THE UNIVERSITY OF MELBOURNE

Towards more accurate and general turbulence models using CFD-driven training on multiple flows

Supervisors: Richard D Sandberg, Andrew S H Ooi, Yaomin Zhao
Presenter: Yuan Fang
28/07/2022

Contents

Contents

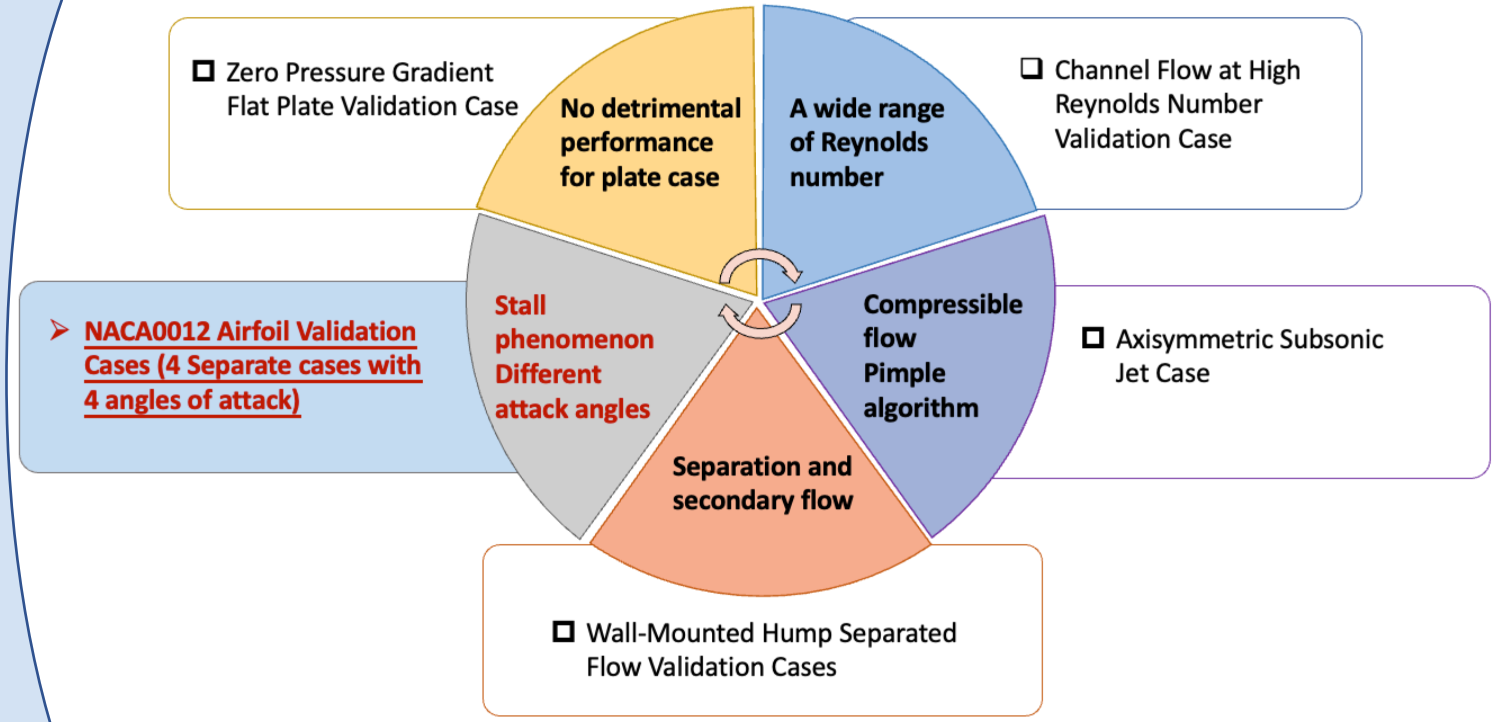
1.1 The research objectives

Numerical cases division

Training cases: plate; channel; jet; hump

Testing cases: NACA 0012 airfoil with 4 angles of attack

- Not enough data at the stall
- Four cases training leads to high computation cost
- Need testing cases



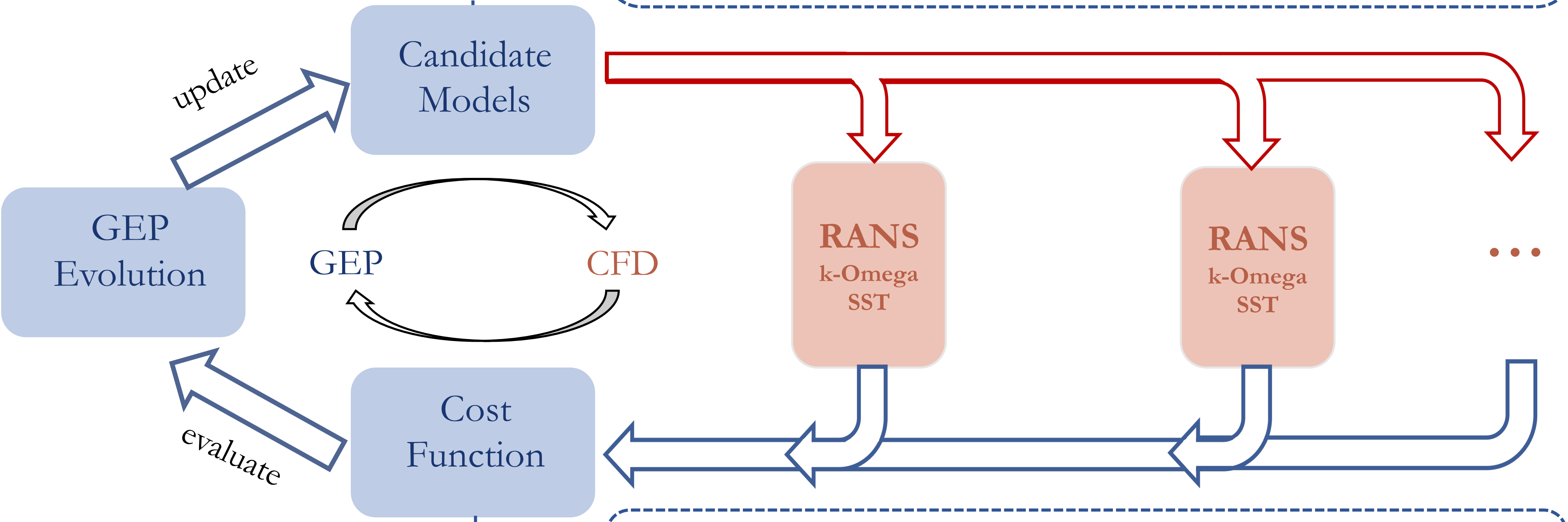
1.2 CFD-driven framework

Single-case CFD-driven framework

$$\tau_{ij} = \frac{2}{3}\rho k \delta_{ij} - 2\mu_T S'_{ij} + \boldsymbol{a_{ij}}\,(V^k_{ij}, I_k)\cdot 2\rho k$$

Based on Pope's theory for 2-D flow:

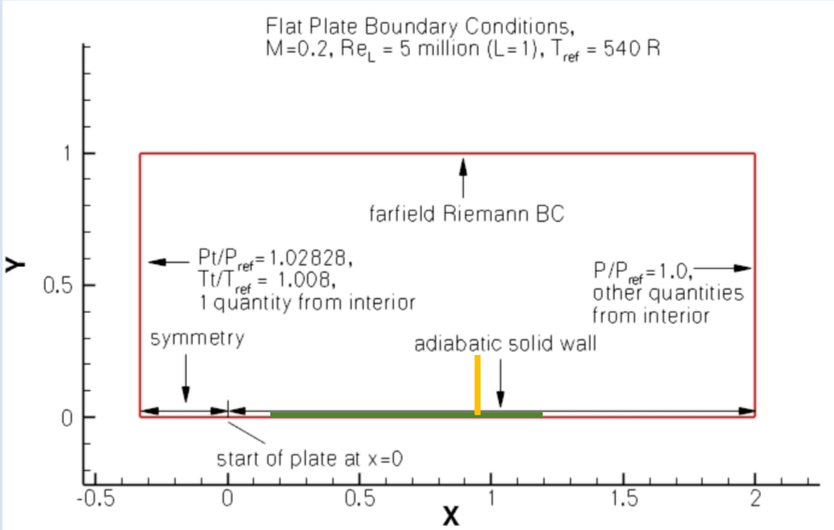
$$\boldsymbol{a_{ij}}\,(V^k_{ij}, I_k) = g1(I_1, I_2)V^1_{ij} + g2(I_1, I_2)V^2_{ij} + g3(I_1, I_2)V^3_{ij}$$



$$\Delta_y = \frac{1}{(y)}\int_0^{y_0} \left(\frac{A_{truth}(y) - A_{RANS}(y)}{\max_y(A)}\right)^2 dy$$

1.3 Single-case training results

1.3.1 Flat Plate



The geometry, boundary conditions for 2D flat plate

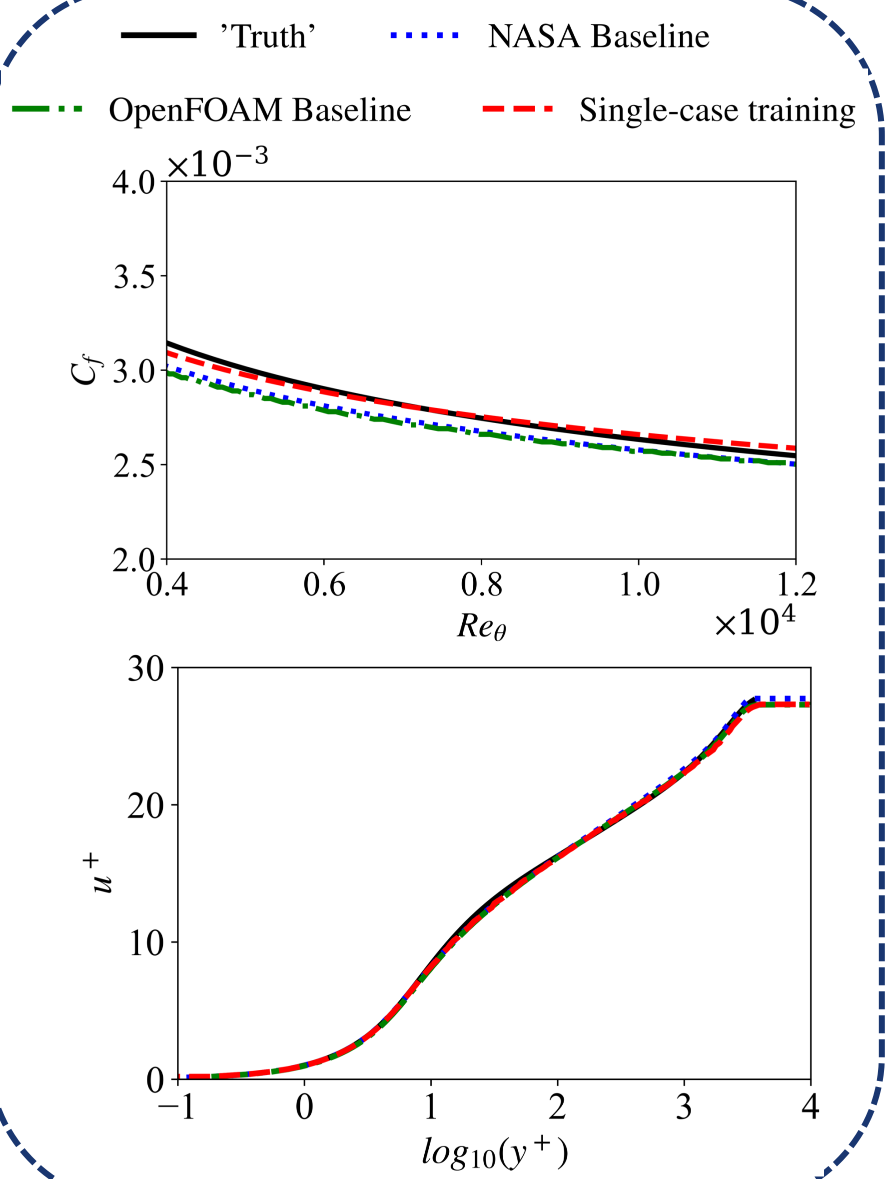
Cost function: friction coefficient along momentum thickness

