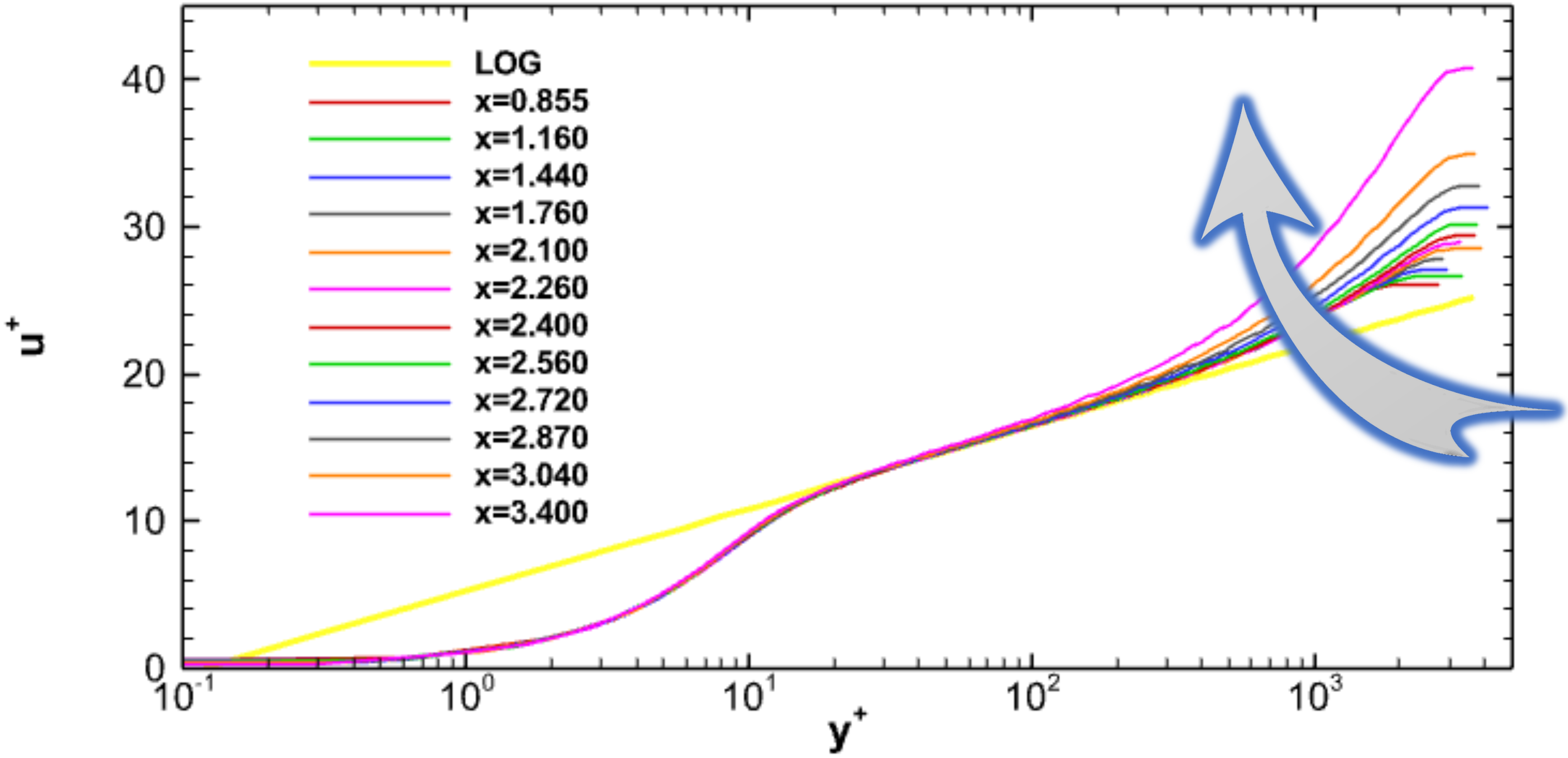
$$\frac{\partial u^+}{\partial y^+} = \frac{\tau / \tau_w}{\left(1 + \kappa y^+ D_\mu\right)\beta} \text{ where } \beta = \frac{1 + \mu_t / \mu}{1 + \kappa y^+ D_\mu}$$

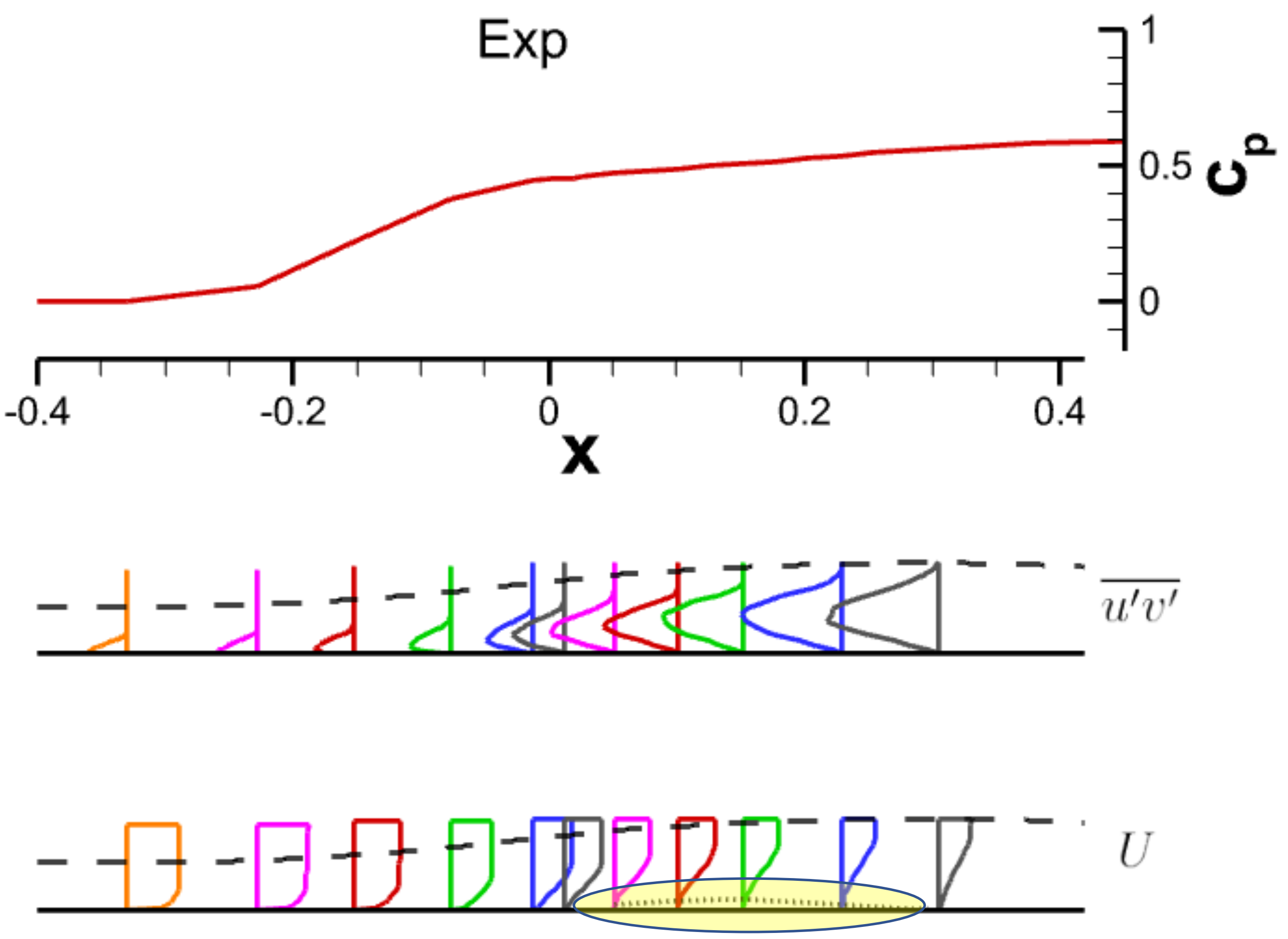


BL with weak APG (no flow separation)
Samuel-Joubert's experiment vs. SA solution

