AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Developing hierarchical augmentations via the "Learning and Inference assisted by Feature-space Engineering (LIFE)" framework

NASA 2022 Symposium on Turbulence Modeling Roadblocks, and the Potential for Machine Learning

July 27-29, 2022

Vishal Srivastava, Karthik Duraisamy

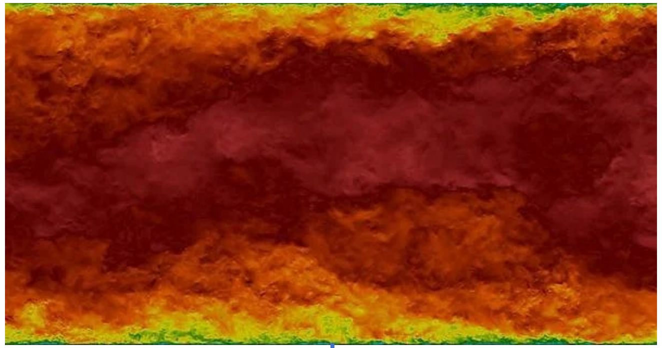
Department of Aerospace Engineering, University of Michigan, Ann Arbor

Computational AeroSciences Laboratory

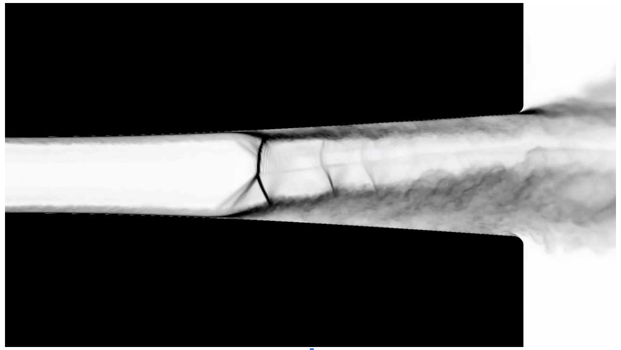
AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Motivation

https://imgur.com/KRJO4I0



https://www.youtube.com/watch?v=cK7g4xio0WU



INFER FUNCTIONAL CORRECTION

$$\mathscr{R}_m(\tilde{\boldsymbol{u}}_m; \boldsymbol{\xi}) = 0$$

Model equations

Model states

Model inputs (Mesh, I.C., B.C.'s, Material properties etc.)

Background – Integrated Inference and Machine Learning

No explicit regularization used in this work

$$\boldsymbol{w}^* = \arg\min_{\boldsymbol{w}} \sum_k \left(\alpha_k \mathcal{C}^k (\boldsymbol{y}^k_{\text{data}}, \boldsymbol{y}^k_{\text{pred}}(\widetilde{\boldsymbol{u}}^k_m; \boldsymbol{\xi}^k)) + \cancel{\lambda_k \mathcal{R}^k (\widetilde{\boldsymbol{u}}^k_m, \boldsymbol{w}, \boldsymbol{\xi}^k)} \right)$$

$$\text{s.t.} \quad \mathscr{R}_{m,\text{aug}}(\widetilde{\boldsymbol{u}}^k_m; \beta(\boldsymbol{\eta}(\widetilde{\boldsymbol{u}}^k_m); \boldsymbol{w}), \boldsymbol{\xi}^k) = 0$$

Model consistency

Holland et al. *(AIAA Aviation, 2019),* ***Sirignano et al.*** *(JCP, 2020),* ***Strofer and Xiao*** *(arXiv, 2021)*

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

- Motivation
- Background
- **LIFE**
- Hierarchical Augmentation
- Summary

Learning and Inference assisted by Feature-space Engineering

Where to augment?

⇒ What is the intended correction?

⇒ How will the augmentation affect cases where correction is not required?

⇒ Physics-based limiting or regularization possible?

How to design feature-space?

⇒ Improve generalizability
- Features chosen by a modeler
- Non-dimensionalized using model quantities

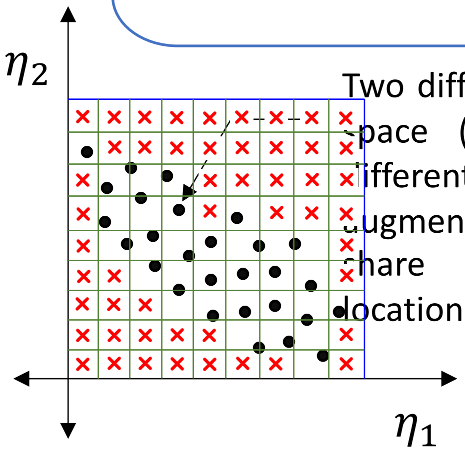
⇒ Ensure predictive accuracy
- Enough features to roughly ensure a one-to-one features-to-augmentation map

⇒ Minimize extrapolation
- Bounded
- Parsimonious set of features

Which function class to use?