Goal: (compare with **theory**)

1) Friction coefficient with x
2) Velocity law at x=0.97

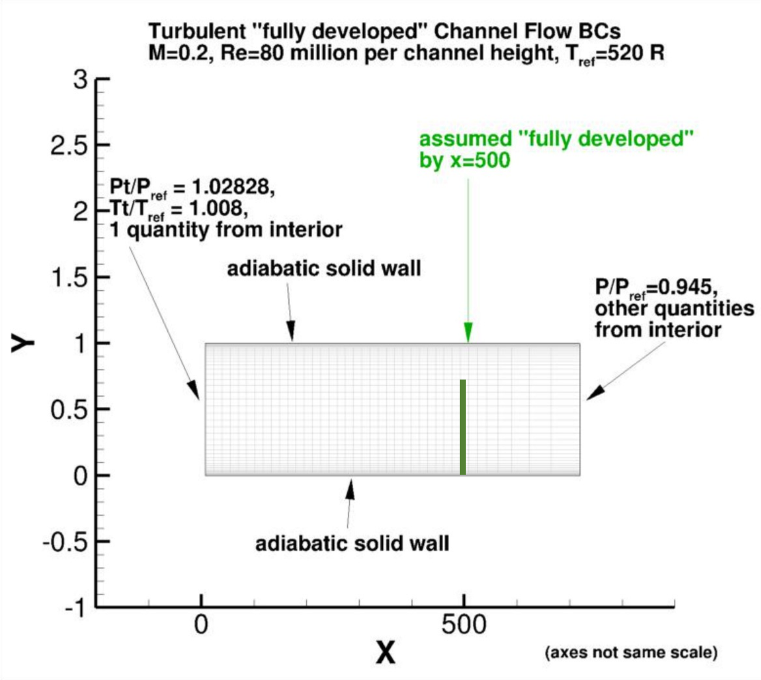
Anisotropic stress

$$\boldsymbol{a_{ij}}\,(V_{ij}^{k}, I_k) = \big(I_1(I_1 - 0.178I_2 - 0.7293)\big)V_{ij}^{1} + (4I_2 + 0.6143)V_{ij}^{2} + (0.089I_1 + 2.05073)V_{ij}^{3}$$



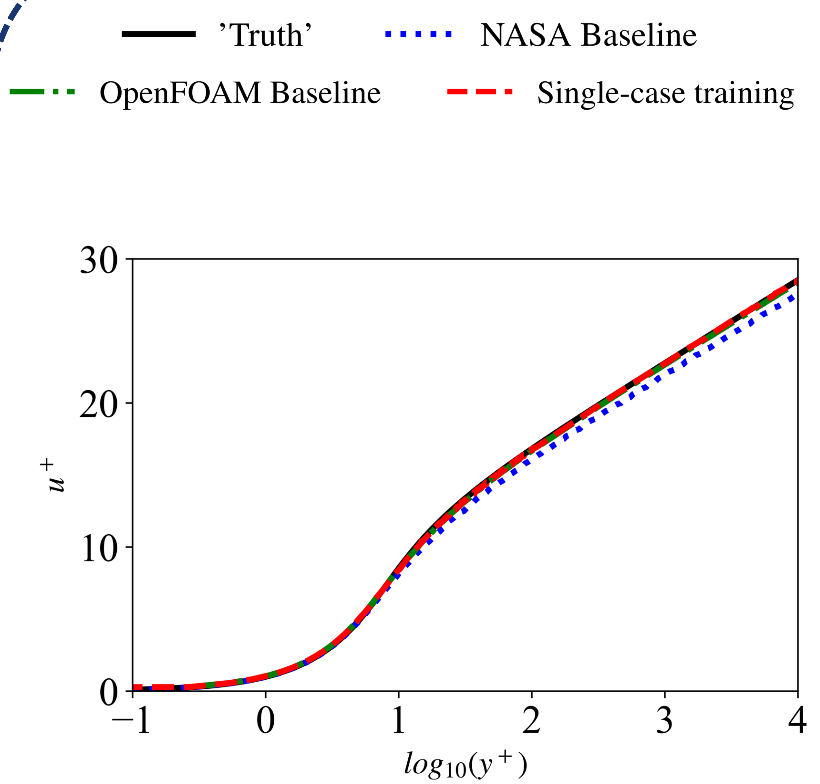
1.3 Single-case training results

1.3.2 Channel Flow at High Reynolds Number



The geometry, boundary conditions of channel flow

Cost function = goal :
the velocity law at x = 500

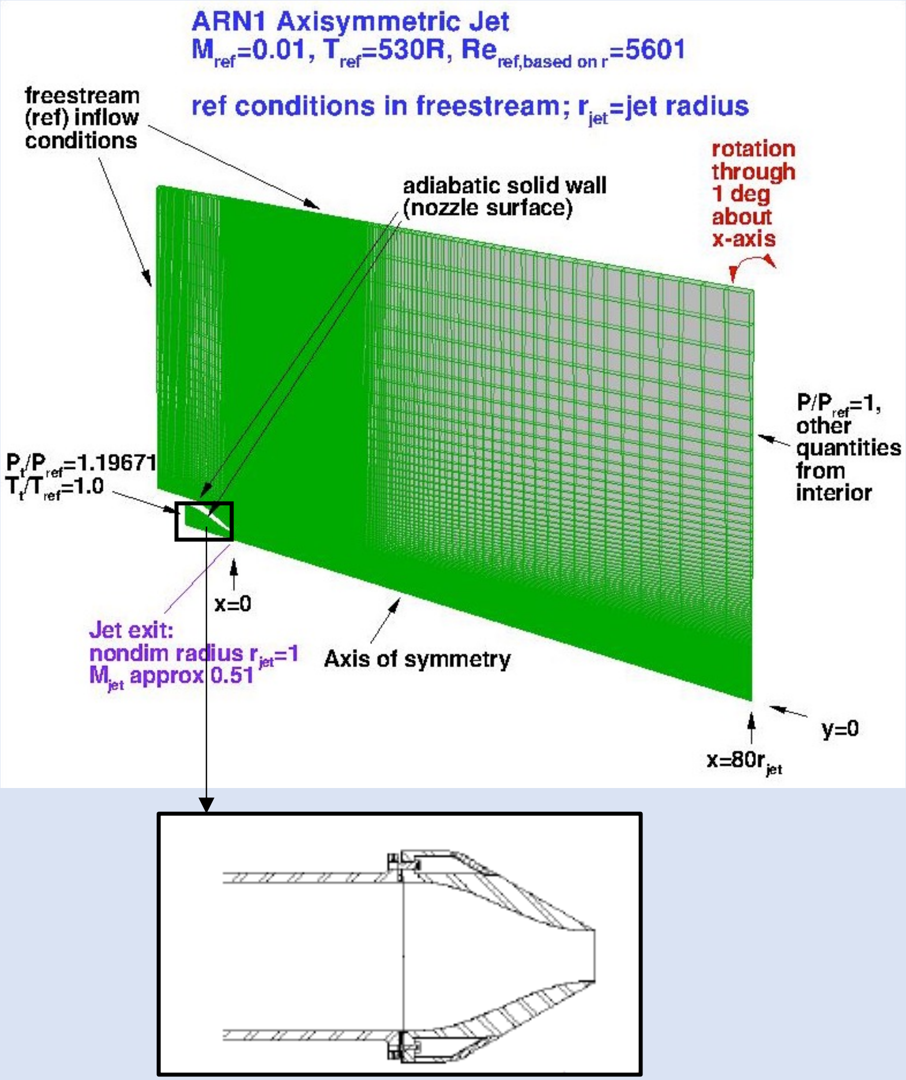


Anisotropic stress

$$\boldsymbol{a_{ij}}\,(V_{ij}^{k}, I_k) = (0.00784535)V_{ij}^{1} + (3I_1 + I_2 + 0.097)V_{ij}^{2} + (I_2)V_{ij}^{3}$$

1.3 Single-case training results

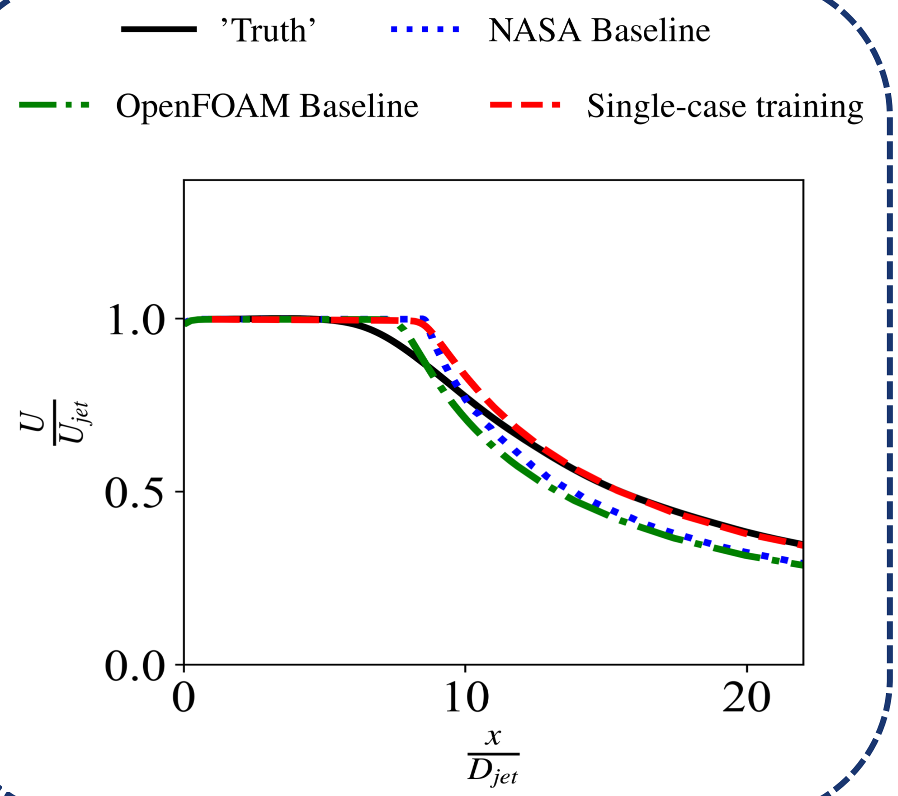
1.3.3 Axisymmetric Subsonic Jet



The geometry, boundary conditions and mesh for Axisymmetric subsonic jet

Cost function :
velocity profiles in the fully turbulent region x/Djet= **15 and 20**

Goal: (compare with experiment)
1) Velocity along x
2) Velocity profiles at 5 locations
3) Shear stress profiles at 5 locations

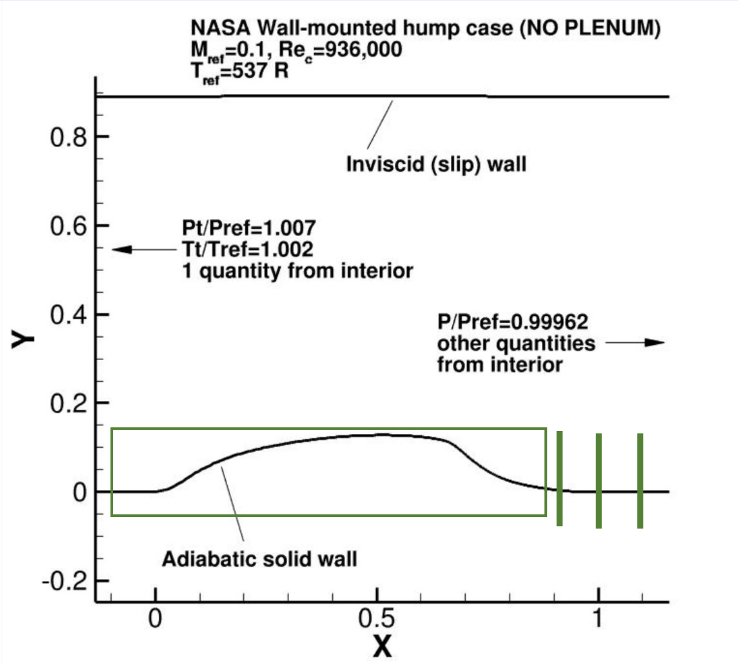


Anisotropic stress

$$\boldsymbol{a_{ij}}\left(V_{ij}^k, I_k\right) = (0.224885 + I_2)V_{ij}^1 + (I_1 + 0.055)V_{ij}^2 + (1.911)V_{ij}^3$$