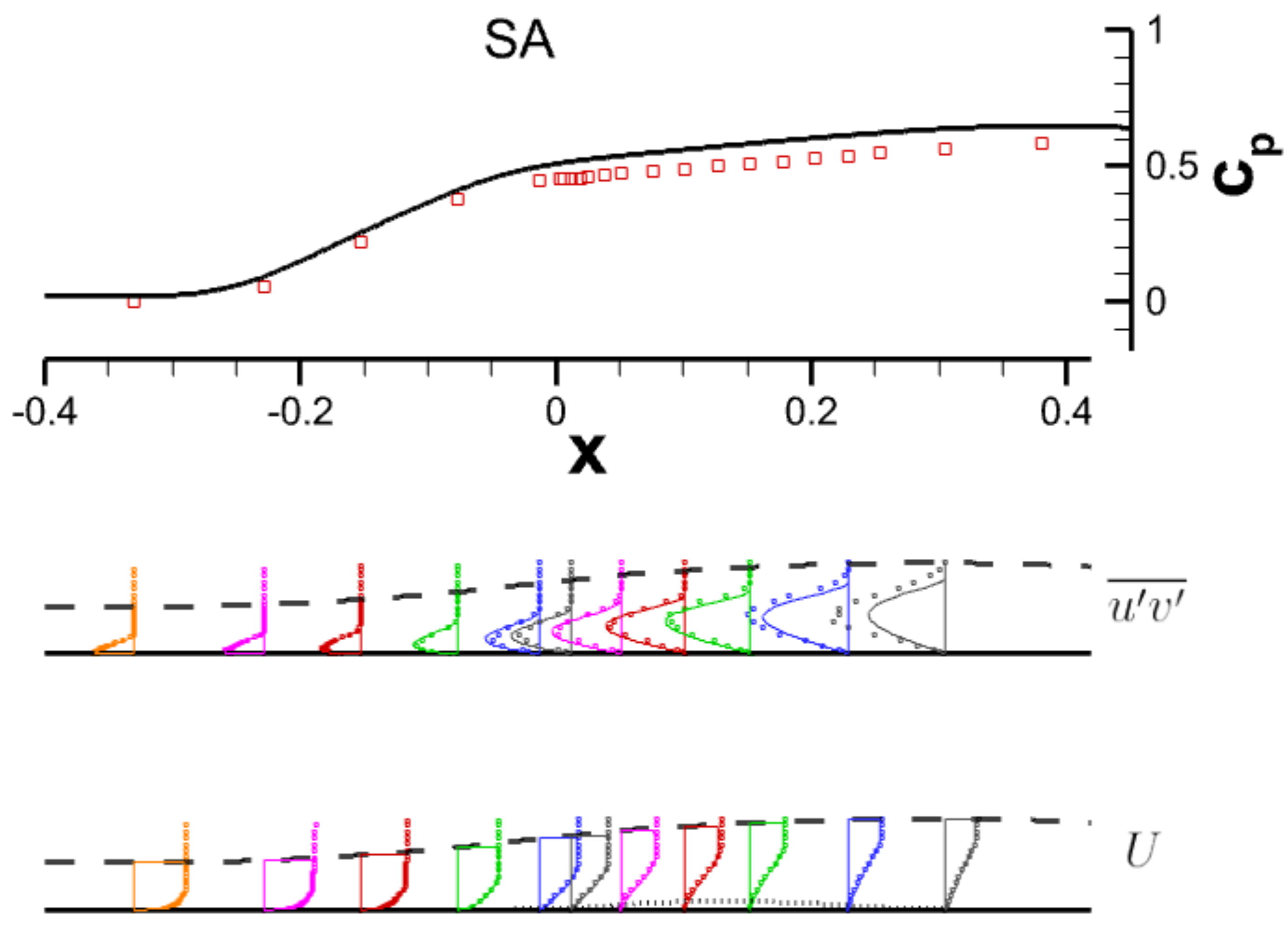


Progressive breakdown of LofW under weak APG





Driver’s Axisymmetric (small) Separated BL (median APG)



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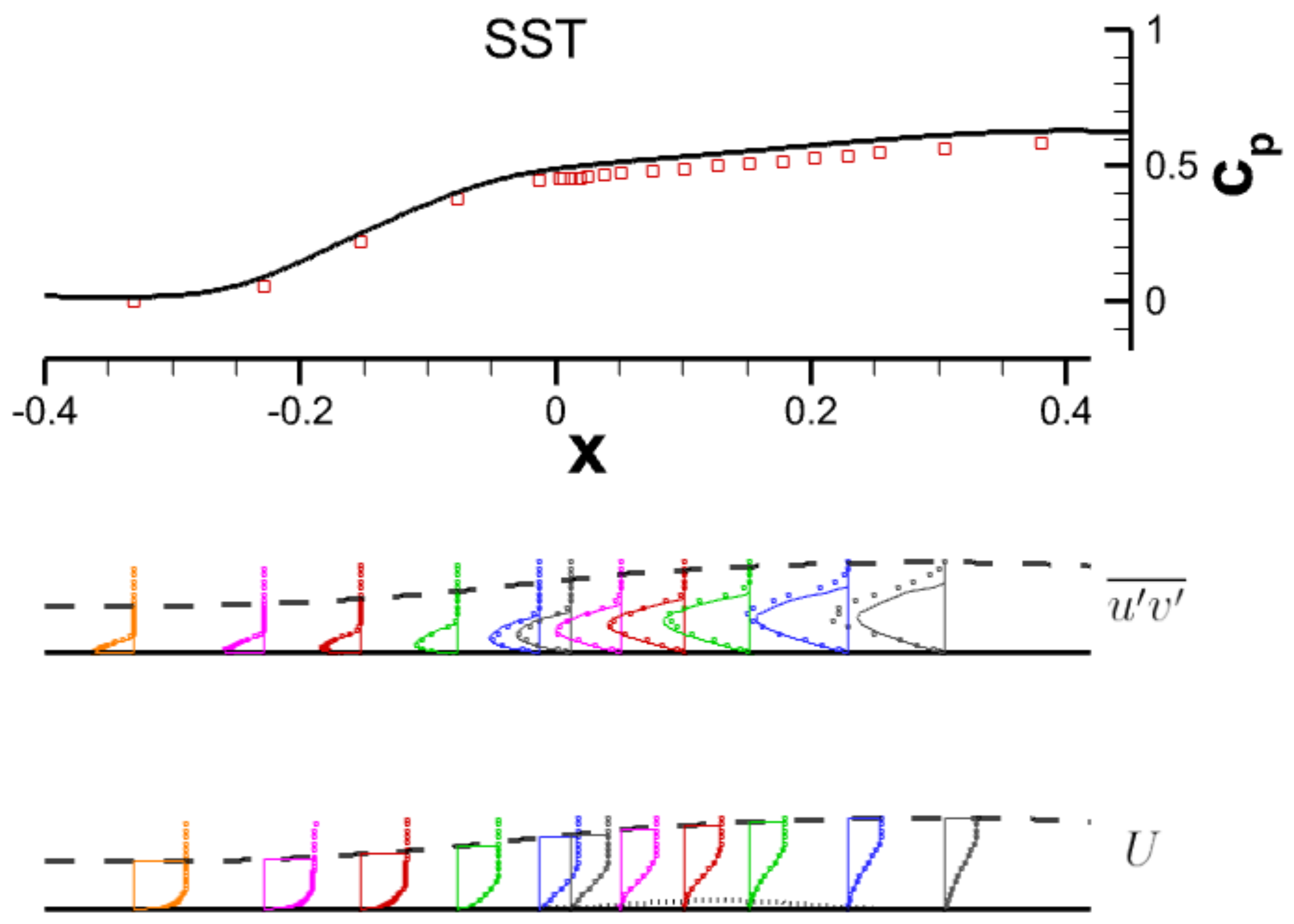
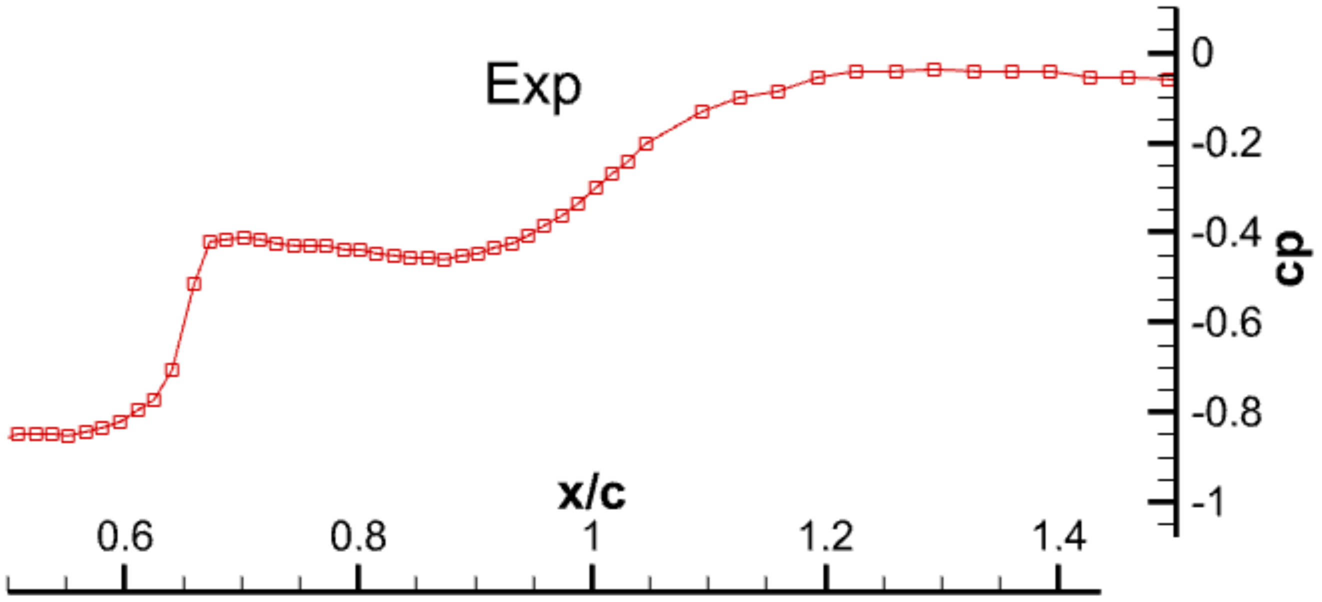
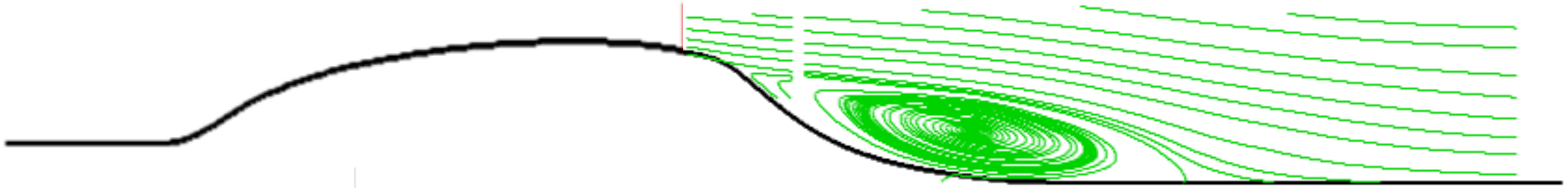
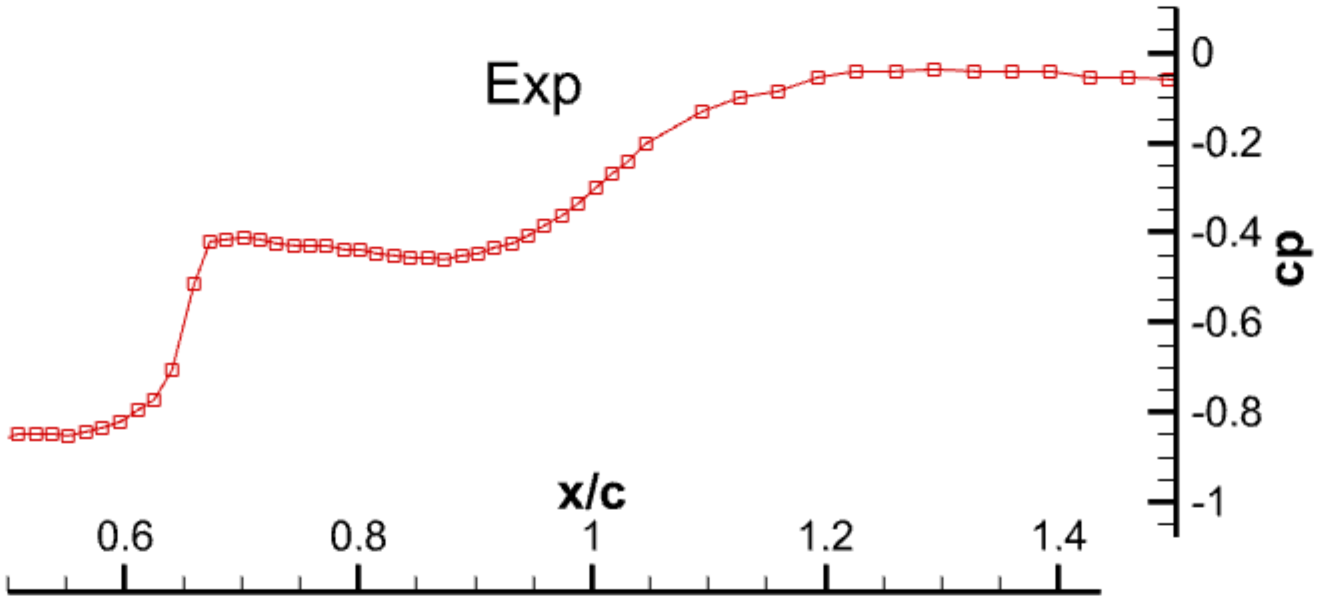