⇒ If available data populates the entire feature-space
- Neural Networks, Decision Trees
- Custom-built functions

⇒ Otherwise
- Localized learning



Two different points in the physical space (from the same case or different cases) requiring different augmentation values should not share the same feature-space location

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Application: Bypass Transition

Adding an augmented bare-bones intermittency equation (inspired by Durbin's model of 2012) to Wilcox's 1988 k-ω model

$$\frac{D\rho k}{Dt} = \boldsymbol{\nabla}\cdot\left(\left(\mu+\sigma_k\mu_t\right)\boldsymbol{\nabla}k\right)+\gamma\left(\mu_t\Omega^2-\frac{2}{3}\rho k\frac{\partial u_i}{\partial x_j}\delta_{ij}\right)-C_{1k}\rho\omega k$$

$$\frac{D\rho\omega}{Dt} = \boldsymbol{\nabla}\cdot\left(\left(\mu+\sigma_\omega\mu_t\right)\boldsymbol{\nabla}\omega\right)+C_{1\omega}\frac{\omega}{k}\left(\mu_t\Omega^2-\frac{2}{3}\rho k\frac{\partial u_i}{\partial x_j}\delta_{ij}\right)-C_{2\omega}\rho\omega^2$$

$$\frac{D\rho\gamma}{Dt} = \boldsymbol{\nabla}\cdot\left(\left(\sigma_{\gamma,\ell}+\sigma_{\gamma,t}\mu_t\right)\boldsymbol{\nabla}\gamma\right)+\rho\Omega\left(\beta-\gamma\right)\sqrt{\gamma}$$

Replaced γ_{max} with augmentation and removed F_γ limiter

Intermittency transport equation smoothens augmentation field into intermittency field

Bounded augmentation: $0\leq\beta\leq 1$ (as intermittency is driven by β)

Feature to help determine transition onset

$$Re_\Omega = \frac{\Omega d^2}{2.188\nu} \qquad \max_d Re_\Omega \approx Re_\theta$$

Physics-informed choice of features

Praisner and Clark (J. Turbomachinery, 2007) gave the correlation

$$\theta_{\rm tr} \approx \sqrt{\frac{7\nu}{9\omega_\infty}}$$

Physics-based non-dimensionalization

Then, we have $$\frac{Re_\theta}{Re_{\theta,\rm tr}} \approx \max_d \frac{Re_\Omega}{U_\infty \theta_{\rm tr}/\nu} \approx \max_d \frac{\Omega d^2 \sqrt{9\omega_\infty}}{U_\infty \sqrt{7\nu}}$$

Freestream quantities are extracted from a constant wall distance.

Applying a conservative bound

$$\eta_1 = \min\left(\frac{d^2 \Omega \sqrt{9\omega_\infty}}{U_\infty \sqrt{7\nu}}, 3\right)$$

Bounded features

Feature(s) to identify laminar/turbulent regions

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Compare ν and ν_t. What about the viscous sublayer, though?

Compare d and ℓ_t to see if d is significantly larger. For k-ω model, $\mathcal{O}(\ell_t) = \mathcal{O}(\sqrt{k}/\omega)$

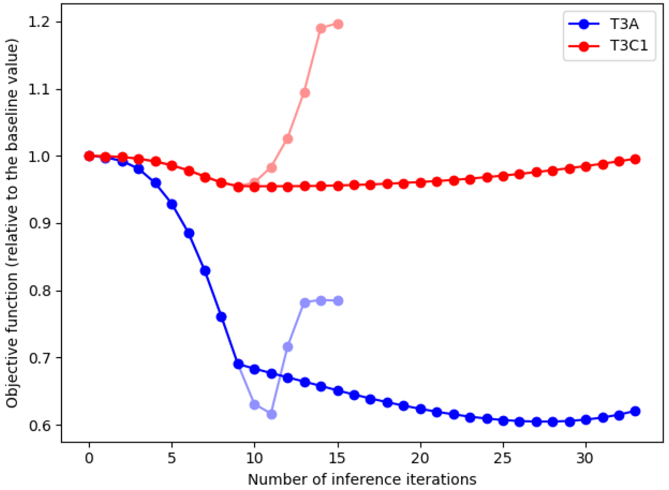
Mathematically bound both the features as:

$$\eta_2 = \frac{d}{d + \sqrt{k}/\omega}$$

$$\eta_3 = \frac{\nu}{\nu_t + \nu}$$

- Too many features over-specify physical conditions and reduce generalizability
- Too few features can result in lower predictive accuracy even for the training cases

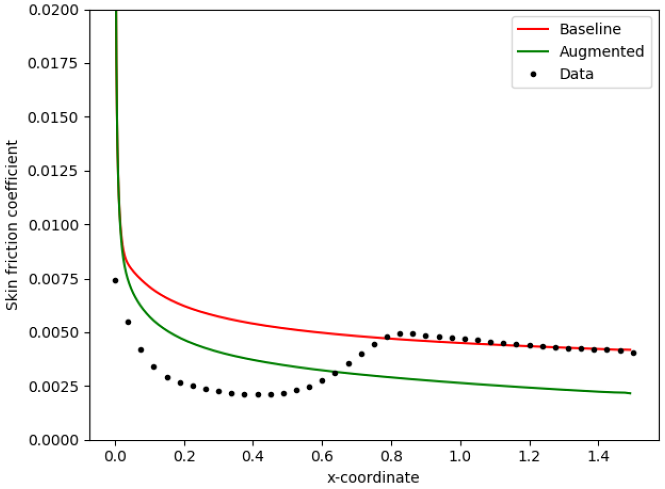
What happens when off-the-shelf NNs are used?



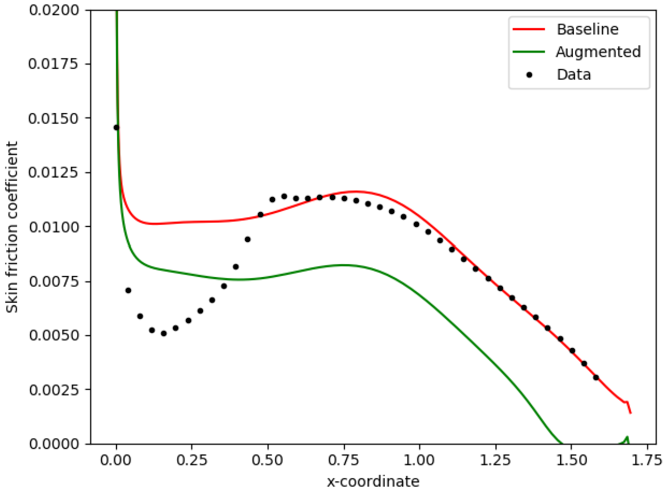
Relative cost reduction w.r.t. inference iterations

$$\mathcal{C}^k = \frac{1}{N^k} \sum_{i}^{N^k} \left(C_f^k(x_i) - C_{f,\text{data}}^k(x_i) \right)^2$$