1.3 Single-case training results

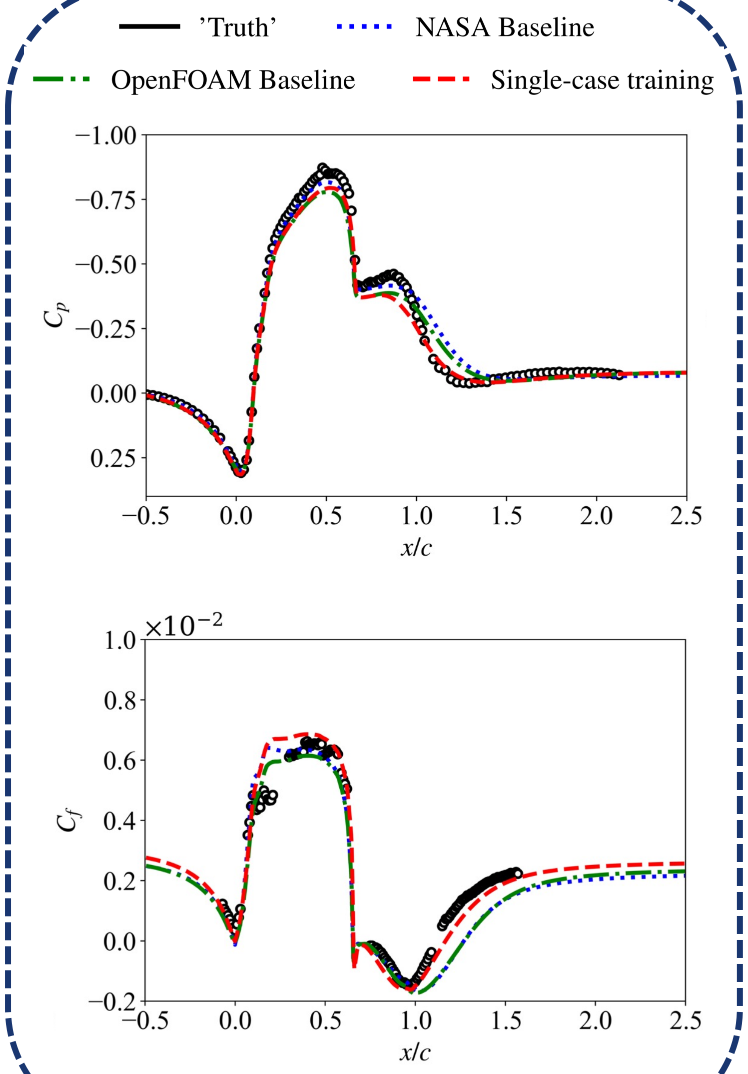
1.3.4 2D NASA Wall-Mounted Hump Separated Flow



The geometry, boundary conditions of hump

Cost function : the sum of velocity profiles near the bubble and pressure along hump

Goal: (compare with experiment)
1) Cp vs. x/c
2) Cf vs. x/c
3) Velocity profiles at 7 locations
4) Shear stress profiles at 7 locations



Anisotropic stress

$$\boldsymbol{a_{ij}}\left(V_{ij}^{k}, I_k\right) = (-0.15 - I_1 - 0.57I_2)V_{ij}^{1} + (-I_1 + I_2 - 2.061)V_{ij}^{2} + (I_1 I_2)V_{ij}^{3}$$

Contents

THE UNIVERSITY OF MELBOURNE

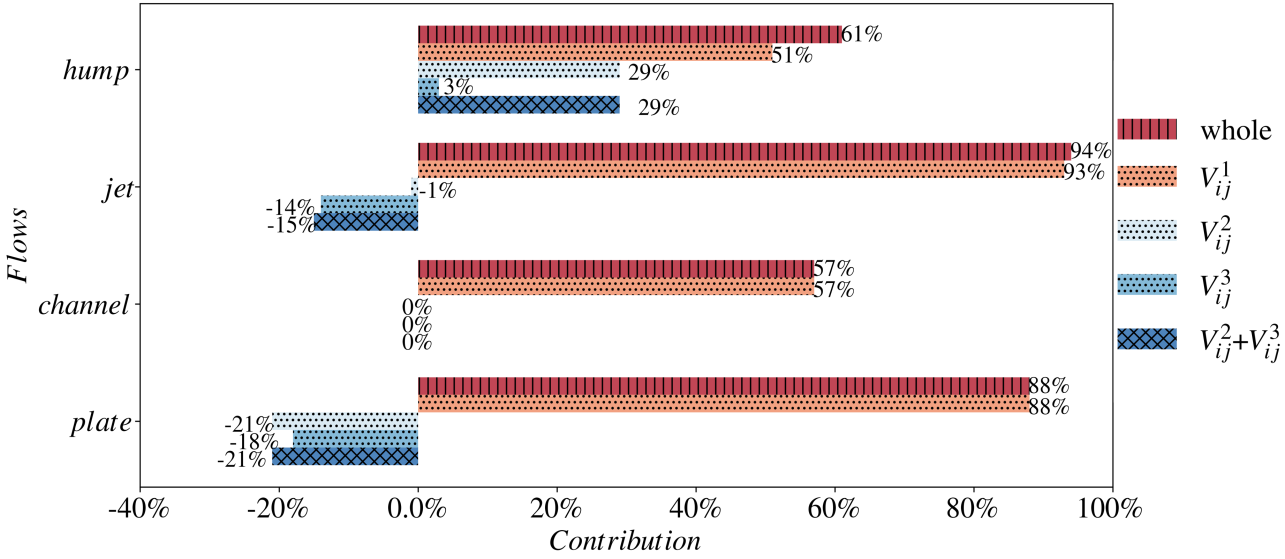
2.1 Difficulty of building a general model

2.1 The open-box expression analysis

➢ $\tau_{ij} = \frac{2}{3}\rho k \delta_{ij} - 2\mu_T S'_{ij} + \boldsymbol{a_{ij}}\,(V_{ij}^k, I_k) \cdot 2\rho k$

Table 1. The nonlinear term of Reynolds stress for every case

Flat Plate Case
$\boldsymbol{a_{ij}}\,(V_{ij}^k, I_k) = (I_1(I_1 - 0.178I_2 - 0.7293))V_{ij}^1 + (4.0I_2 + 0.6143)V_{ij}^2 + (0.089I_1 + 2.05073)V_{ij}^3$
Channel Flow with High Re Number Case
$\boldsymbol{a_{ij}}\,(V_{ij}^k, I_k) = (0.00784535)V_{ij}^1 + (3.0I_1 + I_2 + 0.097)V_{ij}^2 + (I_2)V_{ij}^3$
Axisymmetric Subsonic Jet Case
$\boldsymbol{a_{ij}}\,(V_{ij}^k, I_k) = (I_2 + 0.224885)V_{ij}^1 + (I_1 + 0.055)V_{ij}^2 + (1.911)V_{ij}^3$
Wall-Mounted Hump Separation Flow
$\boldsymbol{a_{ij}}\,(V_{ij}^k, I_k) = (-0.15 - I_1 - 0.57I_2)V_{ij}^1 + (-I_1 + I_2 - 2.061)V_{ij}^2 + (I_1 I_2)V_{ij}^3$



1. Major contribution comes from V_{ij}^1 term

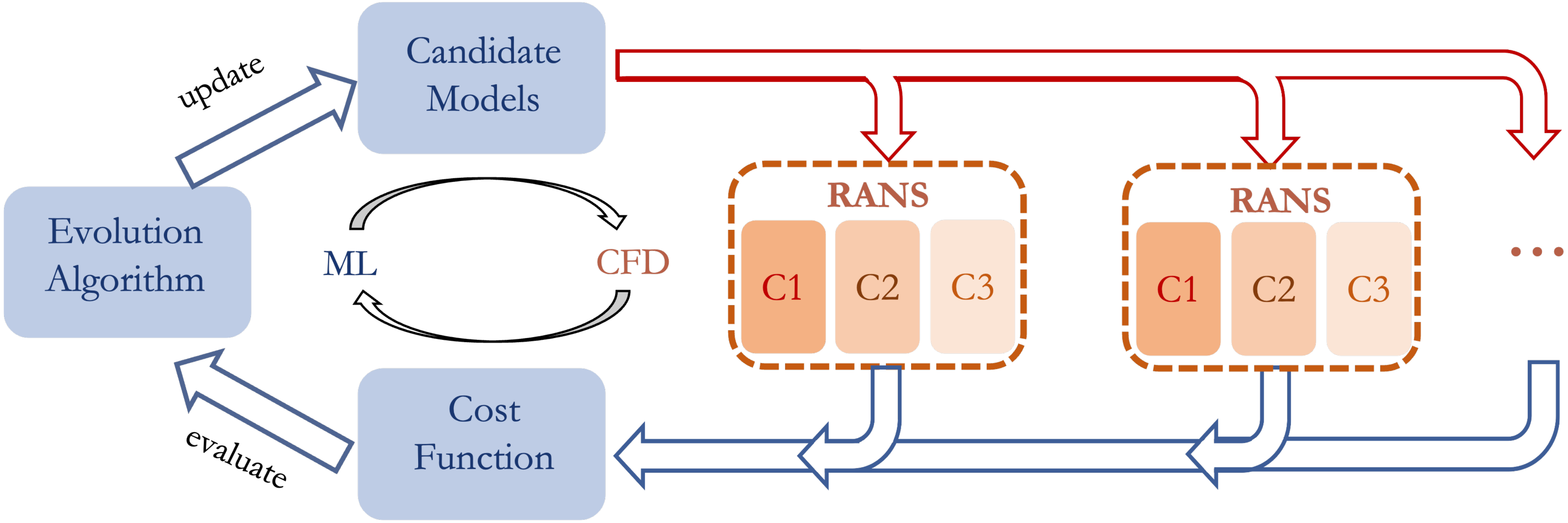
2. The magnitude of I_1 and I_2 are small. Hence, the coefficient inside the V_{ij}^1 term contribute most. **However, both negative and positive values appear**, which leads to compromised results.