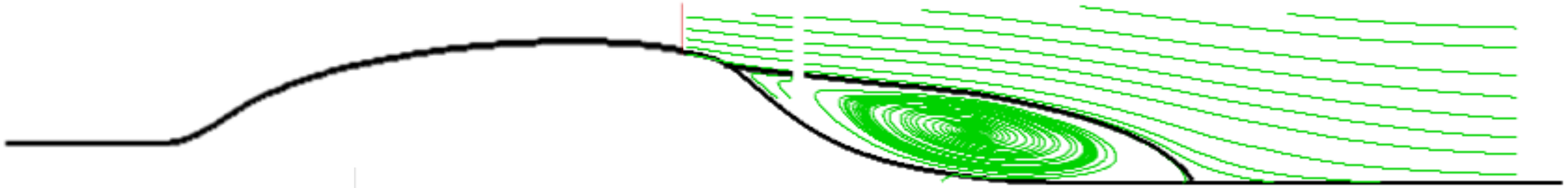
Driver's Axisymmetric (small) Separated BL (median APG)





NASA Wall-Mounted Hump Separated Flow





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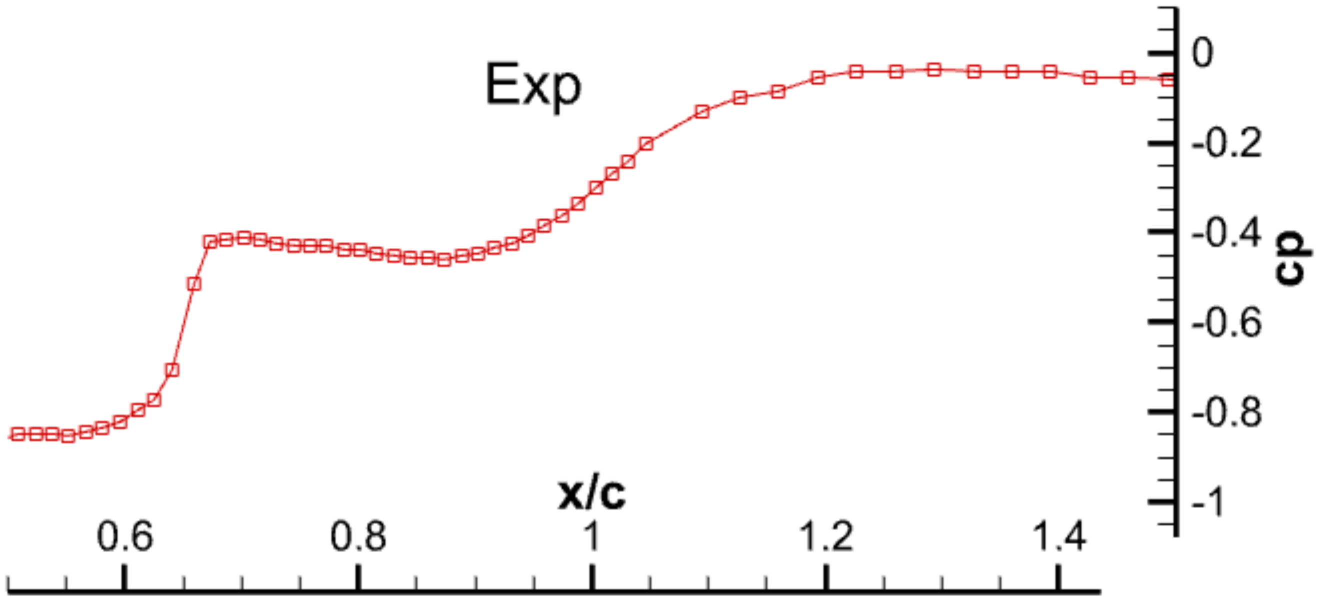
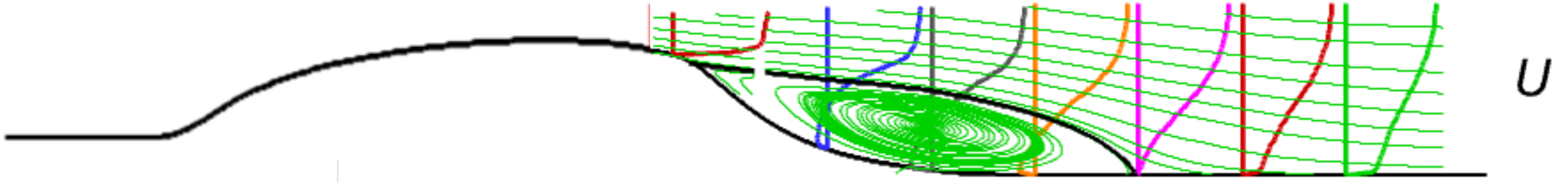
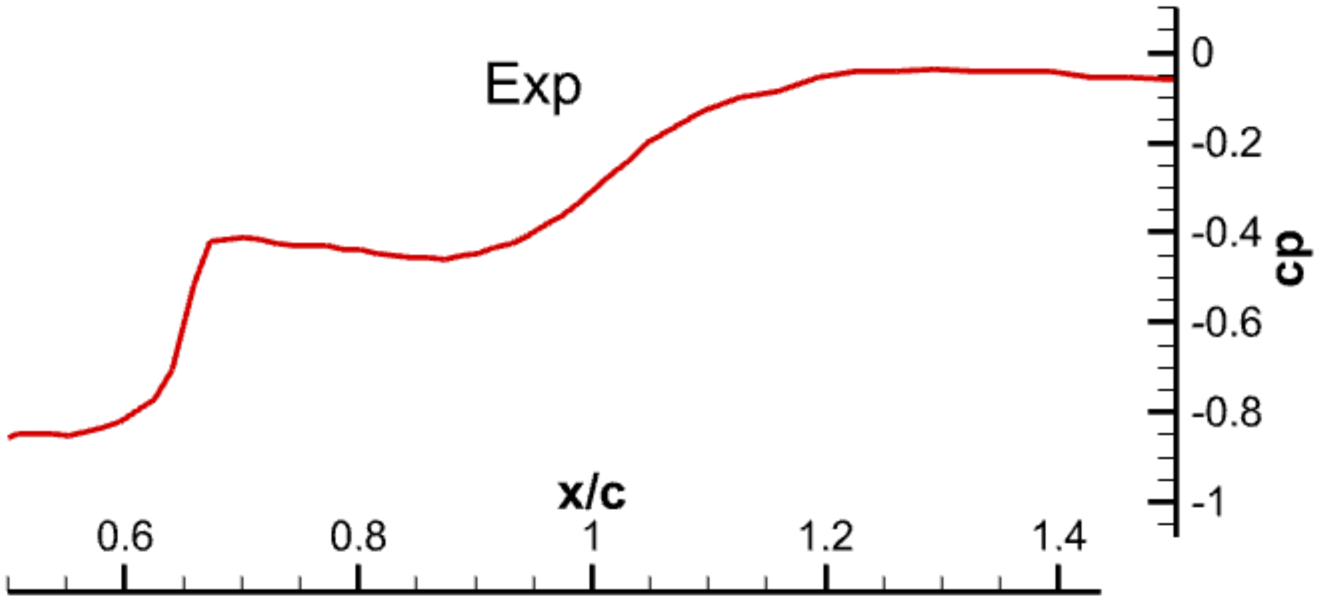
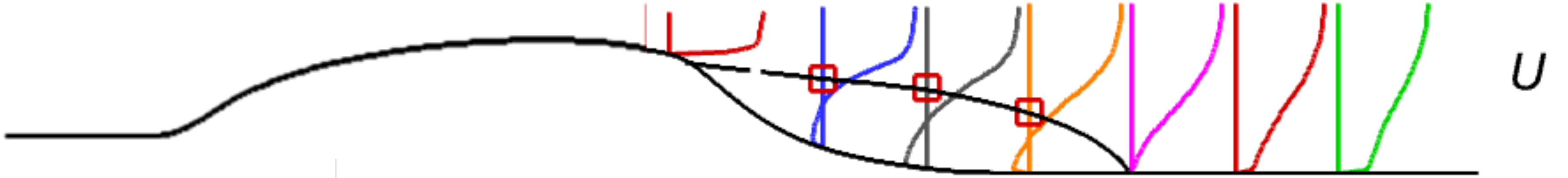




