Challenge entry: SpaRTA with classification

July 2022 - Turbulence Modeling: Roadblocks, and the Potential for Machine Learning

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Outline of methodology

Baseline model $k - \omega$ SST, then:

1. Use k-**frozen approach** to deduce local corrective terms for cases with PIV/LES reference data (`ASJ`, `2DWMH`)
2. Train classifier model, to predict:
   - Cases `2DZP`, `2DFDC`: classifier inactive everywhere
   - Cases `ASJ`, `2DWMH`: active where correction exceeds threshold
   - Case `2DN00`: unknown
3. Train correction model to predict corrections (if classifier active)
   - Symbolic regression for parsimoneous models.

Based on SpaRTA ("Sparse Regression of Turbulence Anisotropy"):

Schmelzer, RPD, and Cinnella (2019). "Discovery of Algebraic Reynolds-Stress Models Using Sparse Symbolic Regression". In: *FTC* 104.2-3, pp. 579–603. DOI: 10.1007/s10494-019-00089-x

1. k-frozen approach

Given **full-field** LES/PIV mean fields:

Main idea (k-frozen)

1. Substitute all *known* LES/PIV quantities ($\boldsymbol{U}^\star, \boldsymbol{\tau}^\star$) into the RANS equations with a baseline model (here SST).
2. Deduce unknown quantities (ω, ν_T) by solving equations.
3. RANS equations are not satisfied exactly $\implies$ residuals are (desired) corrective fields.

Specifically, introduce residual into the SST k-equation (R):

$$U^\star{}_j \partial_j k^\star = \mathcal{P}_k^\star - \beta^* k^\star \omega + \partial_j \left[(\nu + \nu_t \sigma_k) \partial_j k^\star \right] + R,$$

$$U^\star{}_j \partial_j \omega = \frac{\gamma}{\nu_T} \left(\mathcal{P}_k^\star + R \right) - \beta \omega^2 + \partial_j \left[\left(\nu + \sigma_\omega \nu_T \right) \partial_j \omega \right] + $$

$$+ 2(1 - F_1) \frac{\sigma_{\omega 2}}{\omega} \partial_j k^\star \, \partial_j \omega$$

1. k-frozen approach

Given ν_T from above, can specify a "residual" in the anisotropy (b^{Δ}_{ij}) compared to Boussinesq:

$$\tau^{\star}_{ij} = 2k^{\star}\left(b^{\star}_{ij} + \frac{1}{3}\delta_{ij}\right)$$

$$b^{\star}_{ij} = -\frac{\nu_t}{2k^{\star}}(\partial_i {U^{\star}}_j + \partial_j {U^{\star}}_i) + b^{\Delta}_{ij}$$

Verification check: Solve

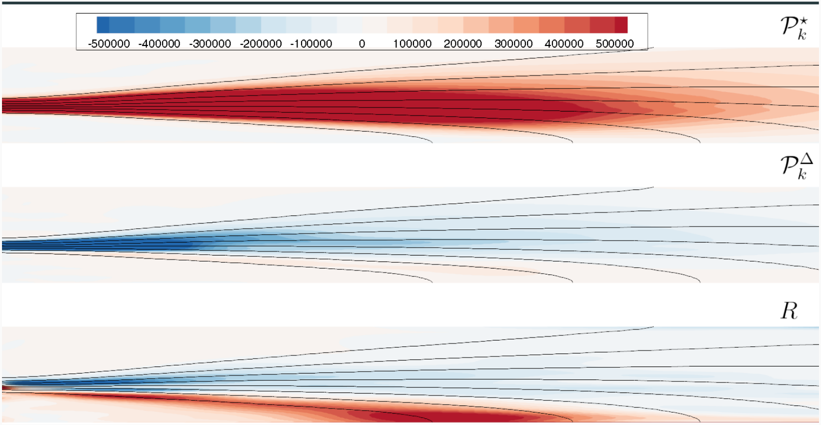
$$U_j \partial_j U_i = -\frac{\partial_i p}{\rho} + \partial_j \nu S_{ij} + \partial_j \nu_T S_{ij} - \partial_j (2k b^{\Delta}_{ij})$$

$$U_j \partial_j k = \mathcal{P}_k + \mathcal{P}^{\Delta}_k - \beta^* k\omega + \partial_j\left[(\nu + \nu_t \sigma_k)\partial_j k\right] + R,$$