2.2 Strategies of building a general model

2.2.1 The framework of multi-case CFD-driven training framework

C1 C2 C3 represent different cases

Different flows with different conditions



- Reduce computation cost: 16 cores for up to 4 days

2.2 Strategies of building a general model

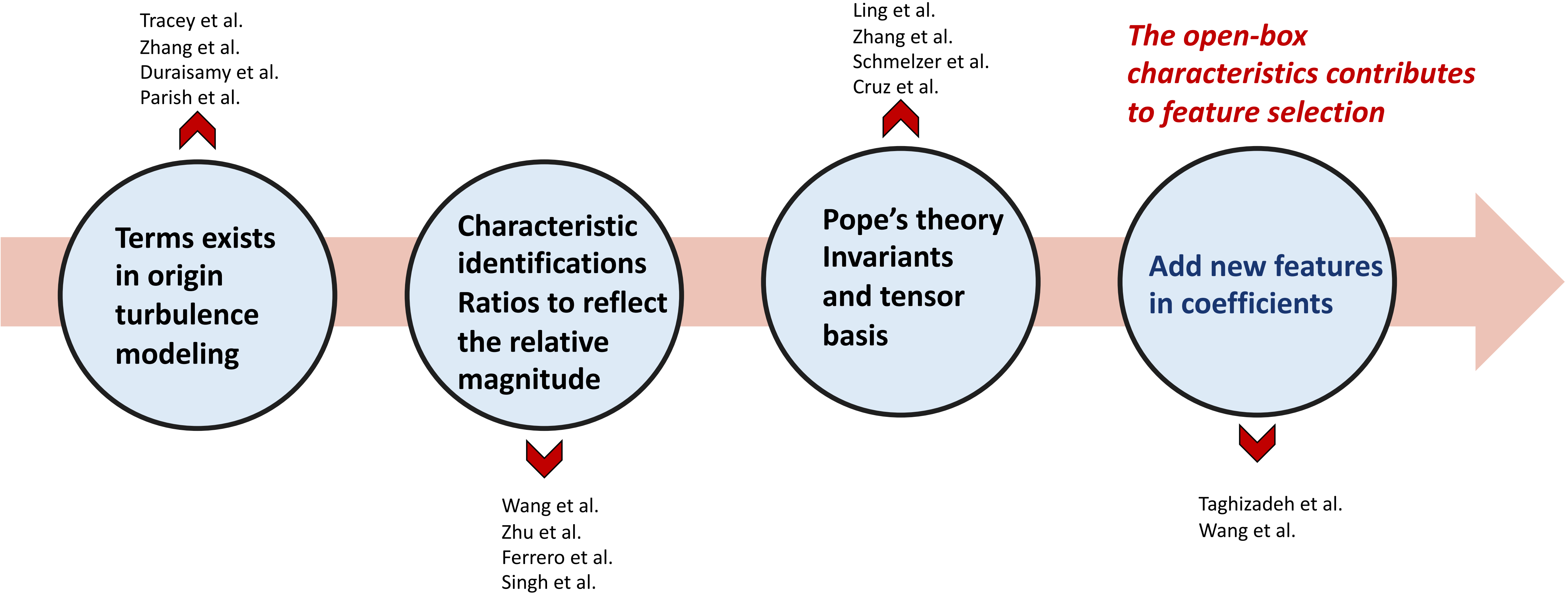
THE UNIVERSITY OF MELBOURNE

2.2.1 Add flow features in the coefficients

Scalar invariants

Nonlinear correction term $a = f(I_i)V_j$ → Tensor bases

The open-box characteristics contributes to feature selection



Literature review of the selection of the input features

THE UNIVERSITY OF MELBOURNE

2.2.1 Add flow features in the coefficients

Table 1: Summary of the added input features

Flow features	Description	Denotation
N1	Reynolds number based on wall distance	$\min(\frac{\sqrt{k}d}{50\upsilon}, 2)$
N2	Pressure gradient along the streamline	$U\frac{\partial P}{\partial x}$
N3	Switch function F_2 in $k-\omega$ SST	F_2

$$F_1 = tanh(arg_1^4); arg_1 = min(max((\frac{\sqrt{k}}{\beta^* \omega y}); \frac{500\nu}{y^2\omega}); \frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}y^2});$$

$$CD_{k\omega} = max(\frac{2\rho\sigma_{\omega 2}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}; 1.0e^{-10})$$

$$F_2 = tanh(arg_2^2); arg_2 = max(2\frac{\sqrt{k}}{\beta^* \omega y}; \frac{500\nu}{y^2\omega})$$

2.2.3 Model an additional turbulence production or dissipation term

$$\rho\frac{\partial k}{\partial t}+\rho U_j\frac{\partial k}{\partial x_j}=\boxed{\tau_{ij}}\frac{\partial U_i}{\partial x_j}-\rho\epsilon+\frac{\partial}{\partial x_j}\left[\mu\frac{\partial k}{\partial x_j}-\frac{1}{2}\rho\overline{u'_i u'_i u'_j}-\overline{p' u'_j}\right]+R$$

Unsteady term, convection, production, dissipation, molecular diffusion, turbulent transport, pressure diffusion

Multi-expression training

$$a_{ij}\,(V_{ij}^{k},I_k)=g1(I_1,I_2)V_{ij}^{1}+g2(I_1,I_2)V_{ij}^{2}+g3(I_1,I_2)V_{ij}^{3}$$

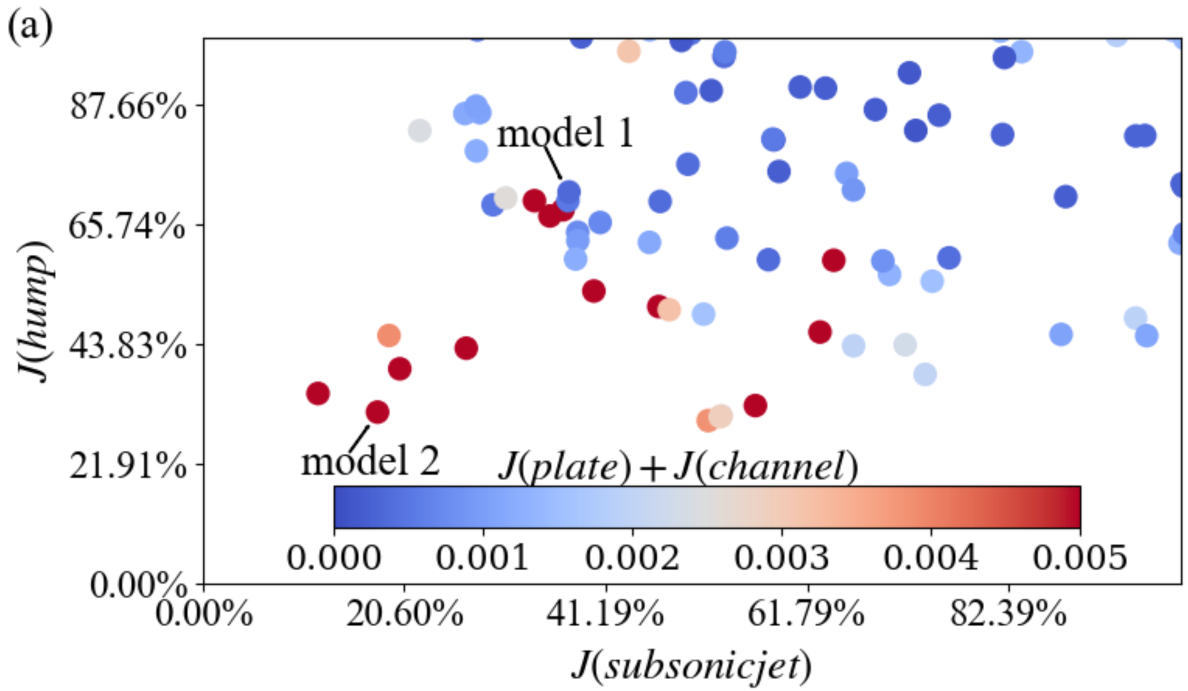
$$R_{ij}\,(V_{ij}^{k},I_k)=g4(I_1,I_2)V_{ij}^{1}+g5(I_1,I_2)V_{ij}^{2}+g6(I_1,I_2)V_{ij}^{3}$$

Schmelzer M , Dwight R P , Cinnella P. Discovery of Algebraic Reynolds-Stress Models Using Sparse Symbolic Regression[J]. 2019.

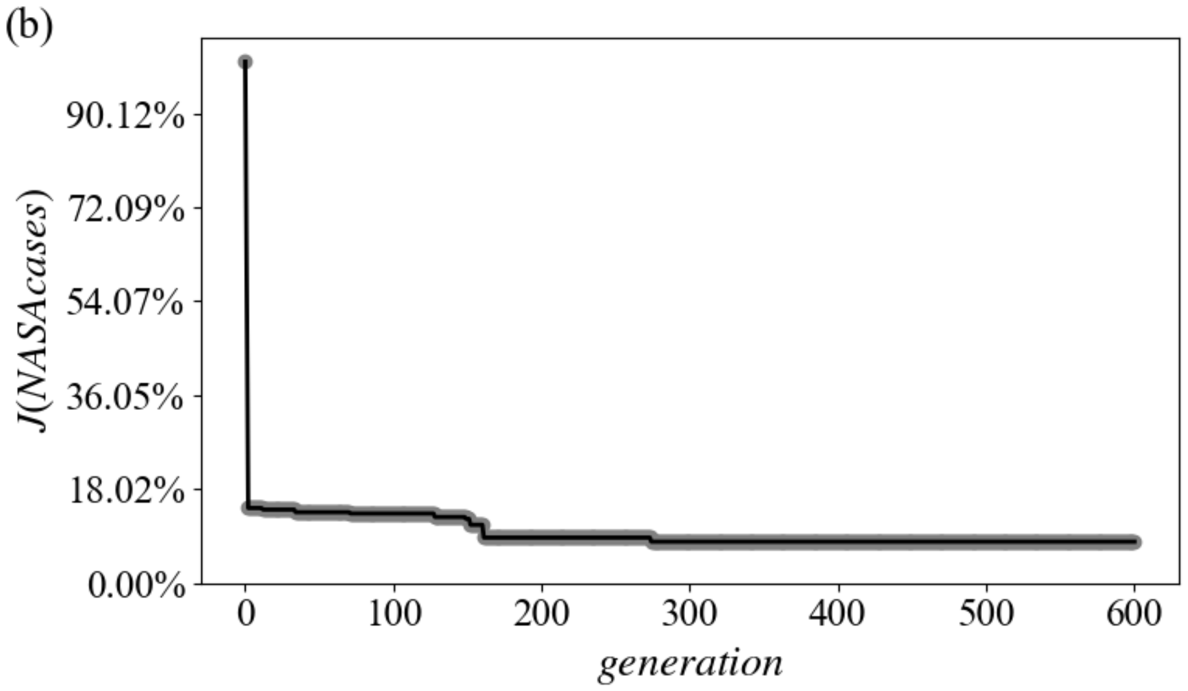
Contents

3.1 Multi-case training result

3.1.1 Models selection according to the uncertainty of ’truth’ for the flat plate case



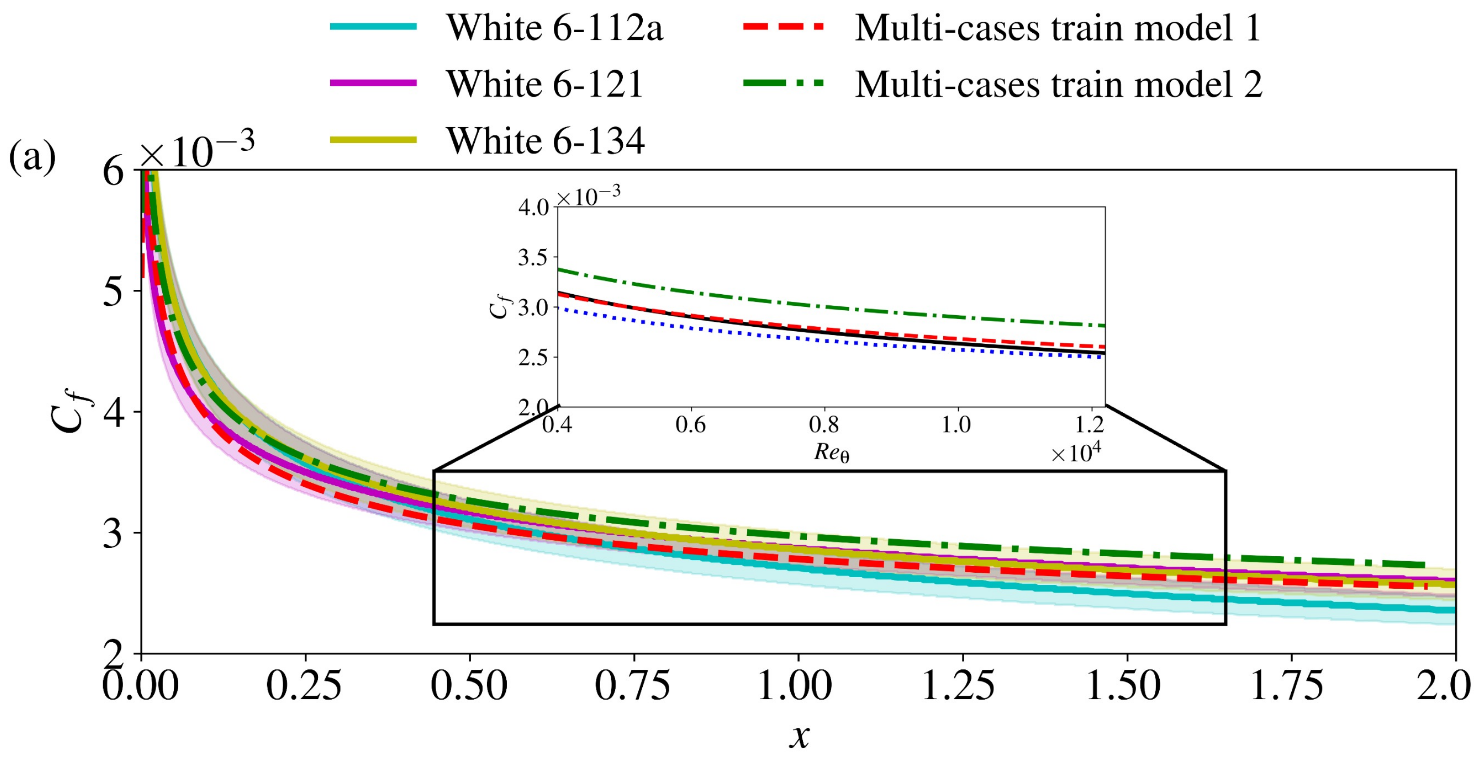
(a) Cost function values for the four cases