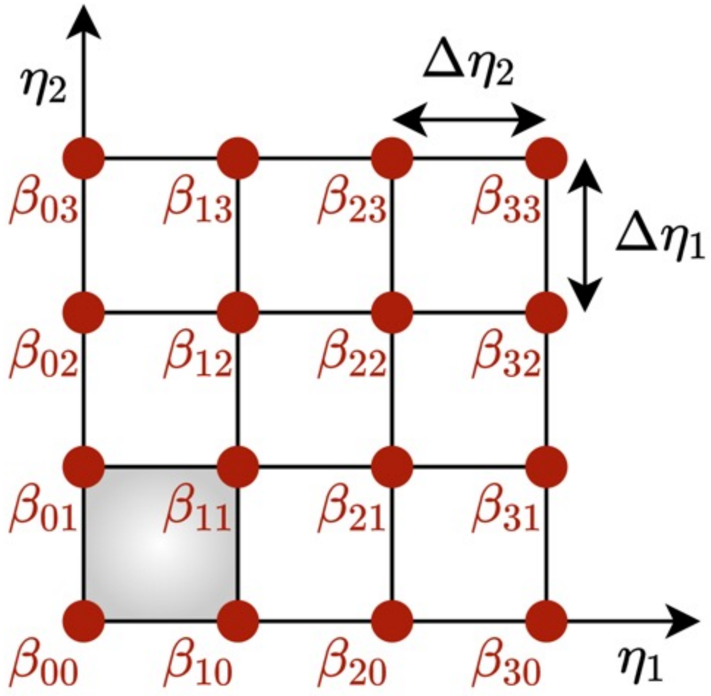
Skin friction coefficient for T3A



Skin friction coefficient for T3C1



Limited data necessitates localized learning

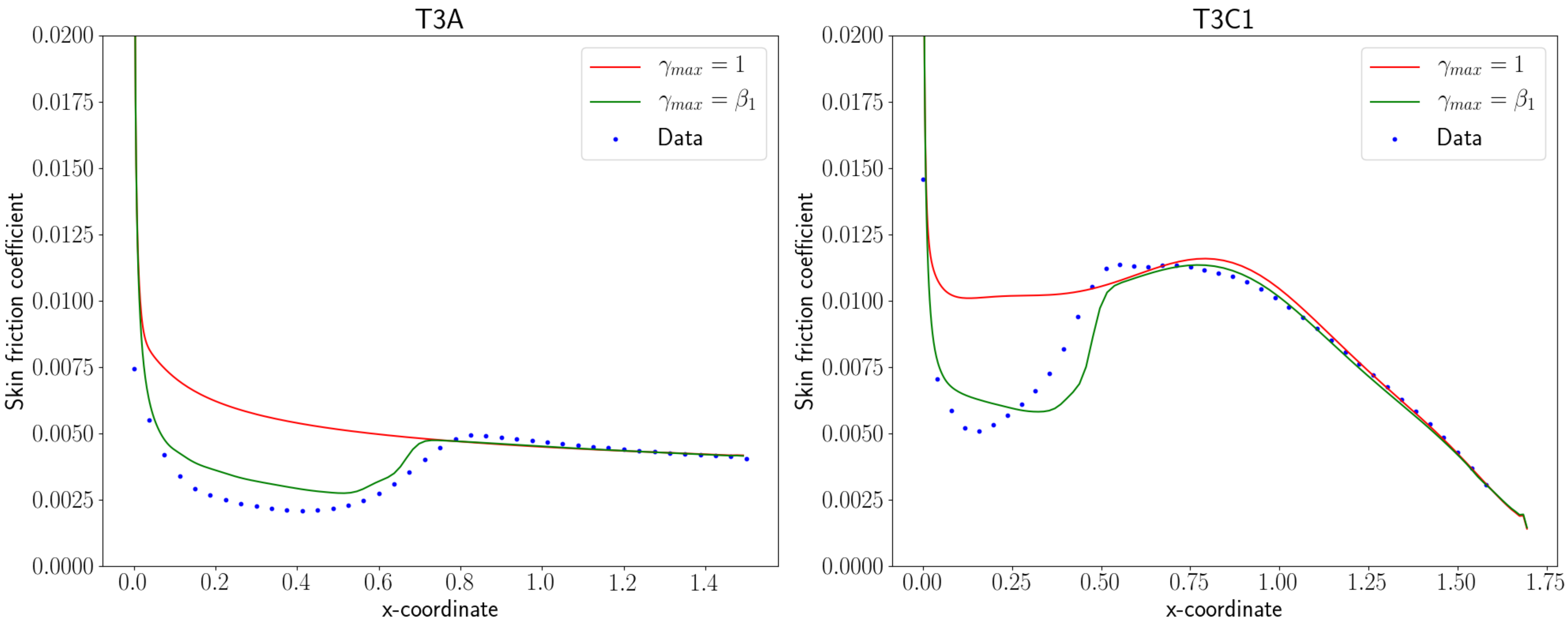


For a feature space location in the shaded region, the multilinear expression reads

$$\beta = \beta_{00} + (\beta_{10} - \beta_{00})\eta_1 + (\beta_{01} - \beta_{00})\eta_2 + \cdots$$
$$(\beta_{11} + \beta_{00} - \beta_{10} - \beta_{01})\eta_1\eta_2$$