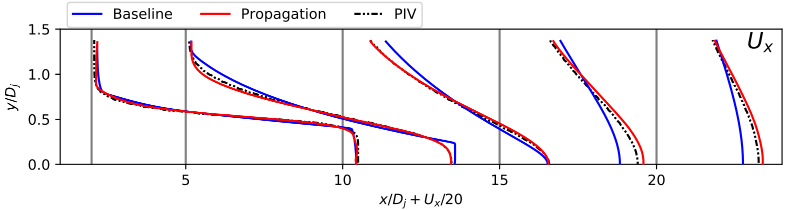
$$U_j \partial_j \omega = \frac{\gamma}{\nu_T}\left(\mathcal{P}_k + \mathcal{P}^{\Delta}_k + R\right) - \beta\omega^2 + \dots$$

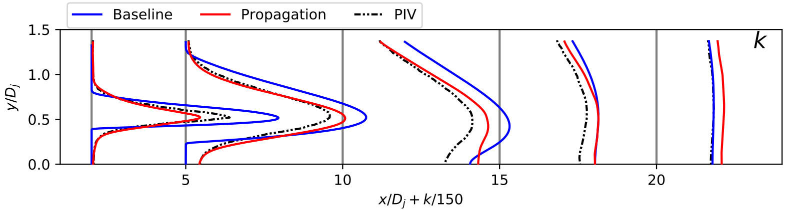
Jet: Corrective fields (ASJ)

- Based on PIV (Bridges and Wernet 2011) - domain limited.
- Inlet ω based on turbulence equilibrium assumption.



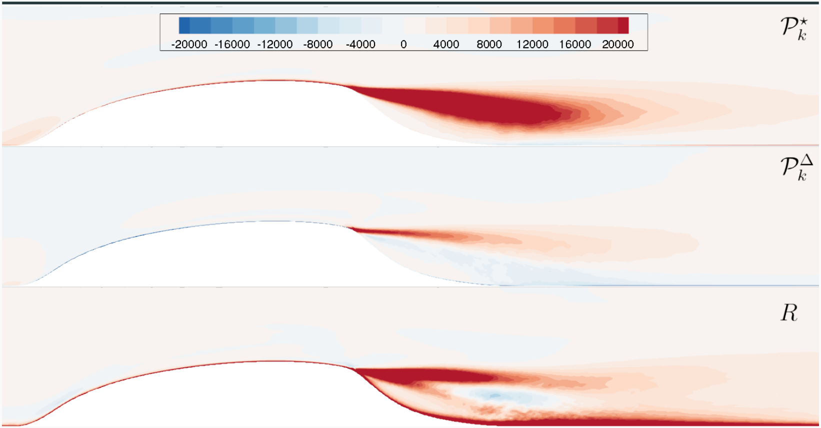
Jet: Effect of corrections on mean-flow (ASJ)



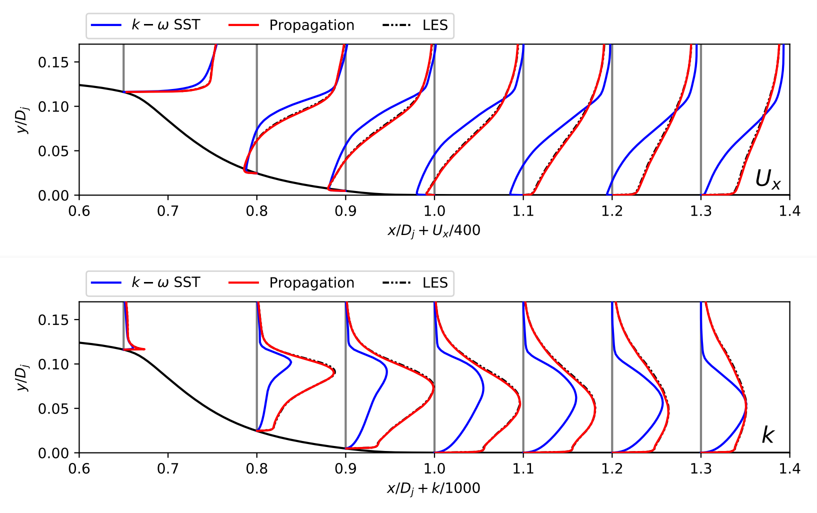


Hump: Corrective fields (2DWMH)