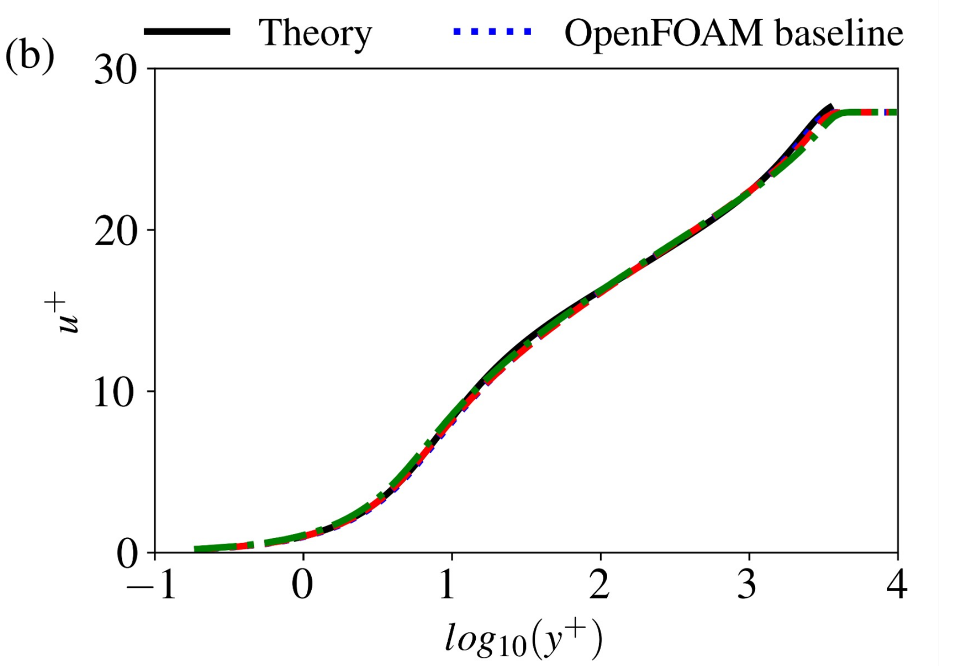
(b) Evolution of the sum of cost function values

3.1 Multi-cases training result

3.1.2 Result of multi-case training for the flat plate



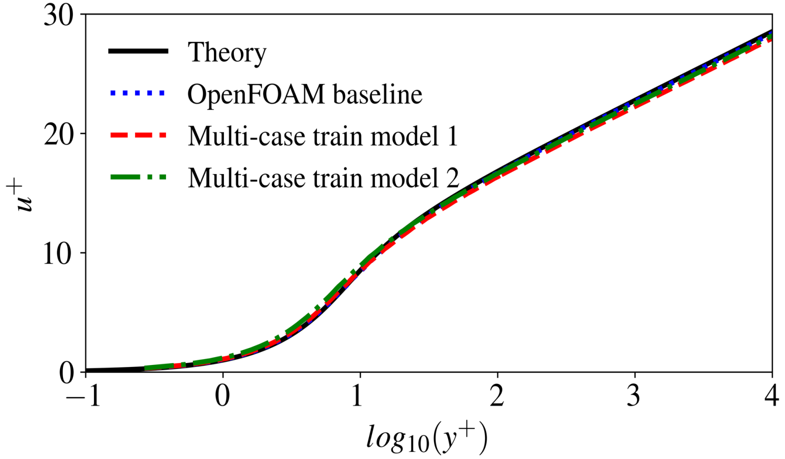
(a) The friction coefficient along plate



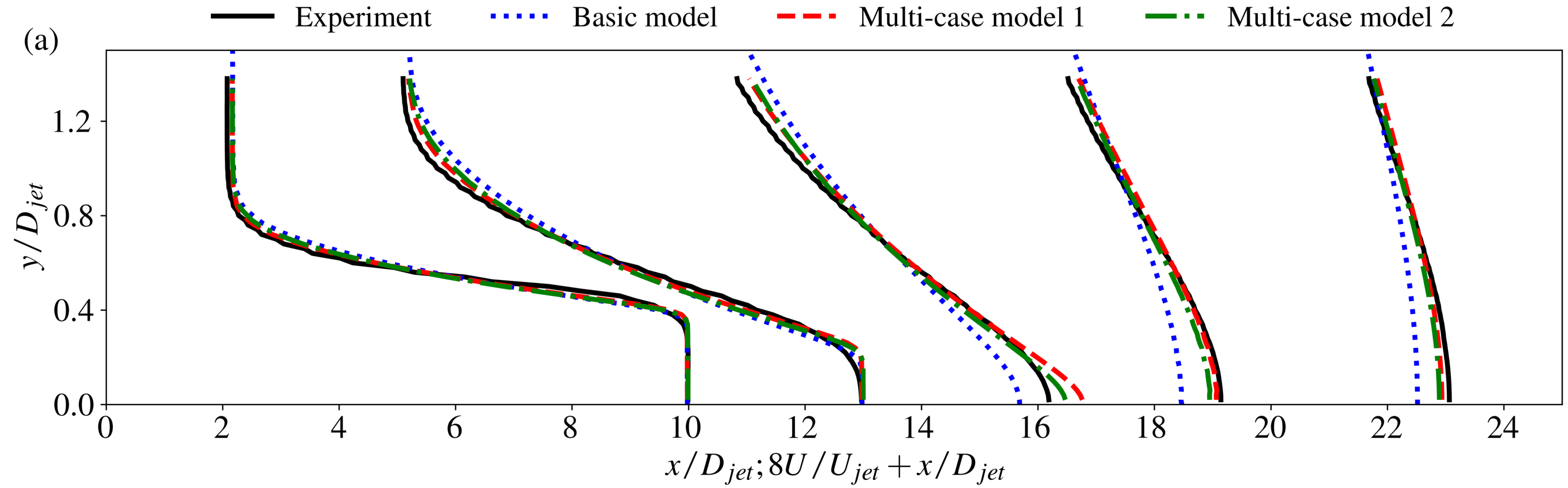
(b) The velocity law at x=0.97

3.1 Multi-cases training result

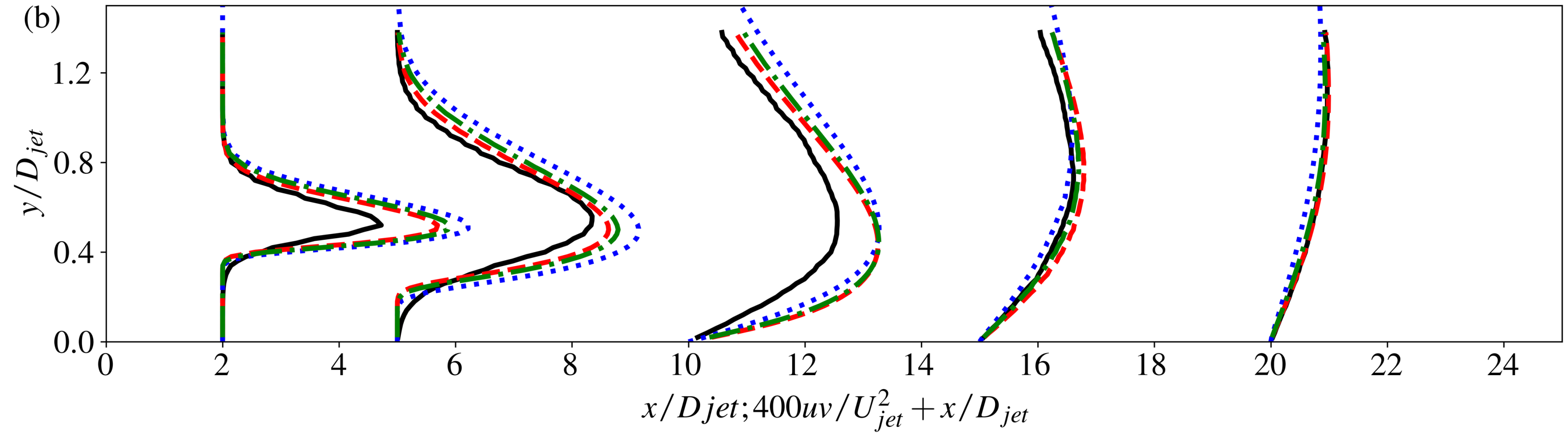
3.1.2 Result of multi-case training for channel and subsonic jet



The velocity law of channel flow at x = 500



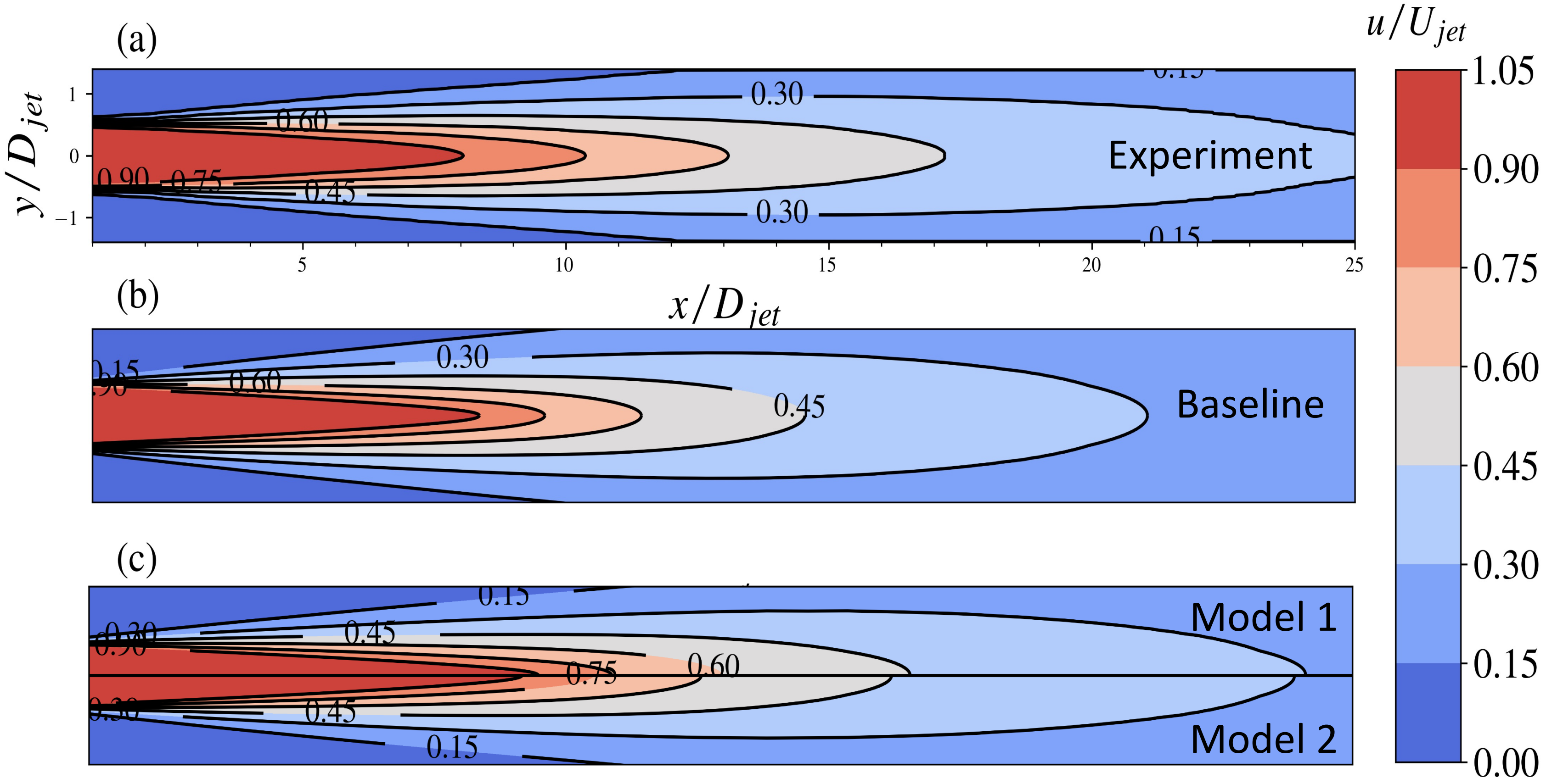
(a) The velocity profiles of subsonic jet at x/Djet = 2, 5, 10, 15, 20