NASA Wall-Mounted Hump Separated Flow





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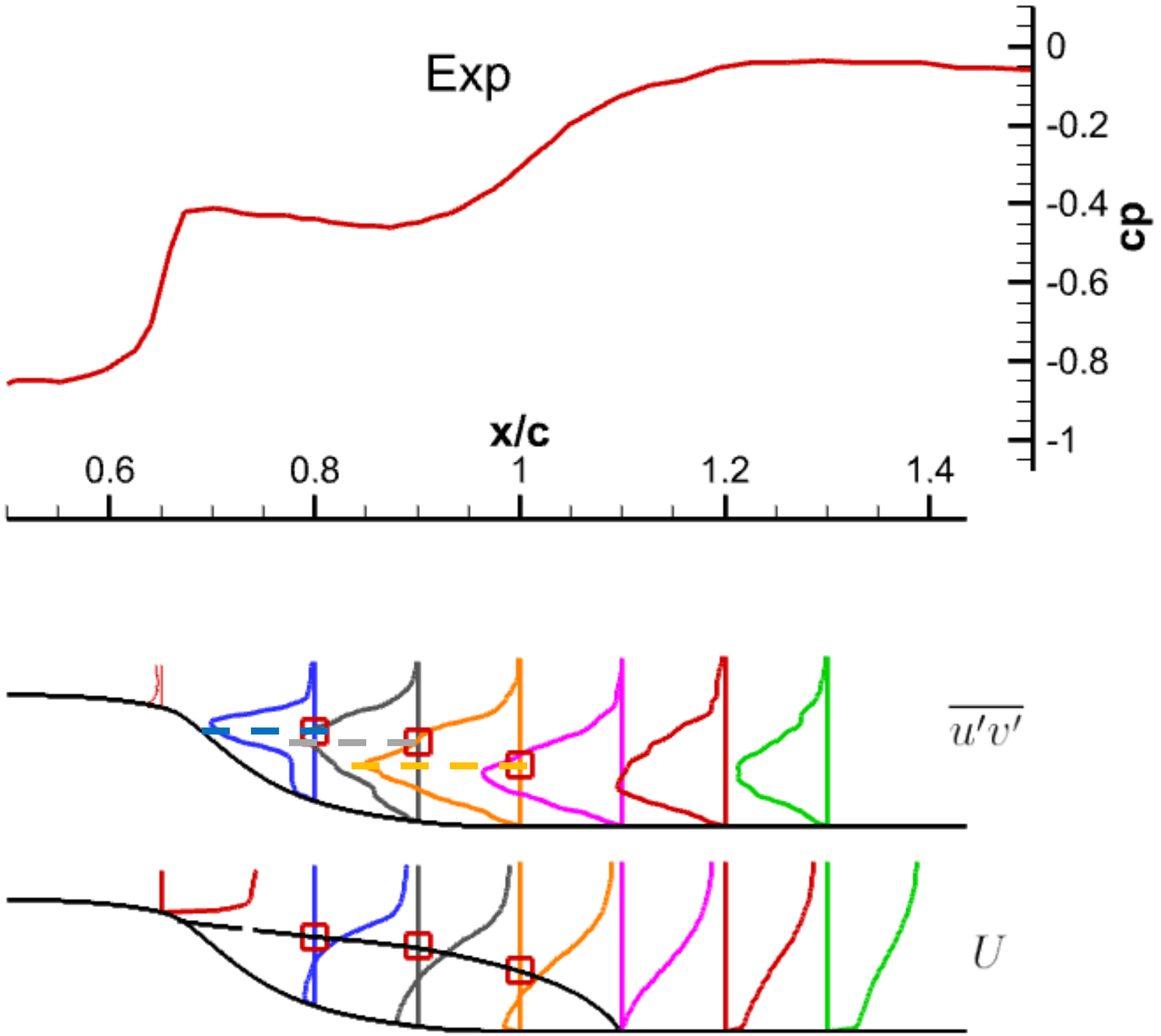

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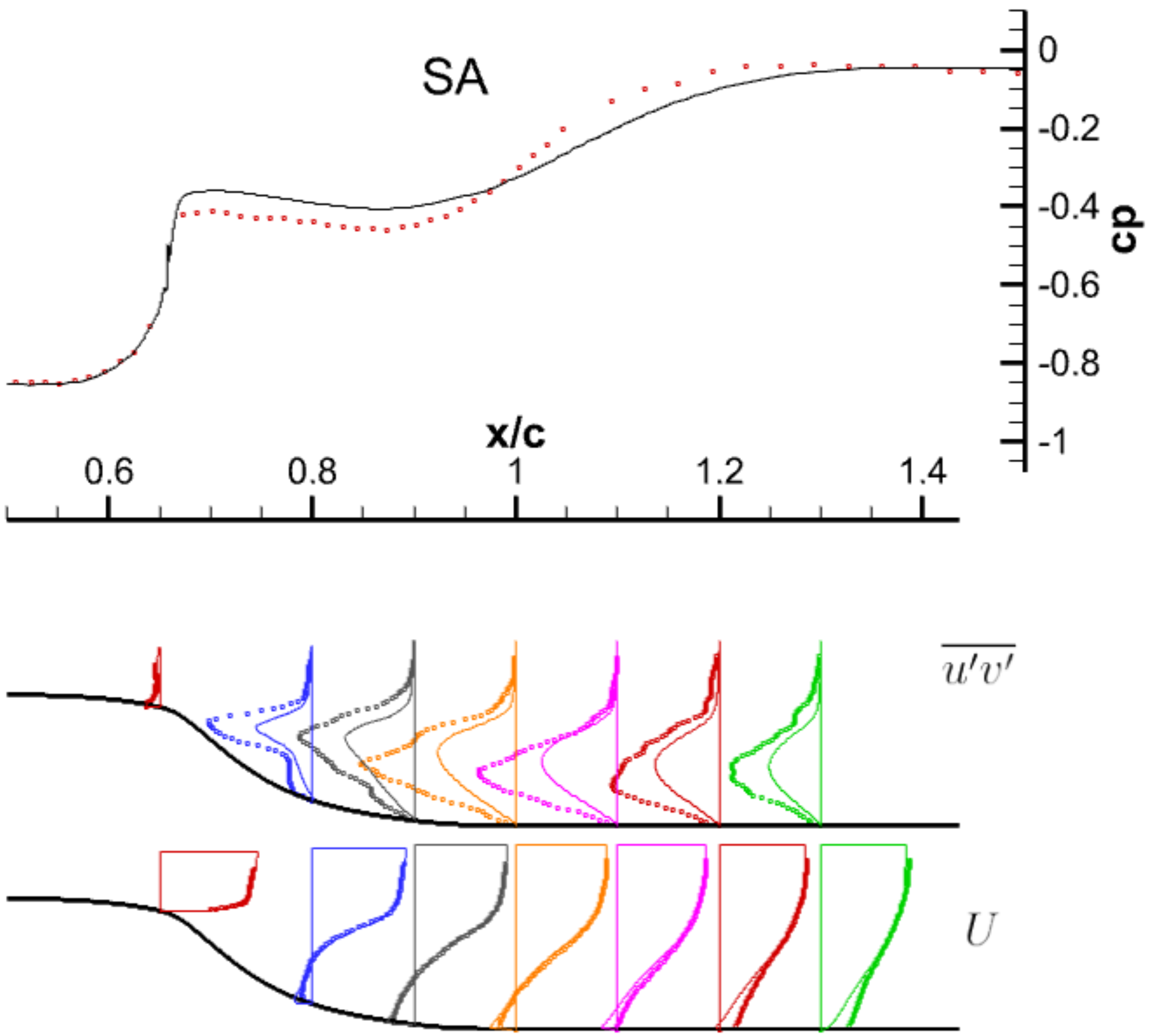
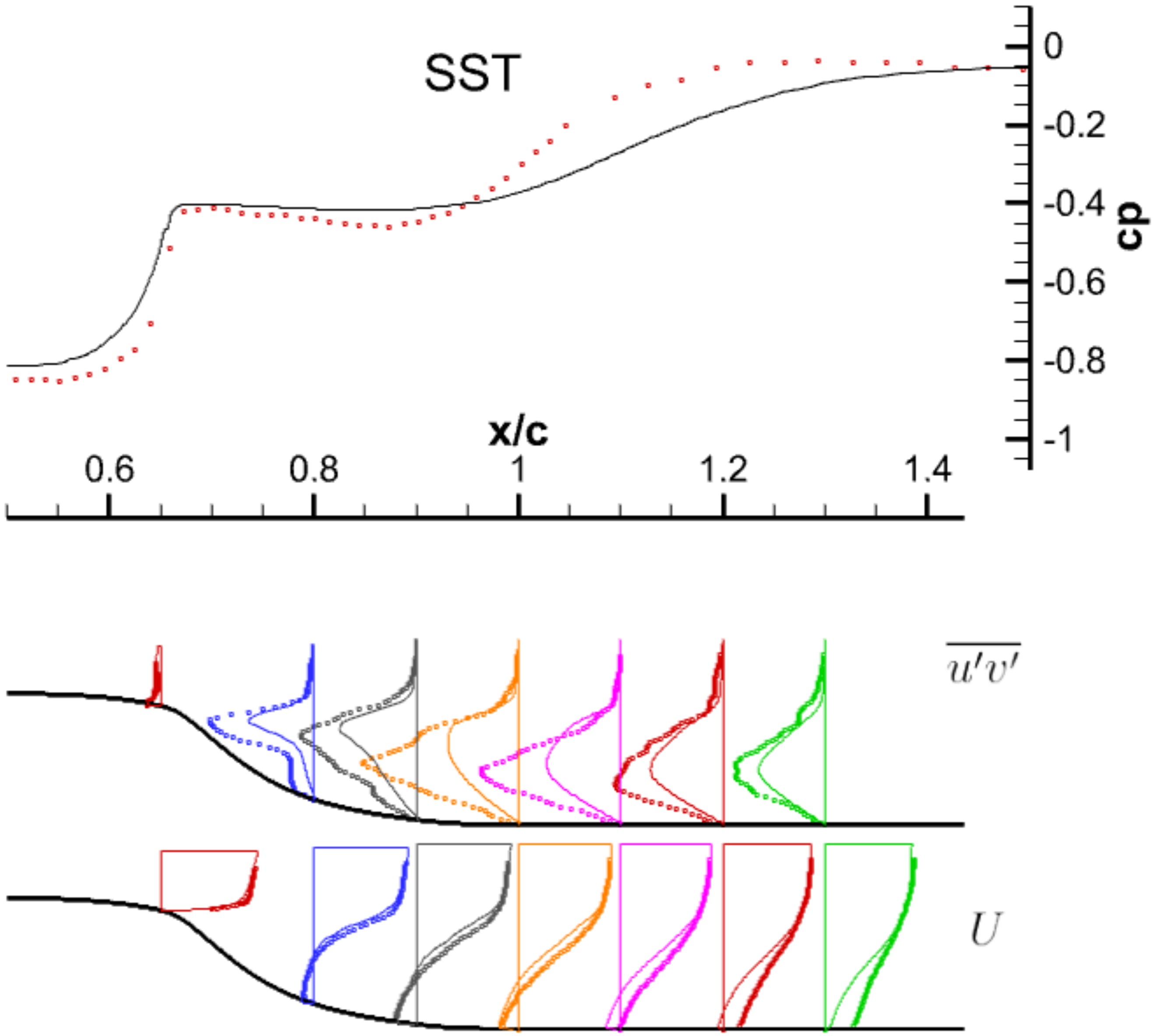
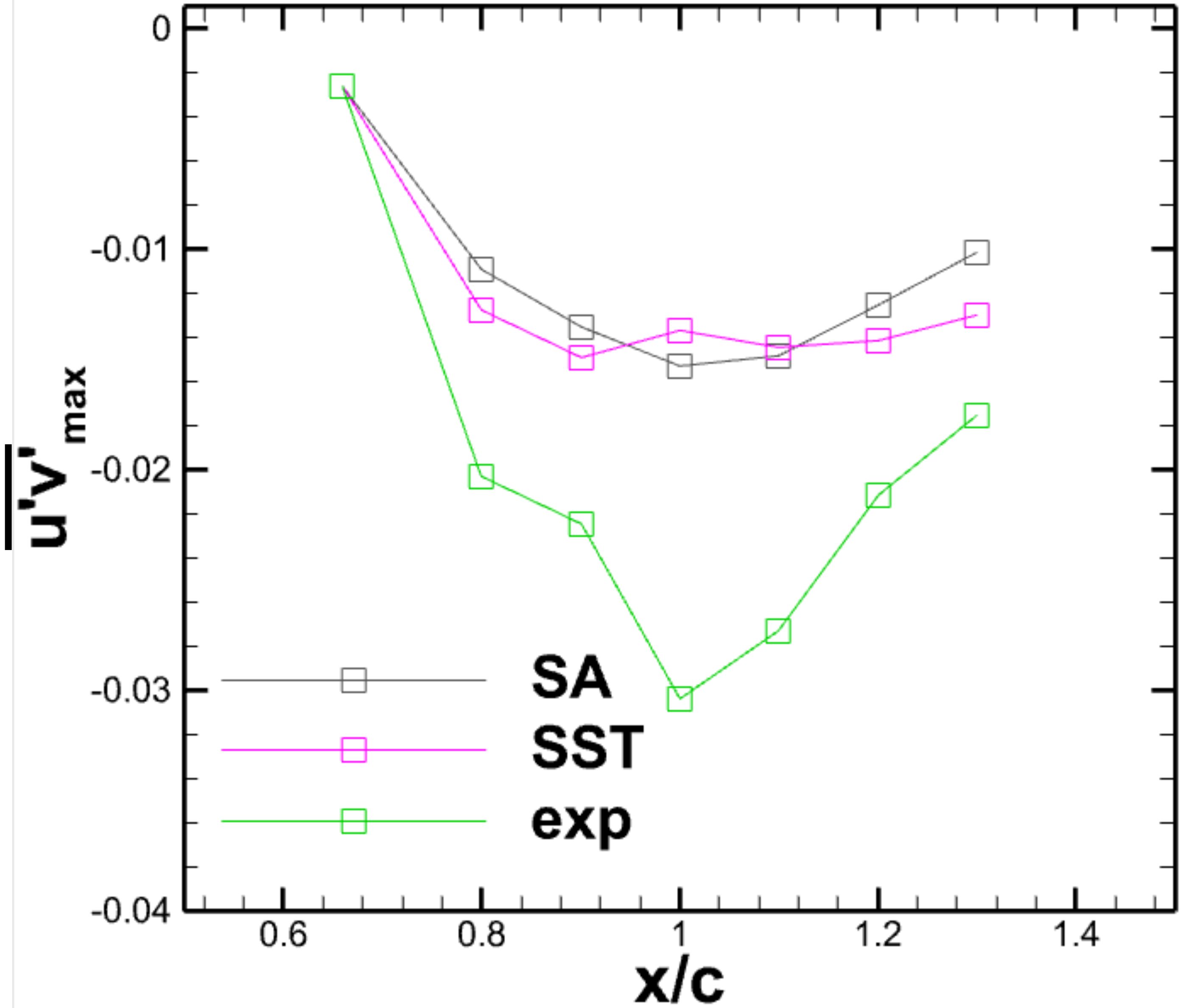