- C^0-continuous
- Susceptible to curse of dimensionality
- Choice of grid resolution is crucial

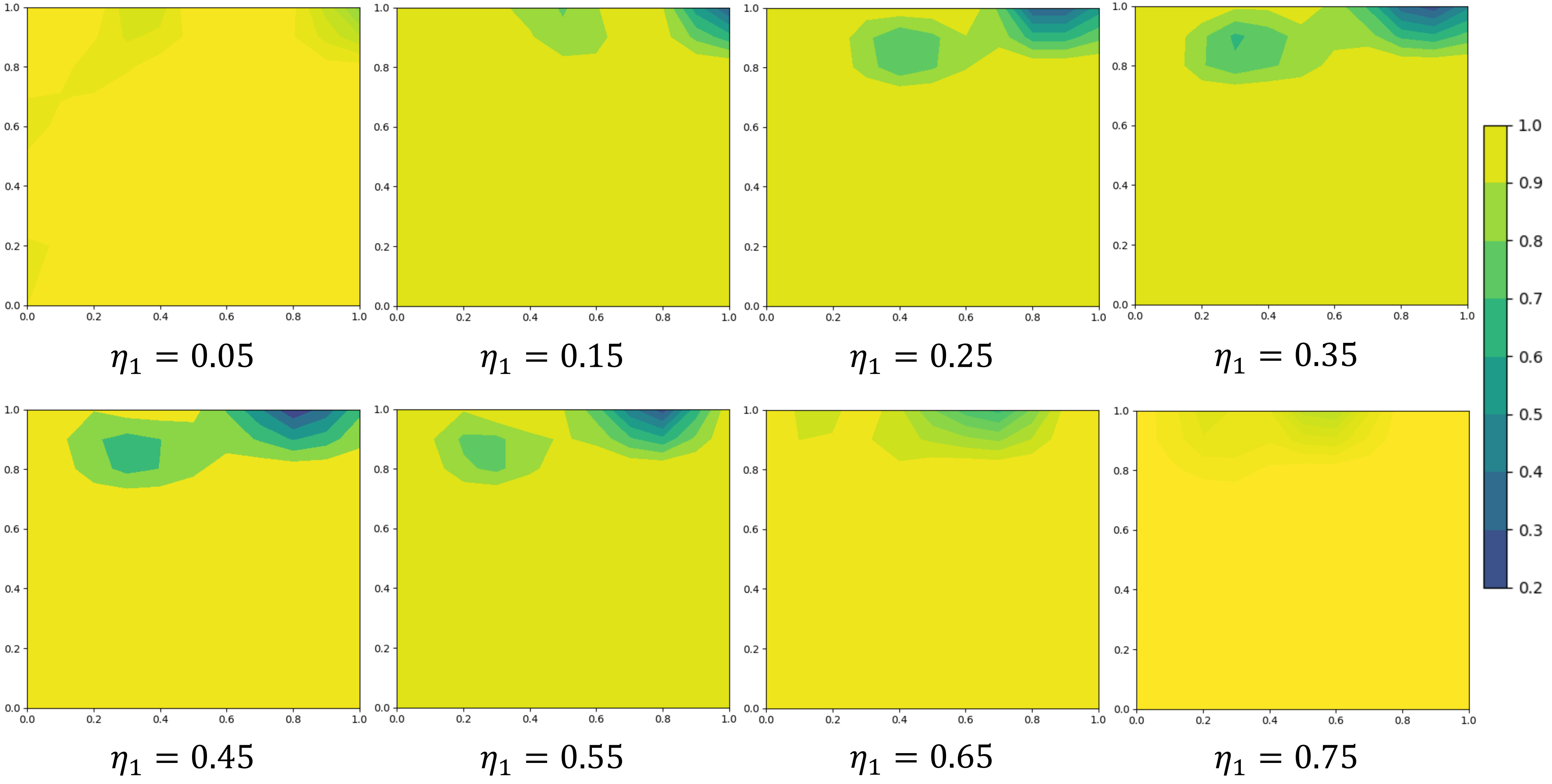
A C⁰-continuous augmentation (Training)



- Feature-space uniformly discretized into 45x15x15 cells
- Excellent solver convergence compared to a discontinuous functional form for the augmentation
- Cost function was the sum squared discrepancy in the C_f profile