(b) The shear stress profiles of subsonic jet at x/Djet = 2, 5, 10, 15, 20

3.1 Multi-cases training result

3.1.2 Flow field result of multi-case training for <u>subsonic jet</u>

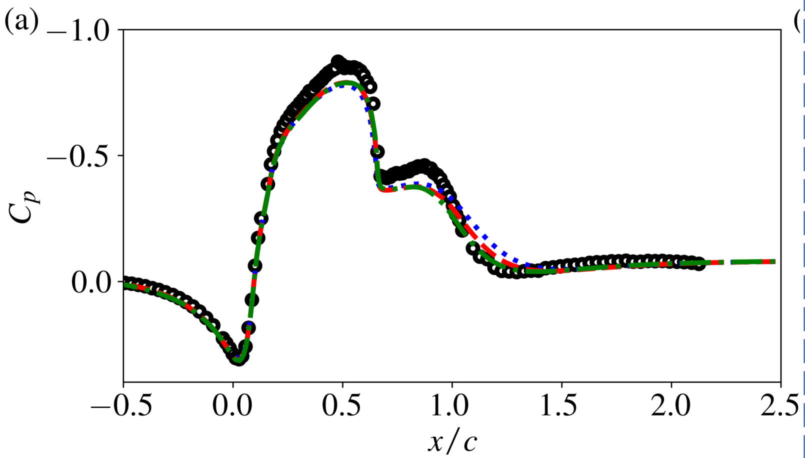


- Both the width and length of jet simulation improved **by reducing the diffusion in the whole computation domain**

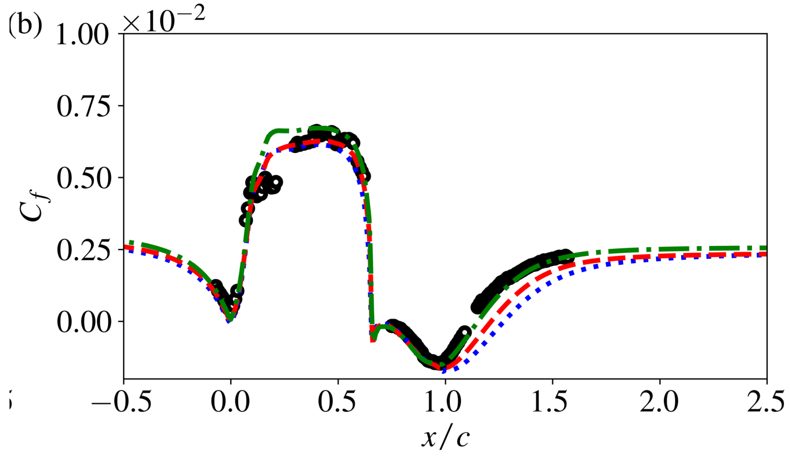
3.1 Multi-cases training result

3.1.2 Result of multi-case training for hump

Along the surface

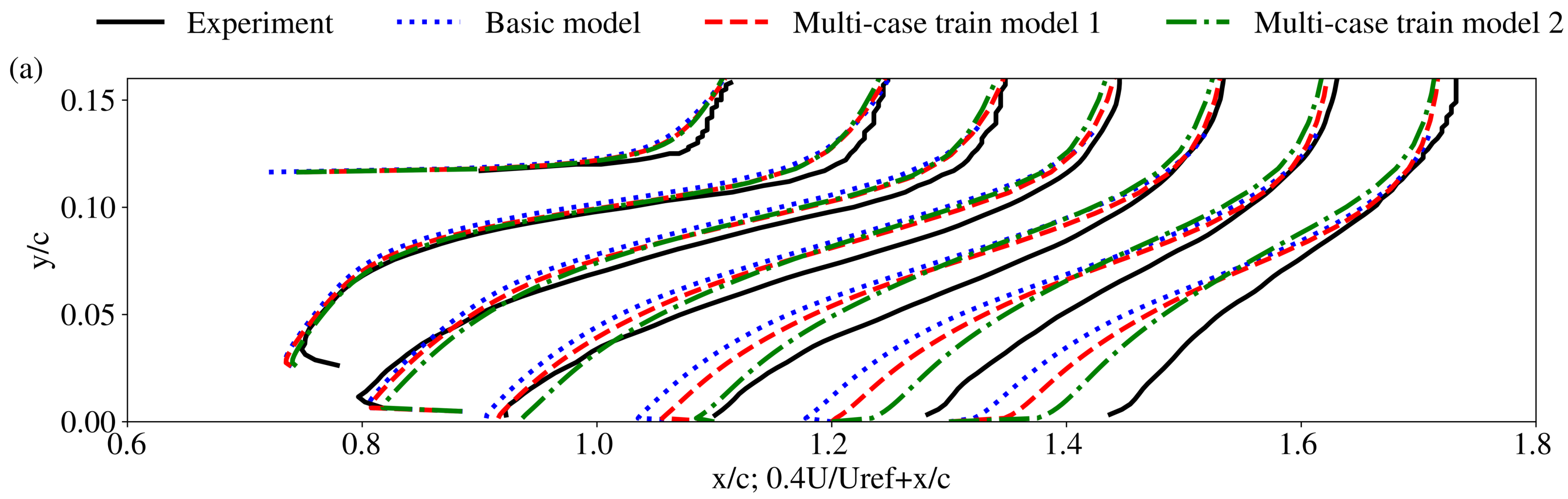


(a)Pressure coefficient along hump surface

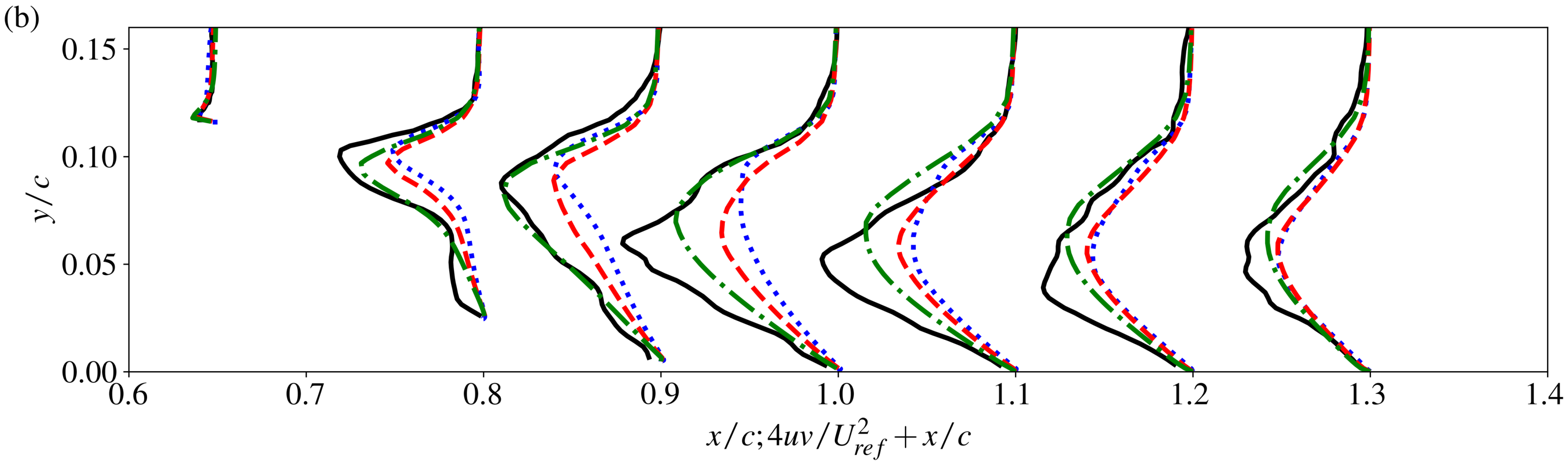


(b) Friction coefficient along hump surface

Profiles at 7 locations



(a) The X-Velocity profiles at x/c = 0.65, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3

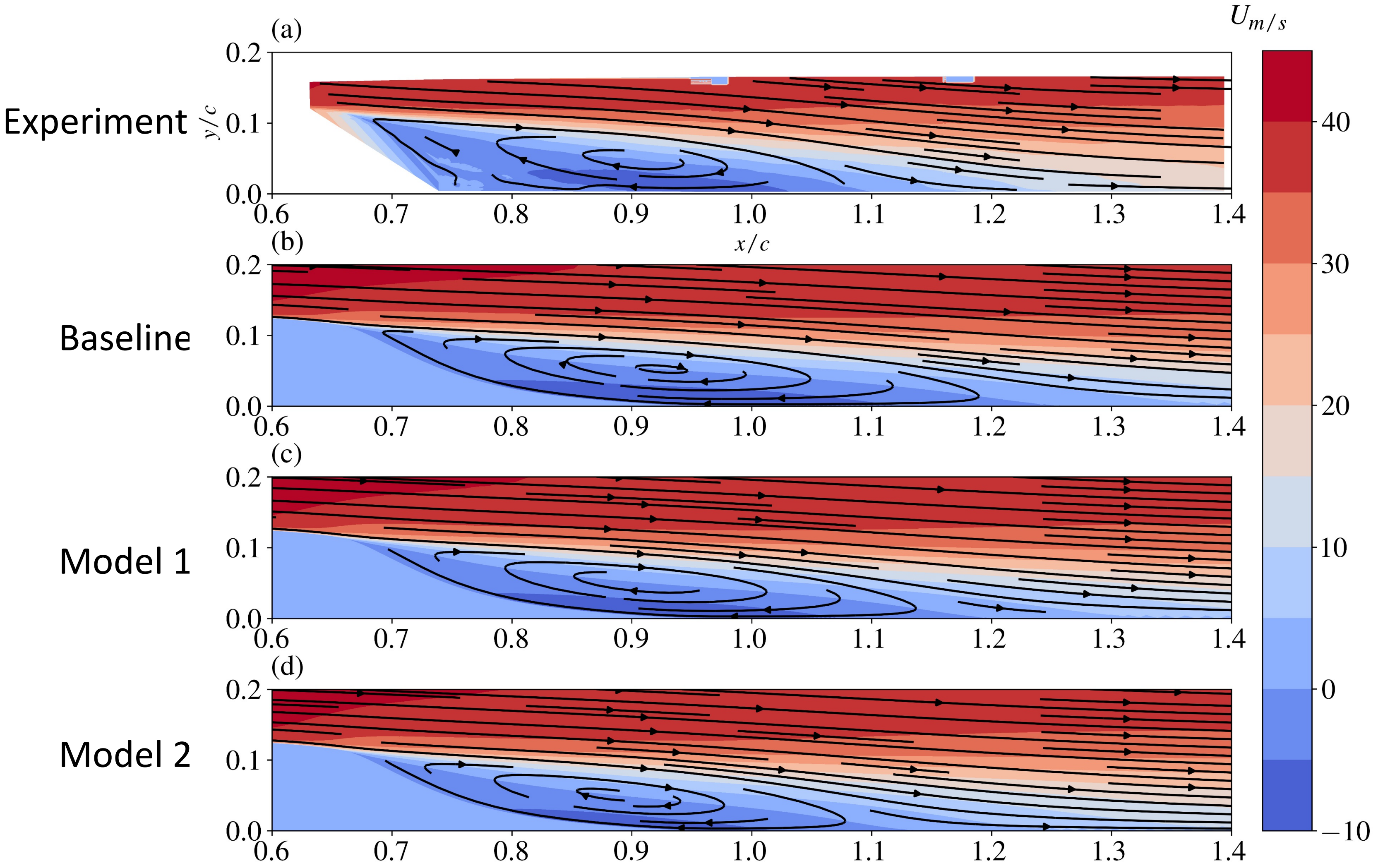


(b)The shear stress profiles at x/c = 0.65, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3