NASA Wall-Mounted Hump Separated Flow

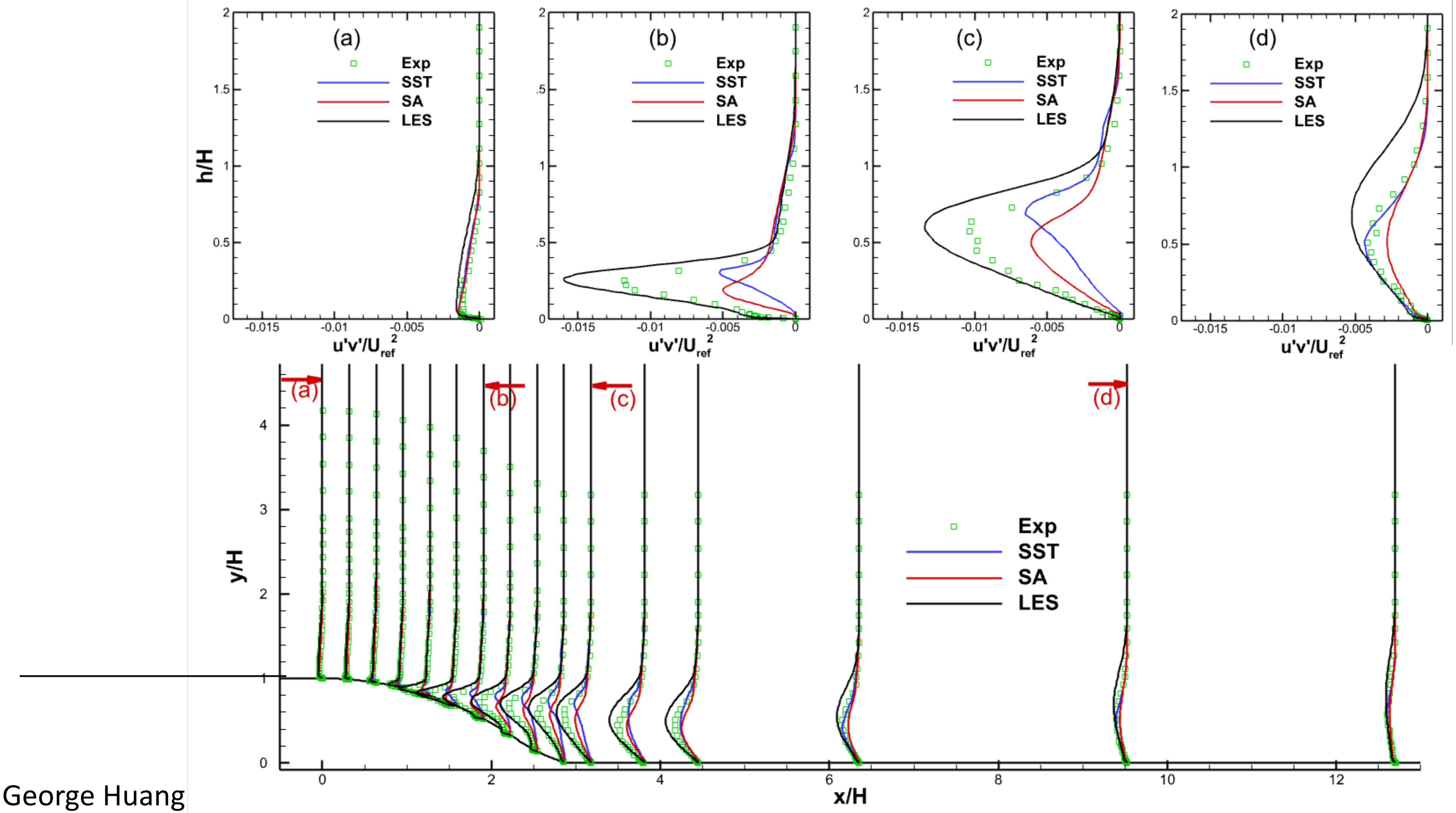


NASA Wall-Mounted Hump Separated Flow

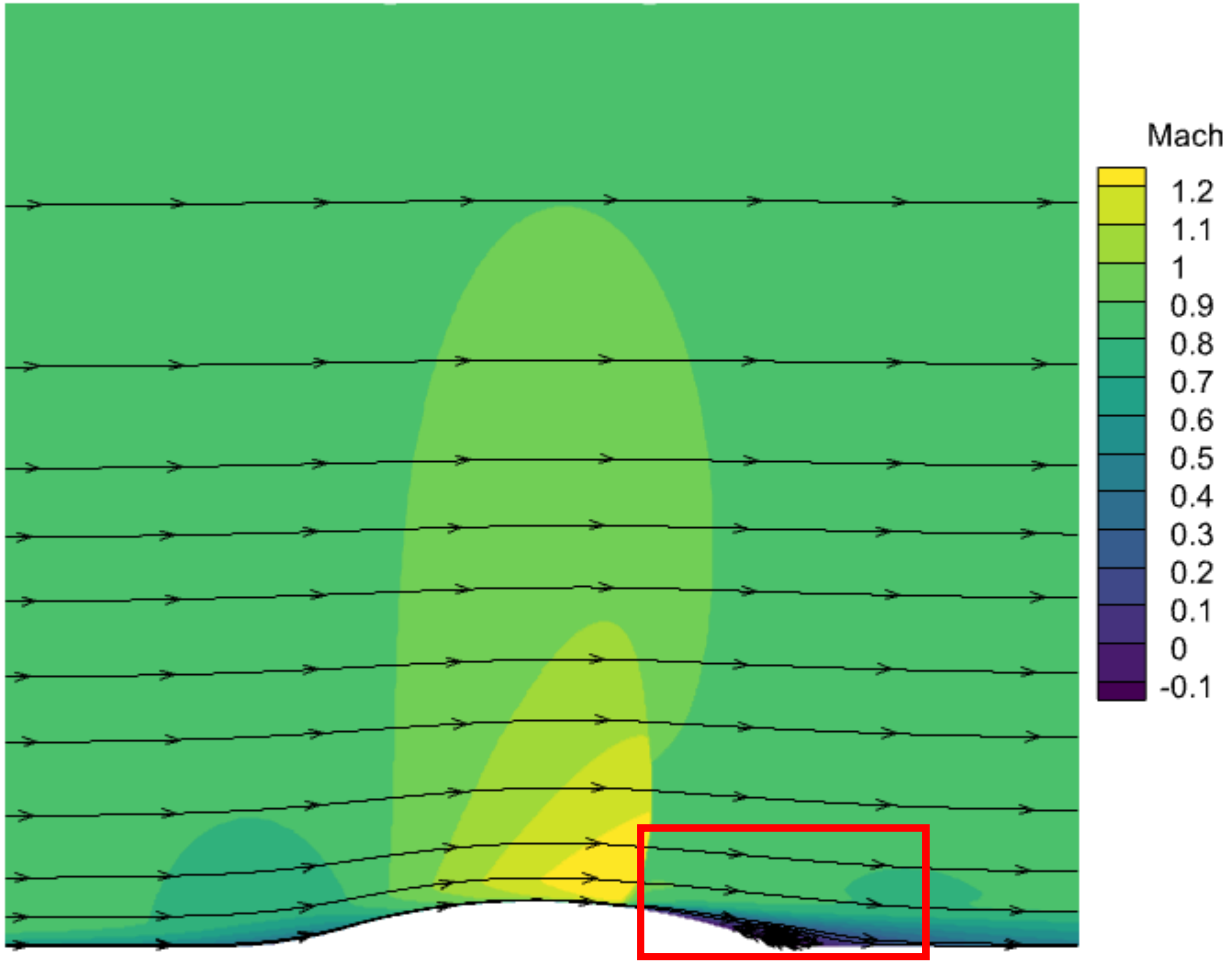
Slingshot effect