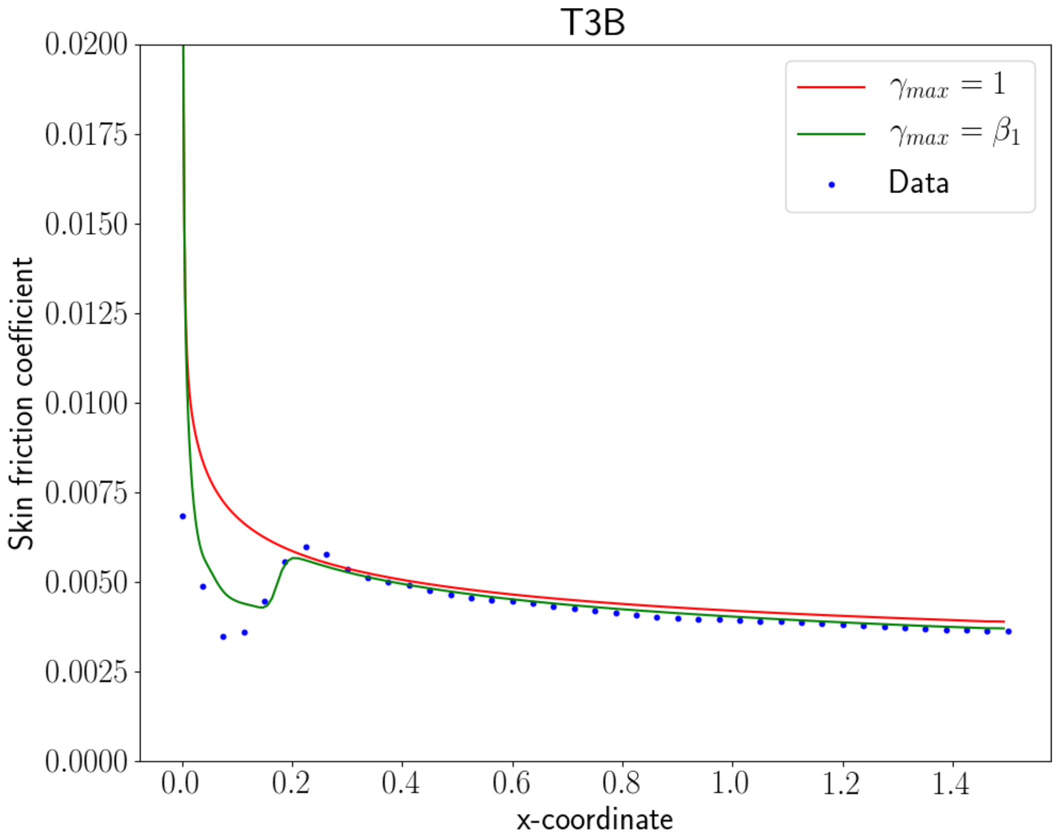
AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

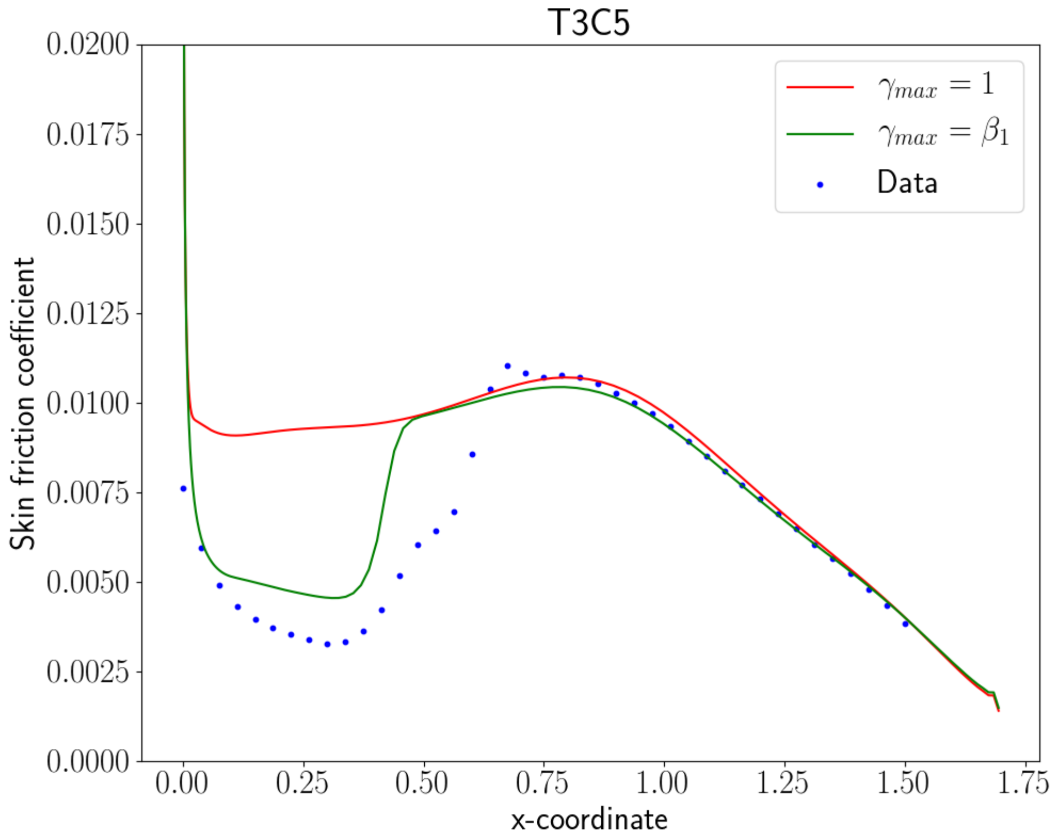
How does the feature-space look?



For all plots: X-axes: η_2 (0 to 1) Y-axes: η_3 (0 to 1)