3.1 Multi-cases training result

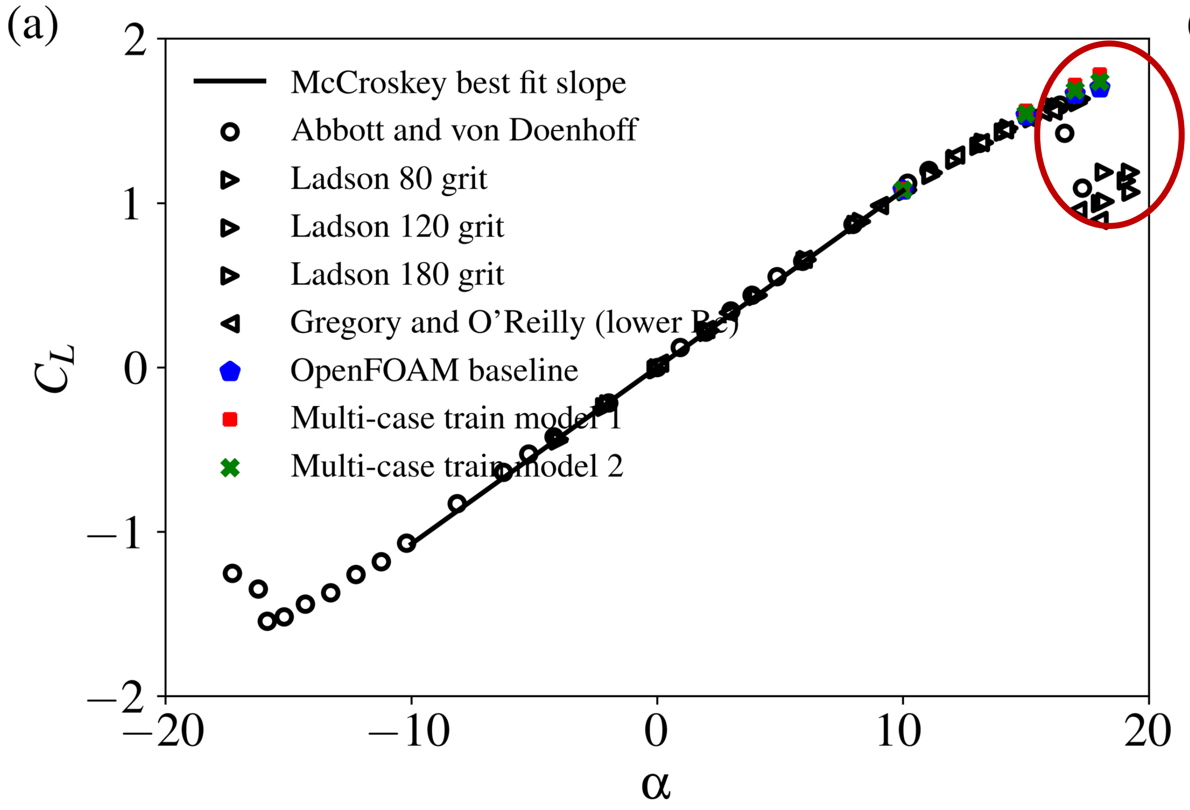
3.1.2 Result of multi-cases training for hump

- The prediction of reattachment location agrees fairly well with the experiment, which is a well-known drawback of the baseline model.

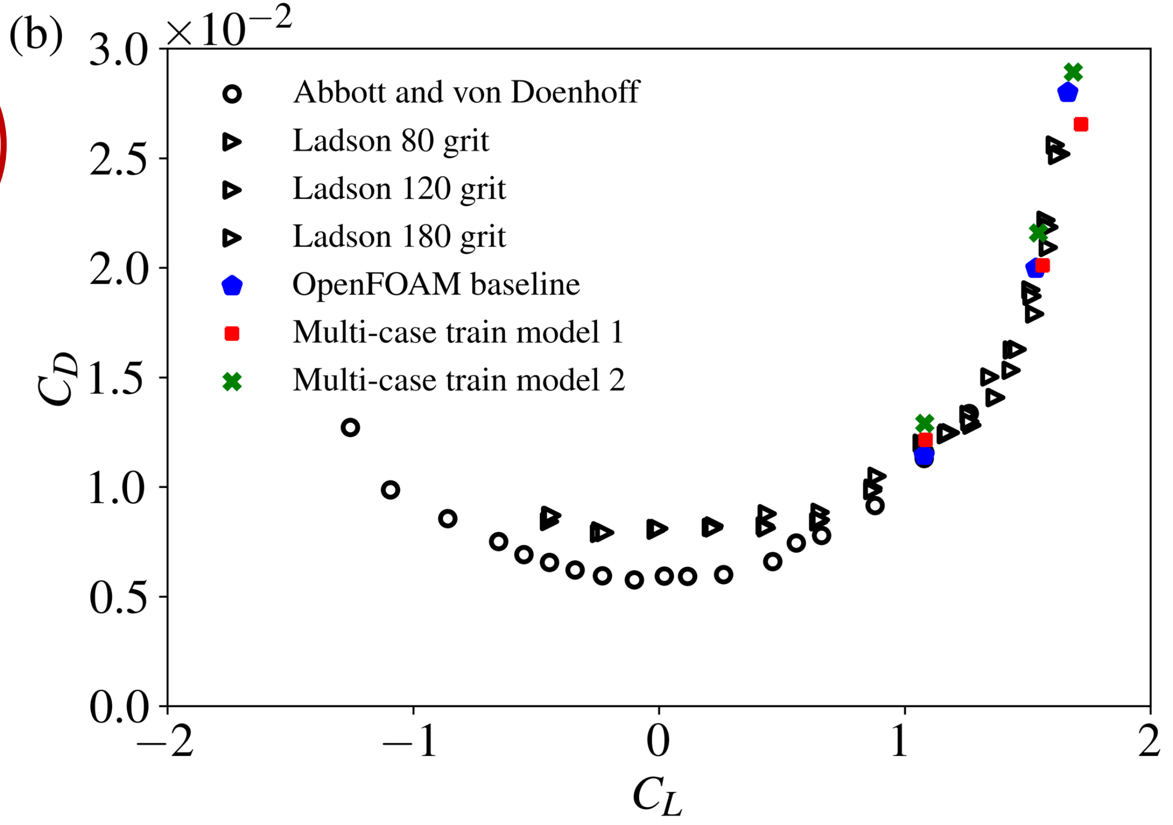


3.2 2D NACA 0012 Airfoil Validation Case (4 separate cases (angles of attack = 10, 15, 17, 18 deg))

Stall phenomenon



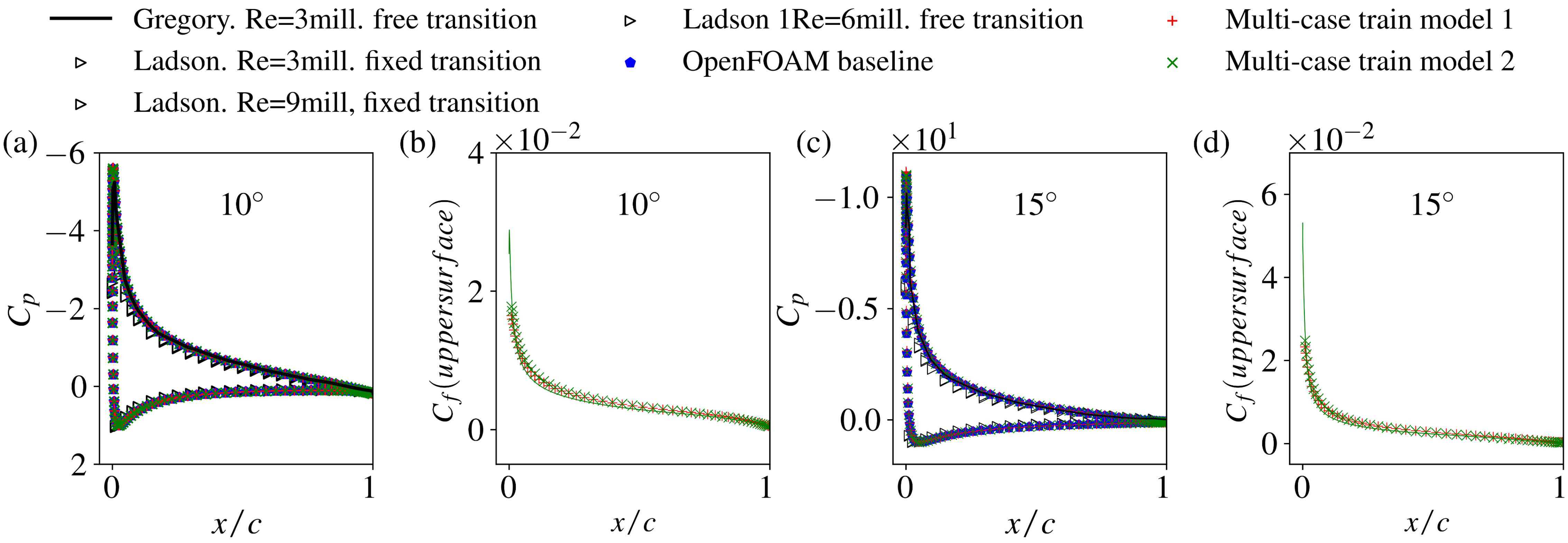
(a) Lift coefficient at different angles of attack



(b) Lift coefficient vs. drag coefficient

3.2 Posterior tests

3.2 2D NACA 0012 Airfoil (4 separate cases (angles of attack = 10, 15, 17, 18 deg))



➢ The built models improve flows with large discrepancies to ‘truth’ while not deteriorating flows outside the training data set.

Contents

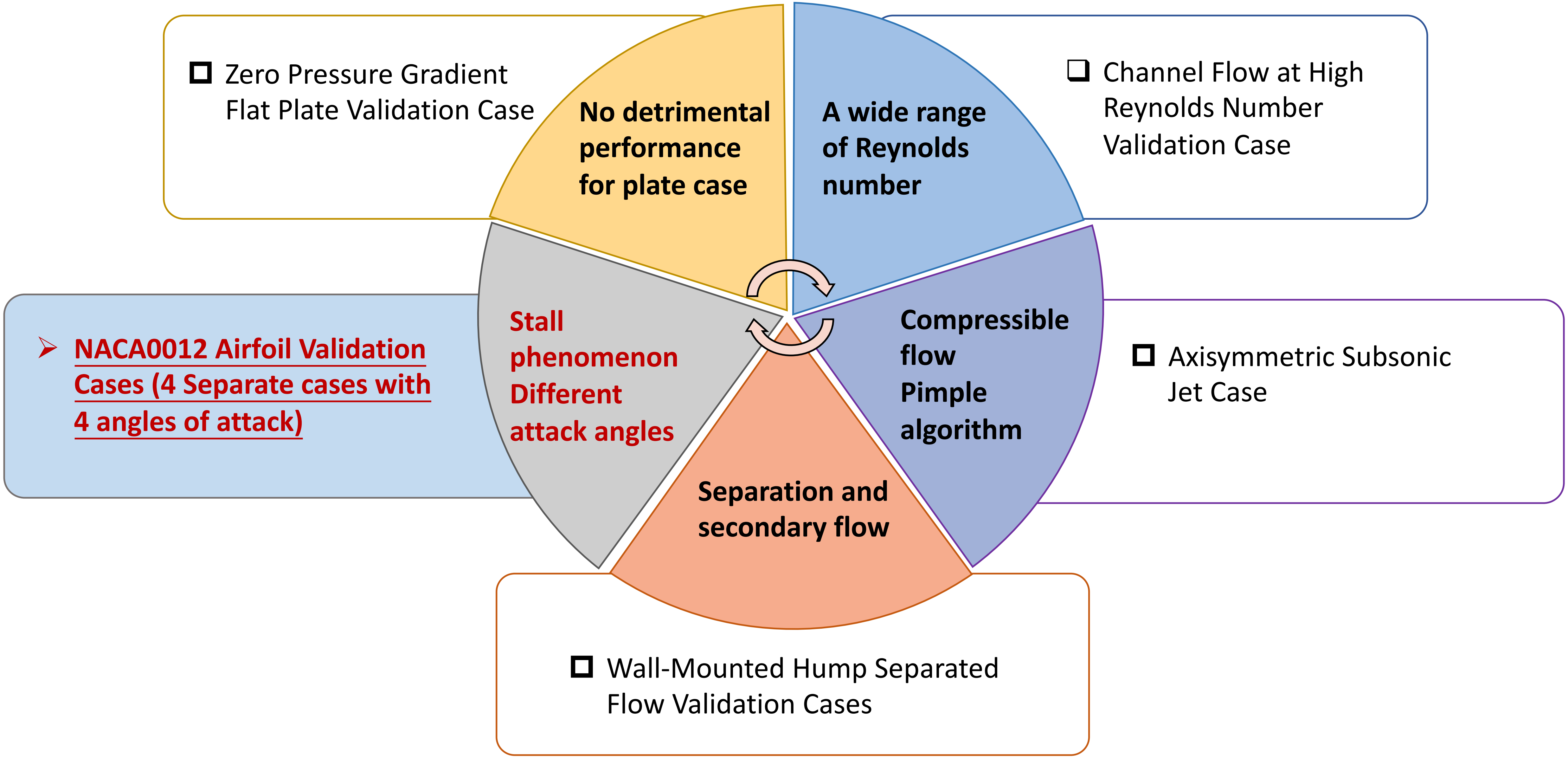
4. Contribution

- **Analyze the difficulties to build a general model by single-case training**
- **Extend the single to multi-case CFD training framework and try to reduce the computation cost**
- **Insert additional flow features to supplement Pope’s theory to capture different trends of corrections.**