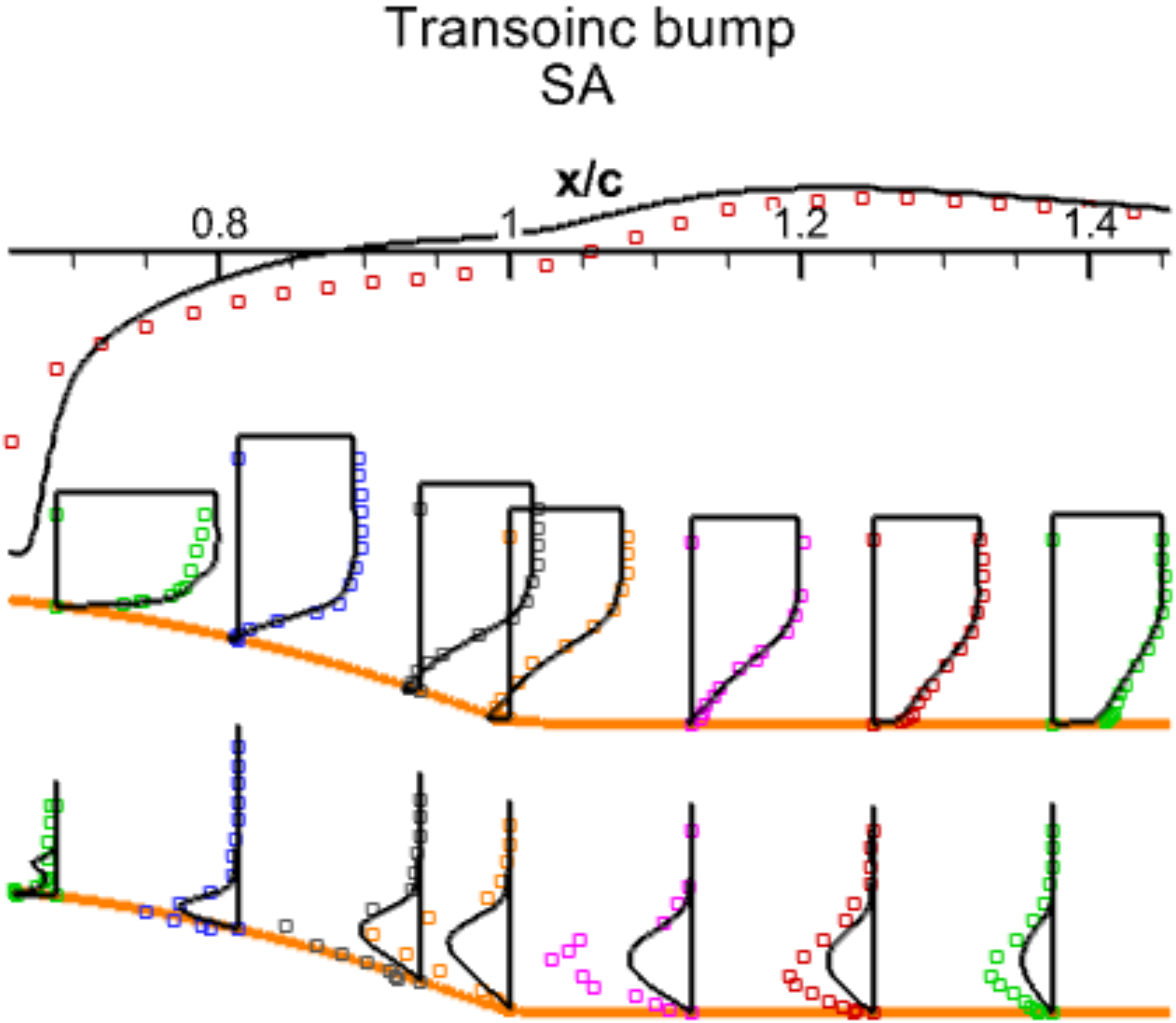


Manchester's TBL separation from a rounded step



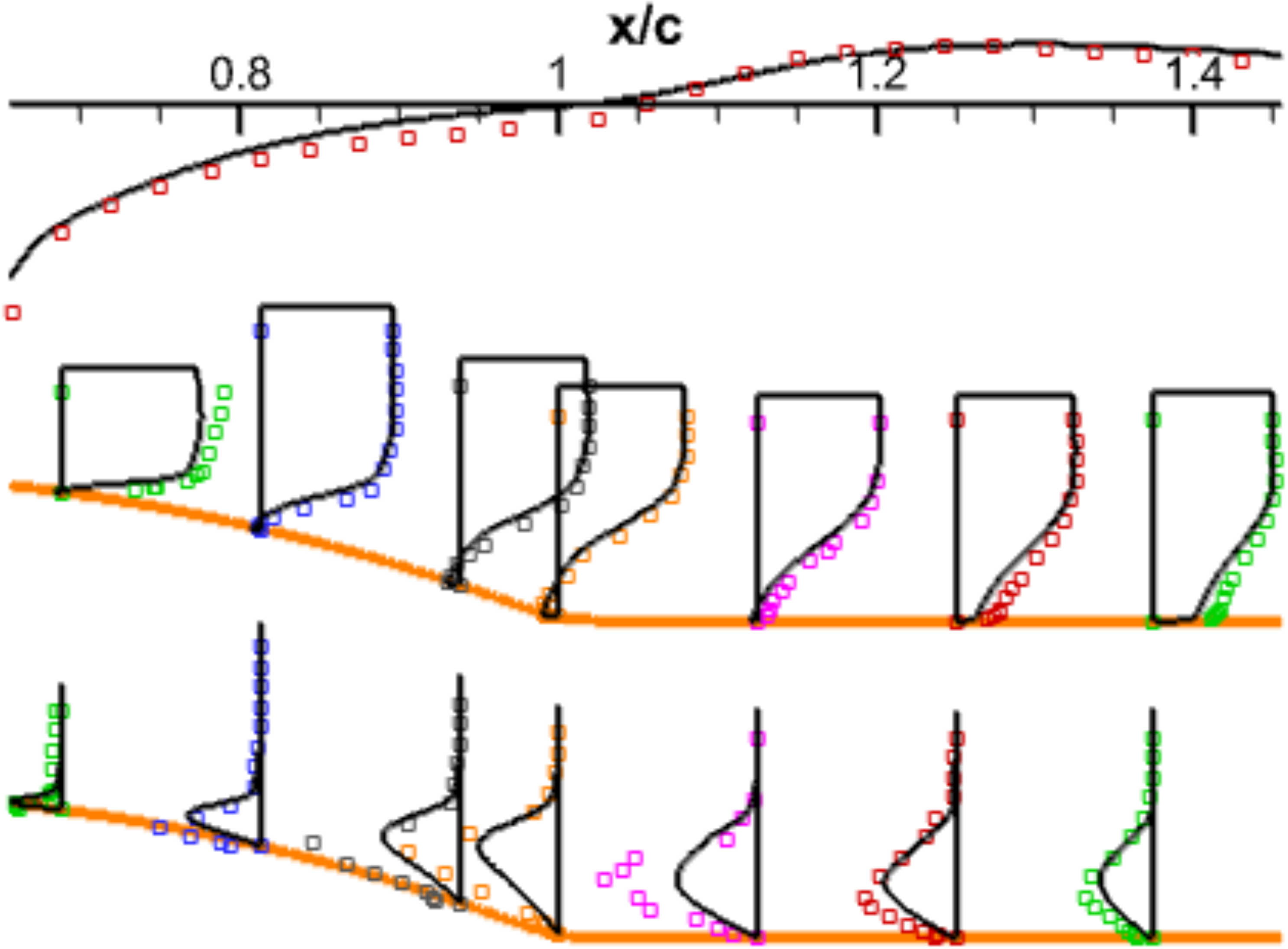
Johnson-Bachalo’s Transonic shock/BL/separation