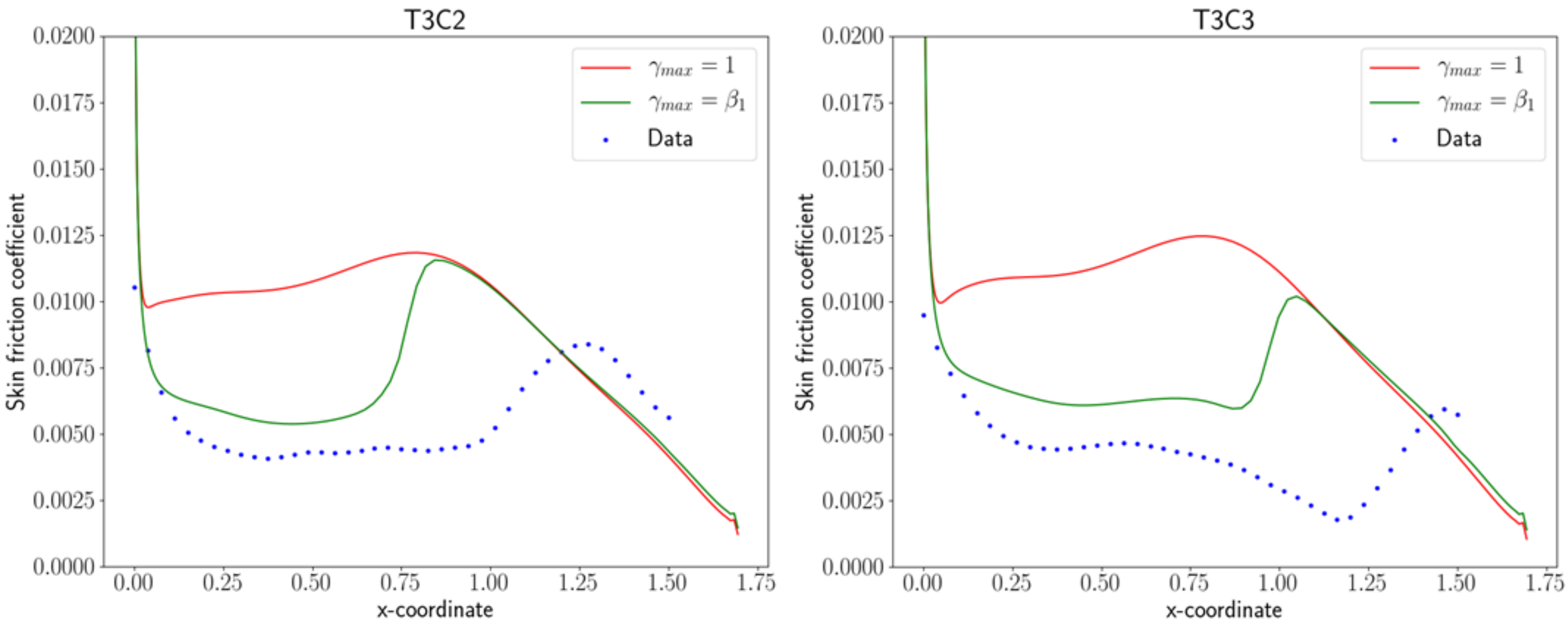
A C^0-continuous augmentation (Testing – FPG)





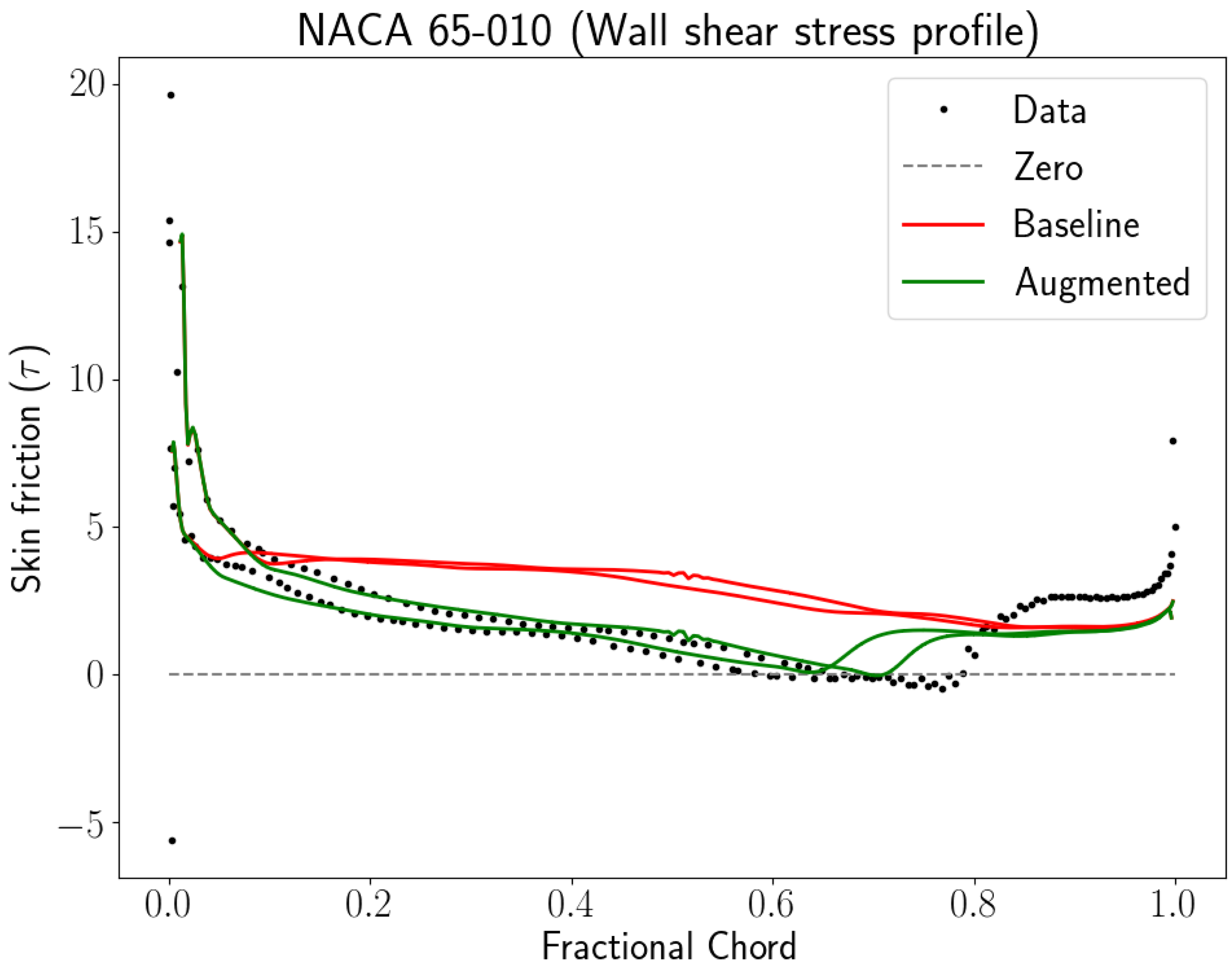
Good generalizability to zero and favorable pressure gradient cases