Other slides

1.1 The research objectives

Testing cases: NACA 0012 airfoil cases with 4 angles of attack



1.1 The research objectives

Numerical cases division

Training cases: plate; channel; jet; hump

Testing cases: NACA 0012 airfoil with 4 angles of attack

- Not enough data at the stall
- Four cases training leads to high computation cost
- Need testing cases

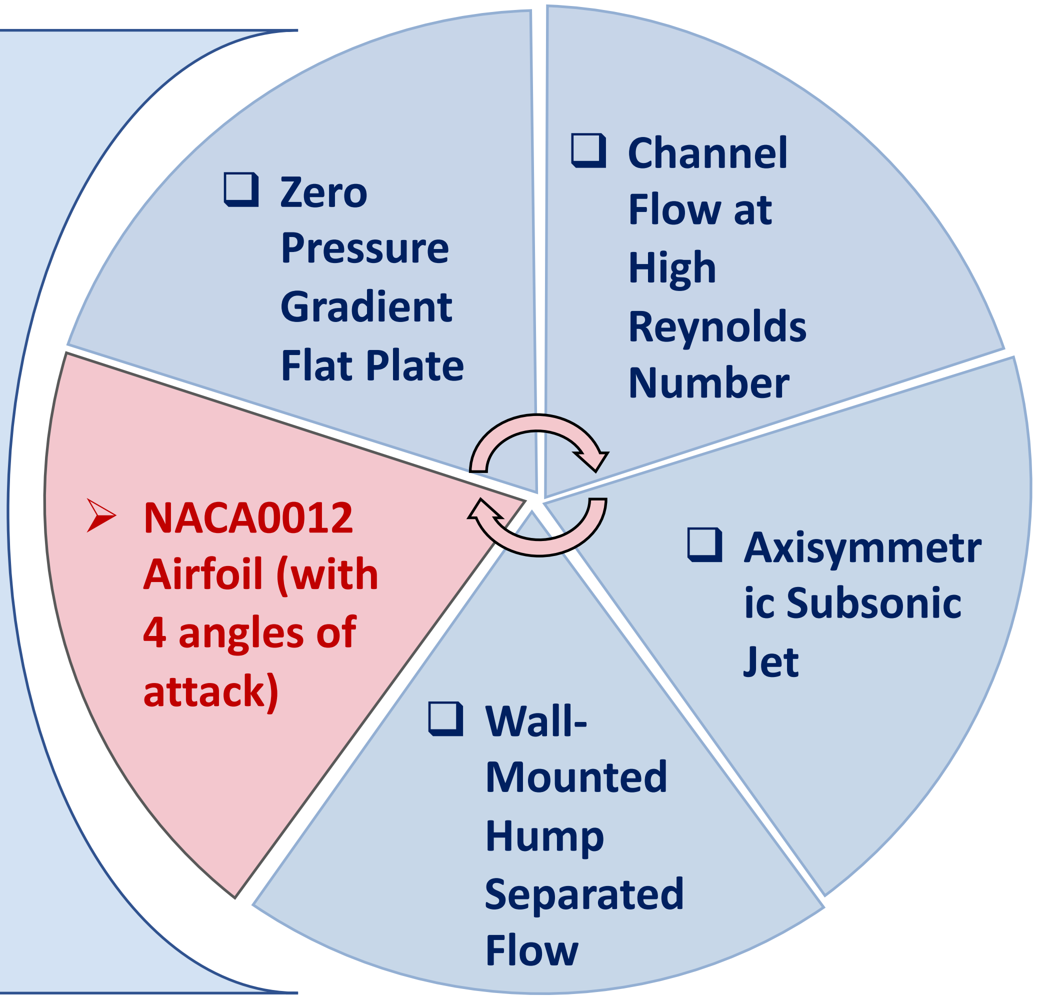


Figure 1: Components of training and testing cases

4. Discussion

THE UNIVERSITY OF MELBOURNE

4.1 Non-dimensionalization and scaling way

Advantages

Disadvantage

1. $\frac{X-\mu}{\sigma}$
   - Disadvantage: Not physical

2. $\frac{X}{\|X\|+\|Y\|}$
   - Advantage: Easy to scale down [-1,1]
   - Disadvantage: Change the feature distribution

3. $\frac{X-min(X)}{max(X)-min(X)}$
   - Advantage: Easy to scale down [0,1]
   - Disadvantages:
     - ❑ Lost the sign information
     - ❑ Ignore the difference among the cases

4. $2\frac{X-min(X)}{max(X)-min(X)}-1$
   - Advantage: Easy to scale down [-1,1]
   - Disadvantages:
     - ❑ Lost the original sign information
     - ❑ Ignore the difference among the cases

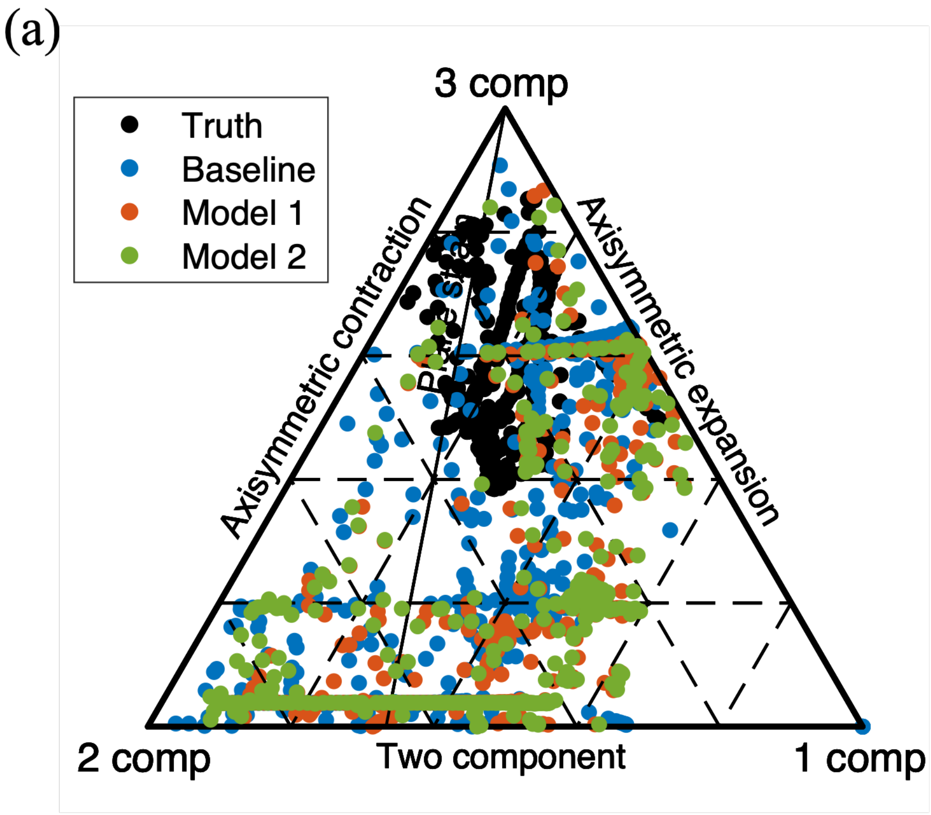
5. $\frac{X}{max(X)}$
   - Advantages:
     - ✓ Easy to scale down [-1,1]
     - ✓ Retain both the physical and sign information
   - Disadvantage: Lost difference among cases

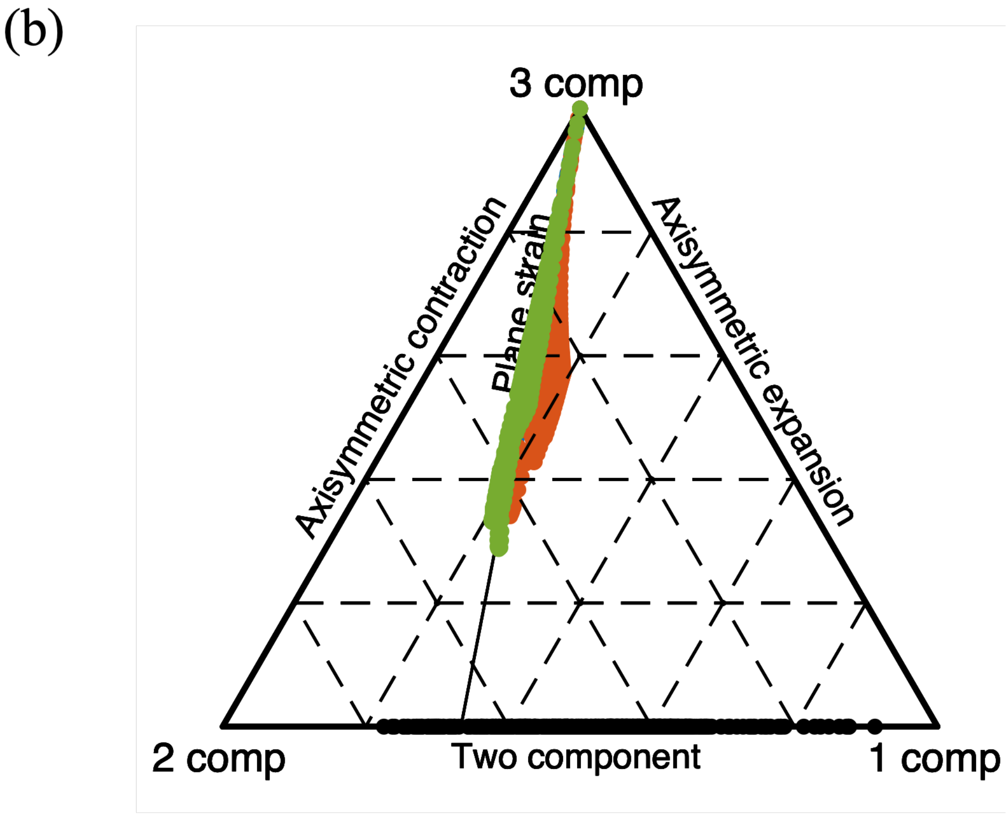
6. $\frac{X}{\rho^m U^n L^k}$
   - Advantages:
     - ✓ Retain sign information
     - ✓ Remain the difference among cases
   - Disadvantages:
     - ❑ Hard to scale down
     - ❑ Lost models' generalibility

?

4. Discussion

4.2 Realizability – Barycentric map





(a) Barycentric map of jet at x/Djet = 2,5,10,15,20

(b) Barycentric map of hump at x/c = 0.65, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3