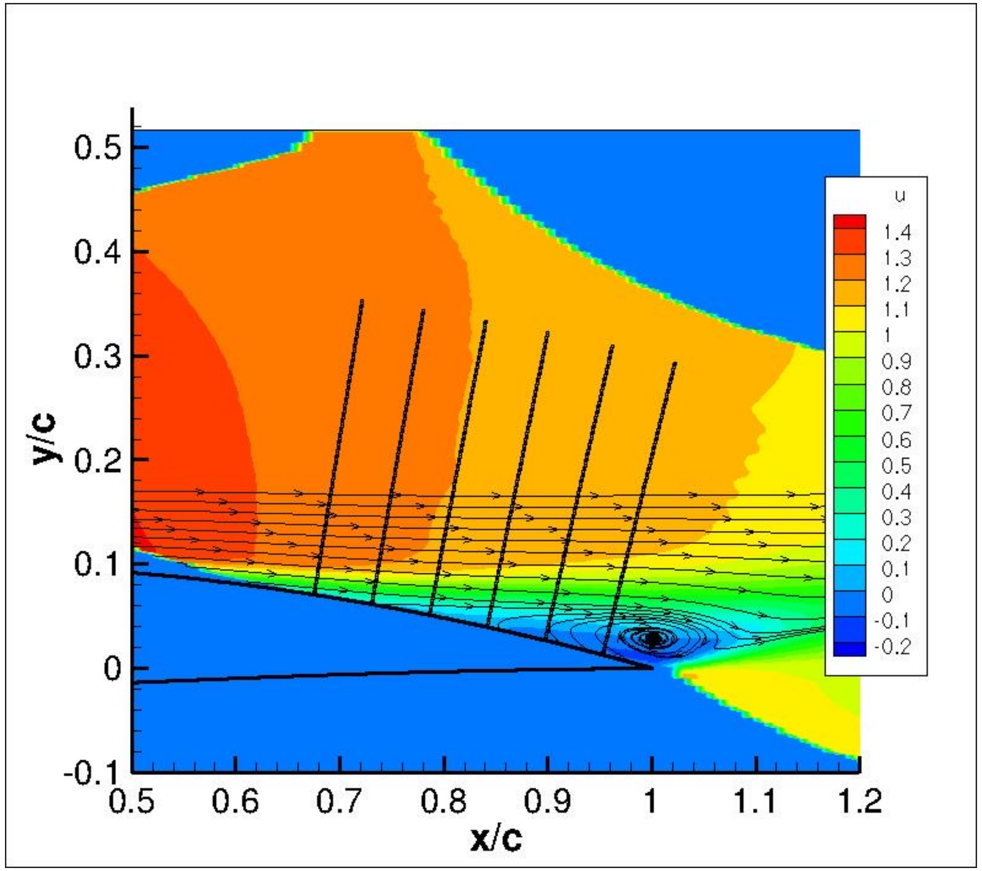


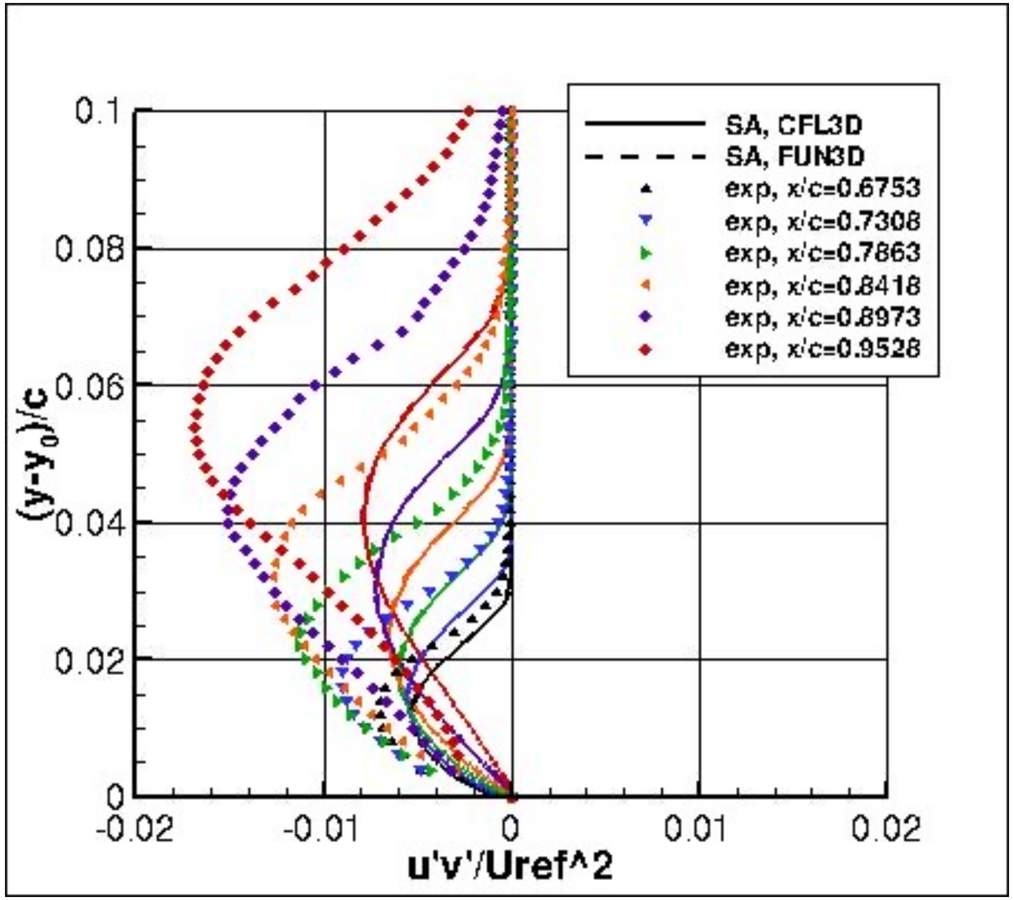
Transoinc bump
SA
x/c
0.8
1
1.2
1.4

Transonic bump
SST



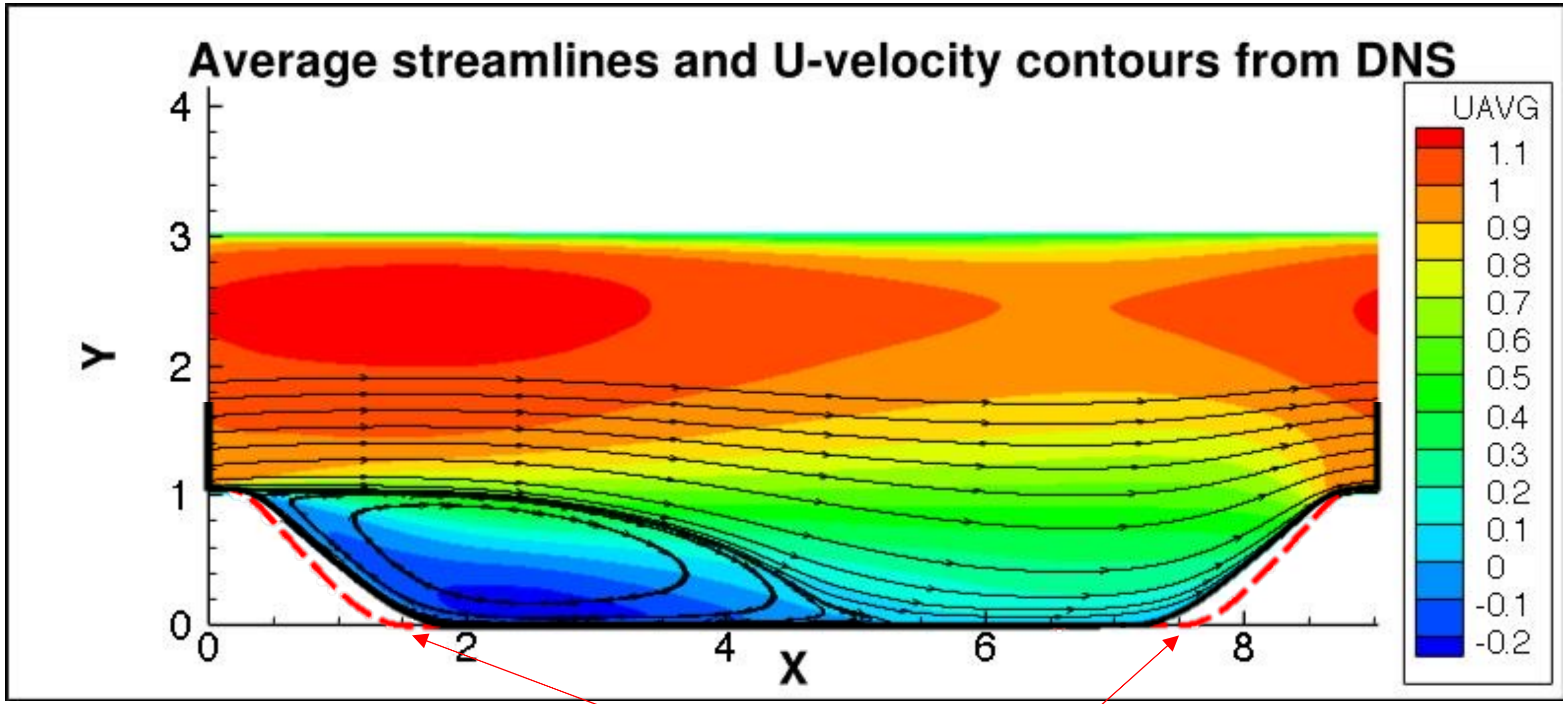
NACA 4412 Airfoil Trailing Edge Separation