A C^0-continuous augmentation (Testing – APG)



Worse predictions for cases involving transition in adverse pressure gradient regions

Prediction on a compressor cascade (LES from RTRC)



- Transition predicted near separation location
- Downstream discrepancy results from inadequacy in underlying turbulence model

Motivation

Background

LIFE

Hierarchical Augmentation

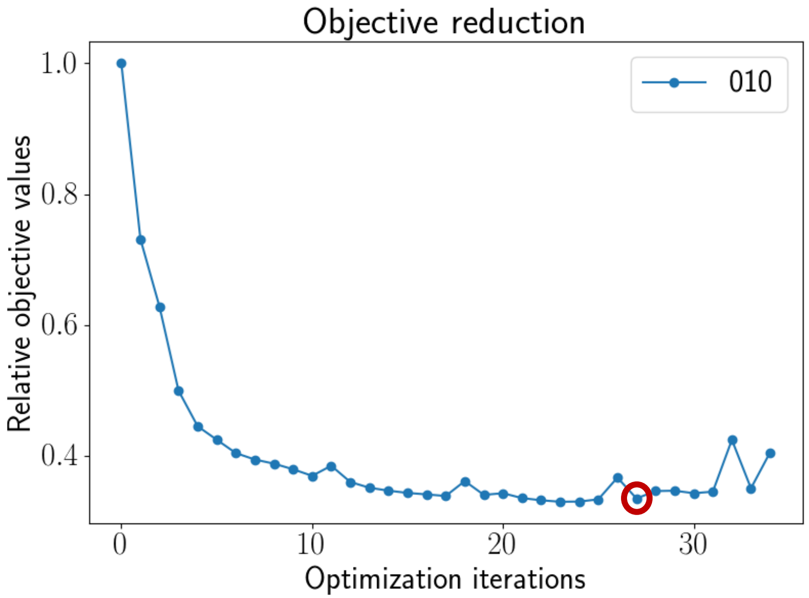
Summary

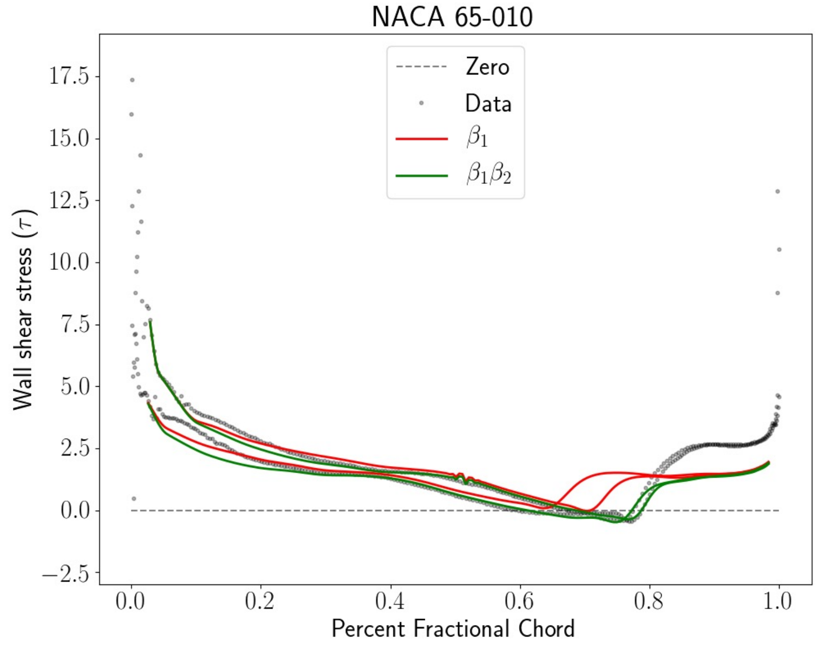
AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Inferring a Hierarchical Augmentation

$$\frac{D\rho\gamma}{Dt} = \nabla\left(\left(\sigma_{\gamma,l}\mu + \sigma_{\gamma,t}\mu_t\right)\nabla\gamma\right) + \rho\Omega(\beta_1\beta_2 - \gamma)\sqrt{\gamma}$$





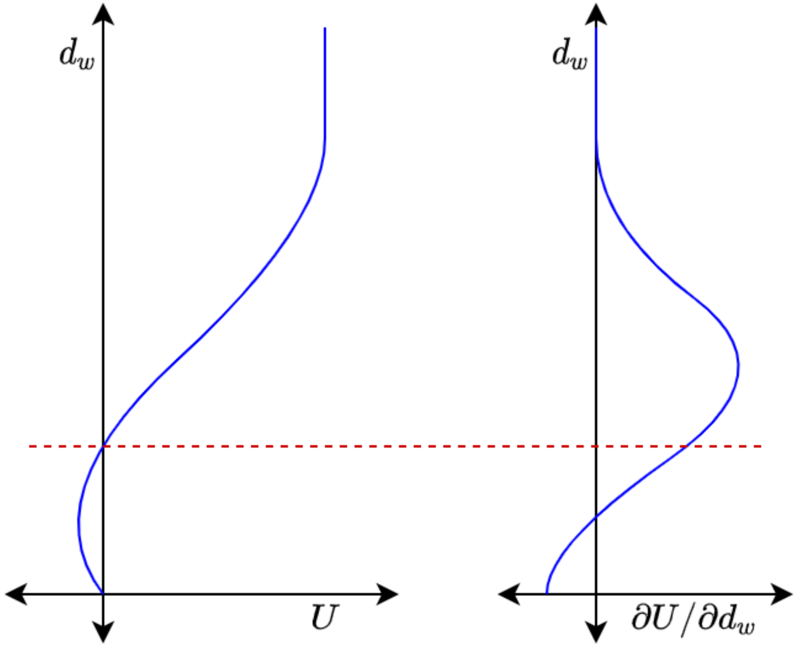
- Feature space uniformly discretized into $30\times10\times10$ cells for hierarchical augmentation
- Cost function was the sum squared discrepancy in the wall shear stress profile
- Using the hierarchical augmentation as is results in poor predictions on the flat plate cases