- Based on LES (Uzun and Malik 2017)
- Data on reduced domain; mesh artifacts present



Hump: Effect of corrections on mean-flow (`2DWMH`)



$\Longrightarrow$ corrective fields R, b_{ij}^{Δ} useful

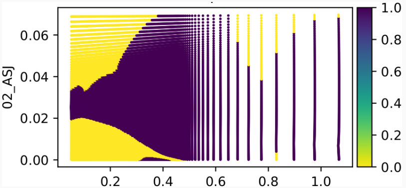
2. Training classifier model - (a) Target

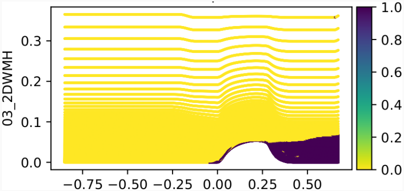
Goal

Identify regions where corrections are necessary

$$\boldsymbol{\sigma}^\star := \begin{cases} 1 & \text{correction needed} \\ 0 & \text{don't correct} \end{cases}$$

1. `2DZP`, `2DFDC`: $\boldsymbol{\sigma}^\star = 0$ everywhere
2. `ASJ`, `2DWMH`: $\boldsymbol{\sigma}^\star = \left\{ |\mathcal{P}_k^\Delta| > 0.2\overline{\mathcal{P}_k^\star} \right\} \cup \left\{ |R| > 0.2\overline{\mathcal{P}_k^\star} \right\}$
3. `2DN00`: Required activation unknown (no training data)





2. Training classifier model - (b) Logistic Regression

Problem: Given local flow-features $\boldsymbol{\theta} \in \mathbb{R}^Q$, find $\sigma(\boldsymbol{\theta}) \approx \boldsymbol{\sigma}^\star$.

Method: Logistic regression

$$\log \frac{\mathbb{P}(\sigma^\star = 1 \mid \boldsymbol{\theta})}{\mathbb{P}(\sigma^\star = 0 \mid \boldsymbol{\theta})} = f(\boldsymbol{\theta}) := \sum_i \alpha_i \psi_i(\boldsymbol{\theta})$$

- Define large dictionary of basis functions $\phi_i(\cdot)$
- Use sparsity-promoting priors to obtain simple models

Steiner, RPD, and Vire (2022). "Classifying regions of high model error within a data-driven RANS closure: Application to wind turbine wakes". In: *Flow, Turbulence and Combustion*. DOI: 10.1007/s10494-022-00346-6