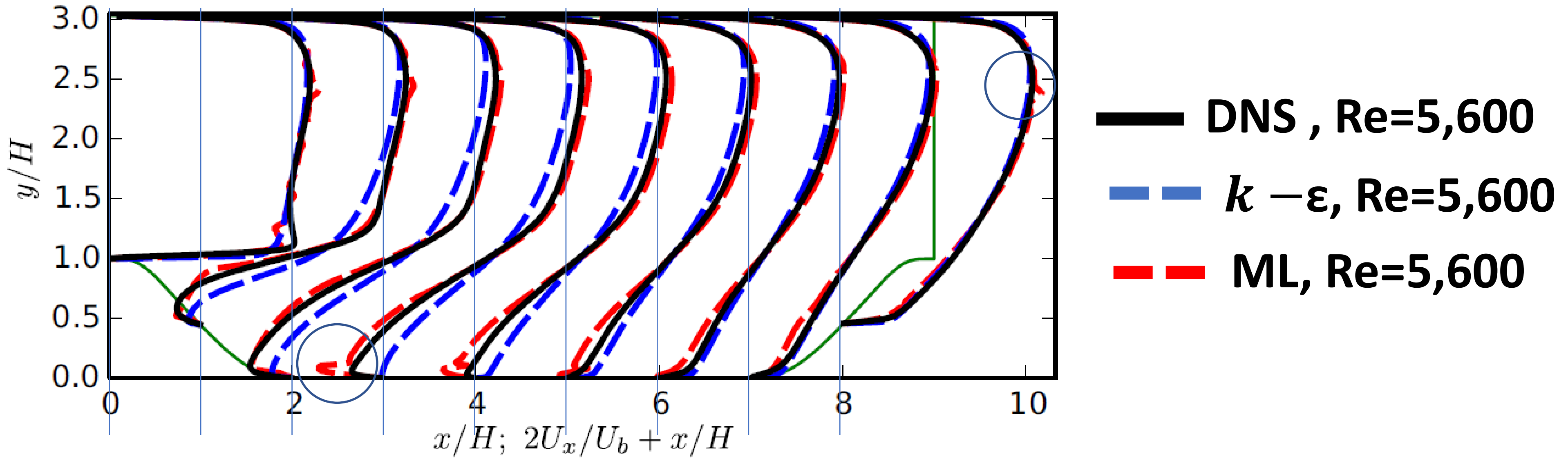


Periodic hill

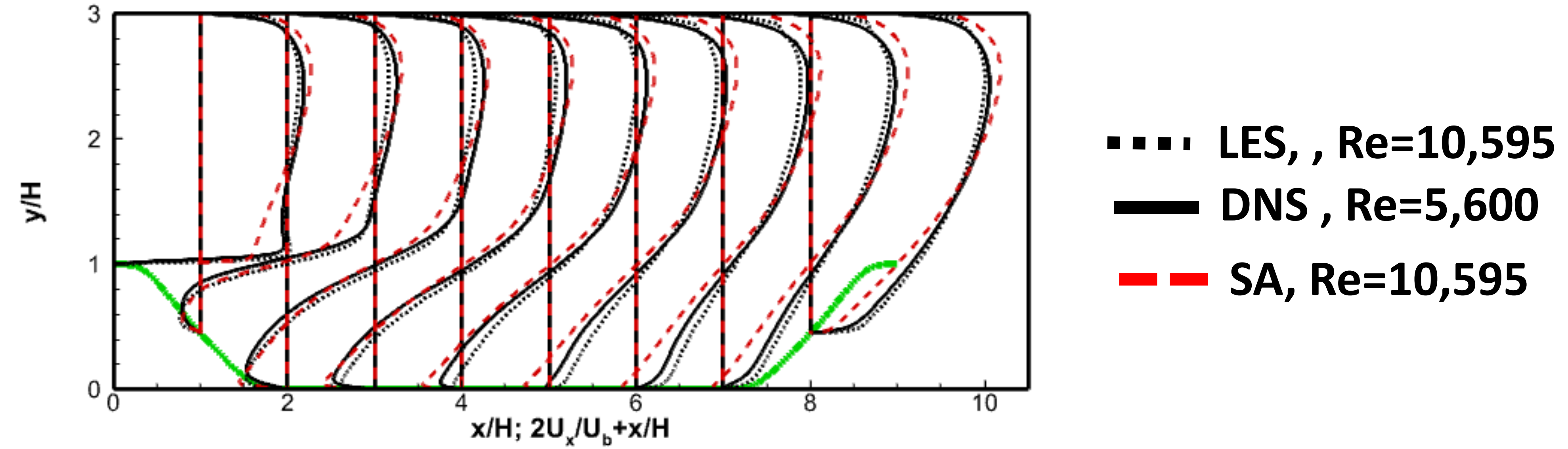
- DNS, Re=5,600
- ML, Re=5,600
- L-S $k - \varepsilon$, Re=5,600
- LES, Re=10,595
- SA, , Re=10,595



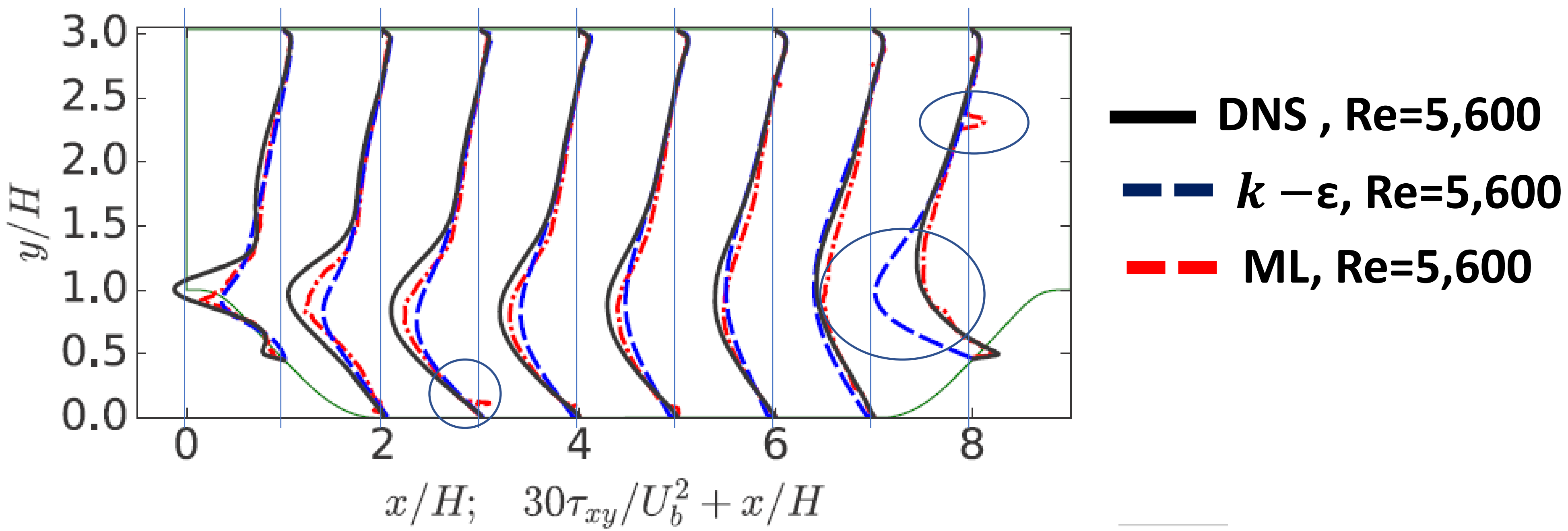
Training geometry

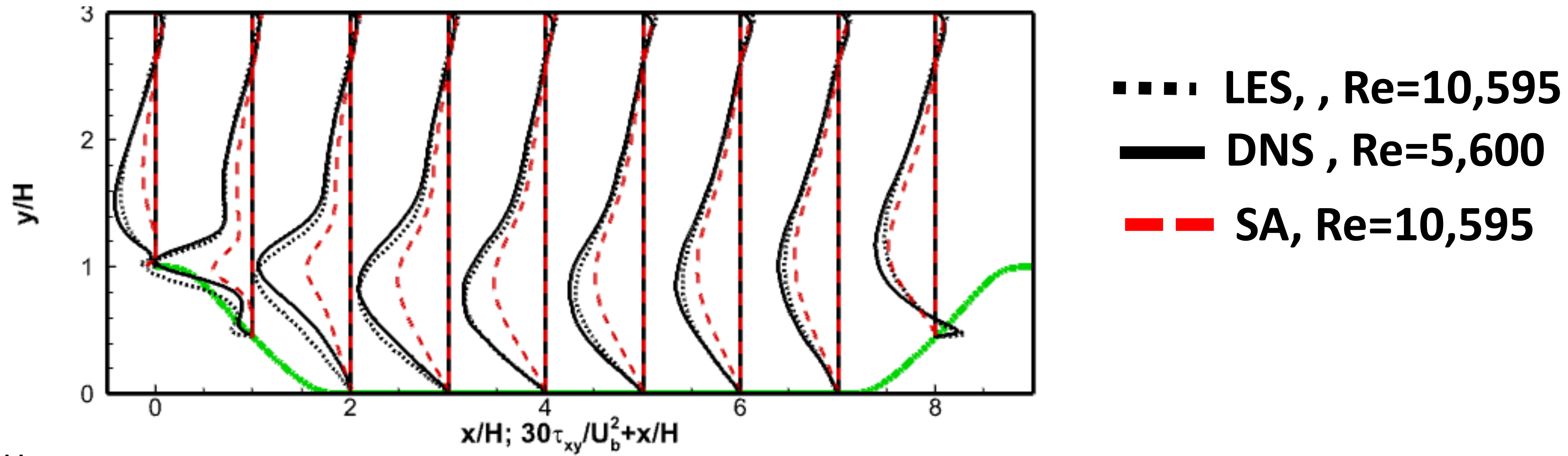


3.0
2.5
2.0
1.5
1.0
0.5
0.0
y/H
0
2
4
6
8
10
x/H; 2Ux/Ub + x/H
DNS , Re=5,600
k −ε, Re=5,600
ML, Re=5,600



3
2
1
0
y/H
0
2
4
6
8
10
x/H; 2Ux/Ub+x/H
LES, , Re=10,595
DNS , Re=5,600
SA, Re=10,595


3.0
2.5
2.0
1.5
1.0
0.5
0.0
y/H
0
2
4
6
8
$x/H; \quad 30\tau_{xy}/U_b^2 + x/H$
DNS , Re=5,600
$k - \varepsilon$, Re=5,600
ML, Re=5,600


3
2
1
0
y/H
0
2
4
6
8
$x/H; 30\tau_{xy}/U_b^2+x/H$
LES, , Re=10,595
DNS , Re=5,600
SA, Re=10,595

My wish lists

- Can ML be less dependent on geometry similarity of the training flows?
- How to extend to high Re #?
  - DNS (or LES) only offers low Re # training data.
- Can ML retain its memory (model’s success is accumulative) ?
- Can ML accommodate flow physics?
- Can ML satisfy asymptotic behaviors of the benchmark flows?
- Slingshot effects and vortex flows deserve further attention.

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