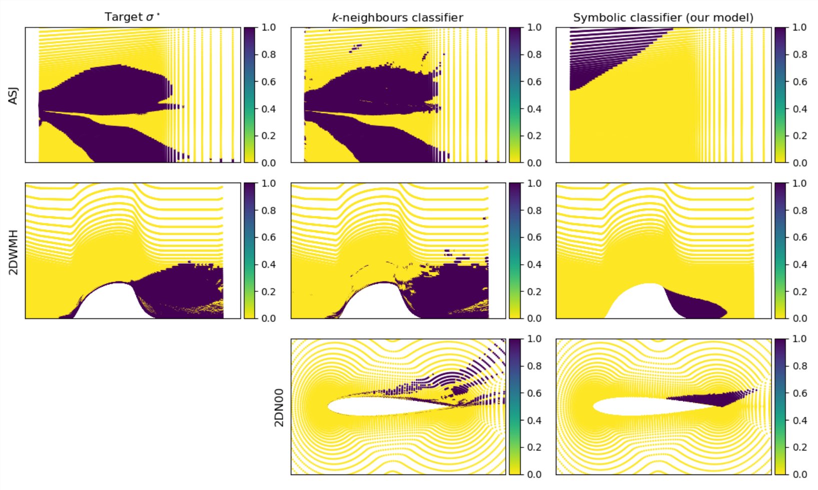
2. Final classifier model

$$\sigma(\boldsymbol{\theta}) := 1/(1+\exp[-f(\boldsymbol{\theta})])$$

$$\begin{aligned}
f(\boldsymbol{\theta}) := 0.02941 & \\
&+ 24.07\,\mathrm{rdiv}(W^2/2.964) && -3.815\,\mathrm{rdiv}(q_{pS}/0.1333) \\
&- 0.7596\,\mathrm{rdiv}(\sqrt{q_{pS}/0.1333}) && -2.869\,\mathrm{rdiv}(q_\gamma/1.847) \\
&- 0.02062\tanh(q_\gamma/1.847) && -0.935\,\mathrm{rdiv}((q_\nu/92.16)^2) \\
&- 0.9397\tanh((q_\nu/92.16)^2) && +3.541\,\mathrm{rdiv}(\sqrt{q_\nu/92.16}) \\
&+ 0.1161\,\mathrm{rdiv}((q_{\mathrm{Re}}/0.5425)^2) && +26.34\,\mathrm{rdiv}(\sqrt{q_{TI}/156.1}) \\
&- 1.995\tanh((q_{\tau k}/0.8177)^2)
\end{aligned}$$

W^2	$:= \mathrm{tr}\,\Omega^2$	Pope's second invariant
q_{pS}	$:= \Vert\partial p\Vert/\Vert U\partial U\Vert$	Pressure to shear ratio
q_γ	$:= Sk/\varepsilon$	Shear parameter
q_ν	$:= \nu_T/100\nu$	Turbulence to molecular viscosity ratio
q_{Re}	$:= 2 - \min\left(\frac{\sqrt{k}d}{50 * nu}, 2\right)$	Wall-distance Reynolds number
q_{TI}	$:= k/2\Vert U\Vert$	Turbulence intensity...
$q_{\tau k}$	$:= U\partial k/\mathcal{P}_k$	Convection to production of k
$\mathrm{rdiv}(q)$	$:= \frac{q}{1+q^2}$	Regularized division

2. Final classifier model - Effectiveness



- *k*-neighbours classifier, targets both ASJ and 2DWMH
- Symbolic classifier, targets 2DWMH only

3. Final correction models

- Trained using sparse regression (Schmelzer, RPD, and Cinnella 2019)
- Training data reduced with classifier: $\{(\boldsymbol{\theta}, R, b^{\Delta}_{ij}) \mid \sigma^{\star} = 1\}$
- Cross-validation to eliminate unstable models

$$b^{\Delta}_{ij}(\cdot) := 0$$

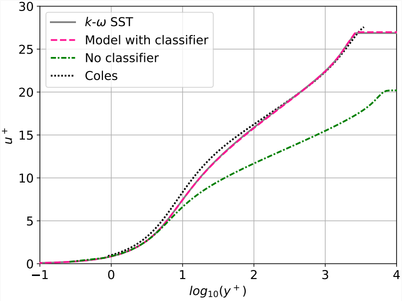
$$R(\cdot) := 0.079\varepsilon \qquad \text{[Coeff. of determination } R^2 = 0.98\text{]}$$

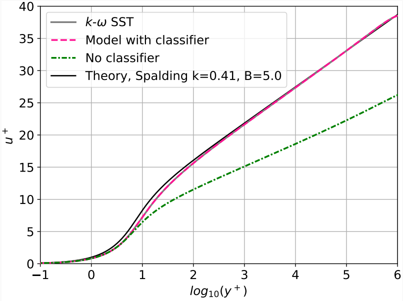
Alternative anisotropy correction (not used in following):

$$b^{\Delta}_{ij}(\cdot) := 5.66\,T^{(2)}_{ij} = \frac{5.66}{\omega^2}\left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}\right)$$

No significant change in results; increased solver instability.

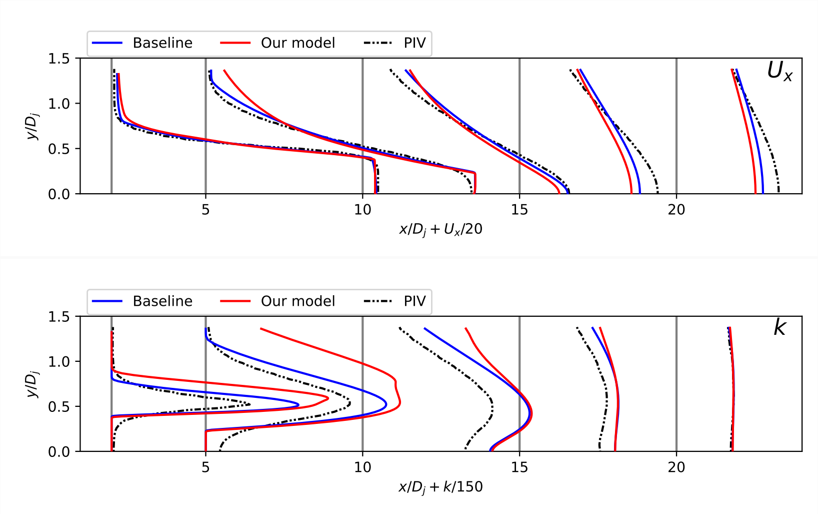
Results - 2DZP, 2DFDC





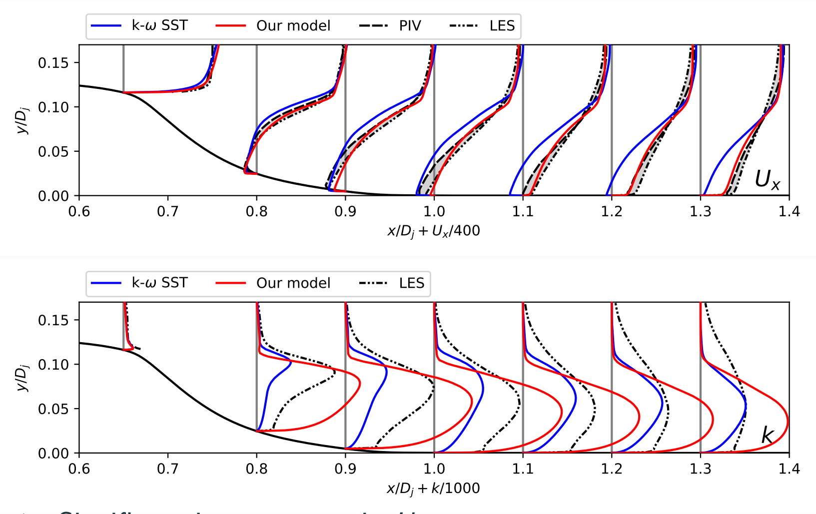
- Predictions identical to baseline $k - \omega$ SST
- Without classifier $\implies$ model correction everywhere.

Results - ASJ



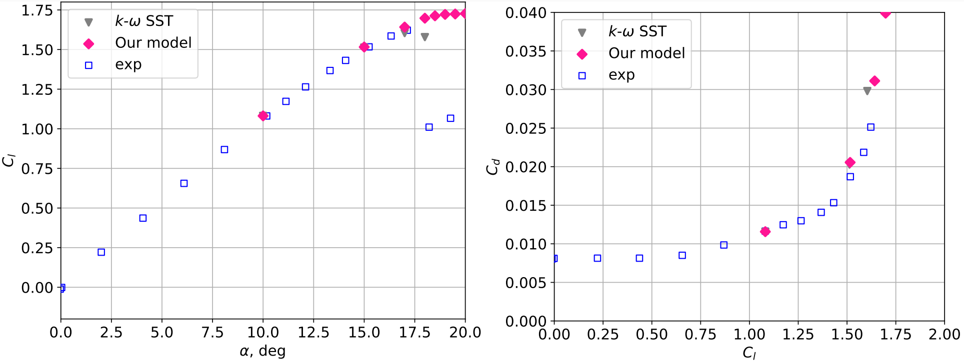
- Slightly worse than baseline - no change to spreading rate

Results - 2DWMH



- Significant improvement in U_x
- Insufficient mixing near top of shear layer

Results - 2DN00



- **Significantly** delayed seperation
- Consequence of reduction of dissipation

Conclusions

- Derived a minor correction to SST, locally active.
- Score card:
  - 2DZP → Identical predictions to SST ✔
  - 2DFDC → Identical predictions to SST ✔
  - ASJ → Slightly worse ●
  - 2DWMH → Significantly better ✔
  - 2DN00 → Overestimated stall angle/$C_{L,\max}$ ✗

Further work:

- Multi-class classifier for different kinds of corrections
- Increase size of training sets - e.g. consider multiple seperated flows
- Ideas welcome...

References

- Bridges and Wernet (2011). "The NASA Subsonic Jet PIV Dataset". In: *NASA Technical Report* NASA/TM-2011-216807.
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