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

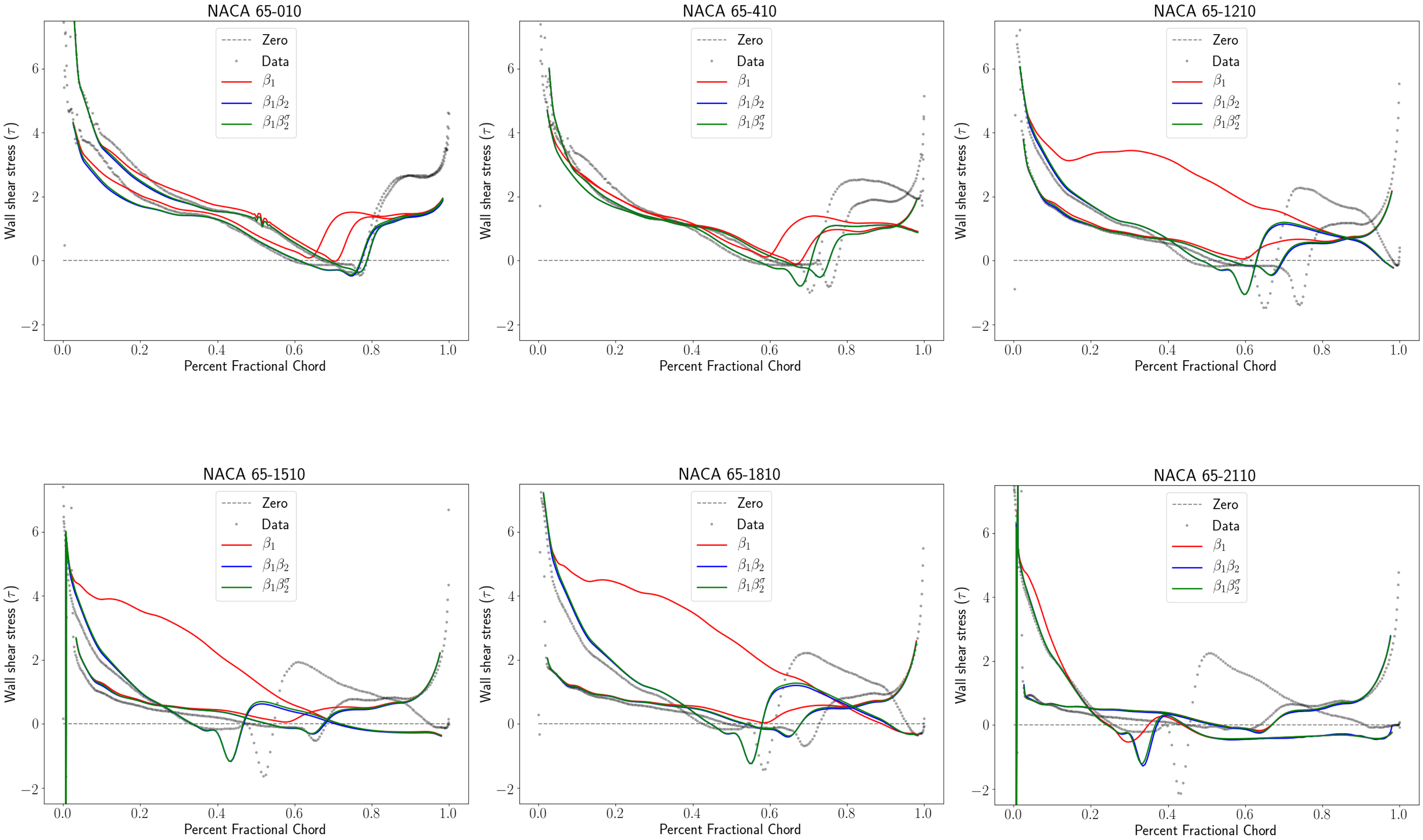
Designing a physics-informed blending function

$$\frac{D\rho\gamma}{Dt} = \nabla\left(\left(\sigma_{\gamma,l}\mu + \sigma_{\gamma,t}\mu_t\right)\nabla\gamma\right) + \rho(\beta_1\beta_2^{\sigma} - \gamma)\sqrt{\gamma}\Omega$$

$$\sigma = \frac{1}{1+\exp\left(-\frac{f_\sigma + 0.05}{0.003}\right)} \qquad f_\sigma = \left(\frac{\sqrt{\nu/\Omega}(\boldsymbol{n}_w\cdot\nabla)\Omega}{\sqrt{\nu/\Omega}|(\boldsymbol{n}_w\cdot\nabla)\,\Omega| + \Omega}\right)\left(\frac{\omega}{\sqrt{2}\Omega+\omega}\right)$$



Predictions using the hierarchical augmentation



AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

- Motivation
- Background
- LIFE
- **Hierarchical Augmentation**
- Summary

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Motivation

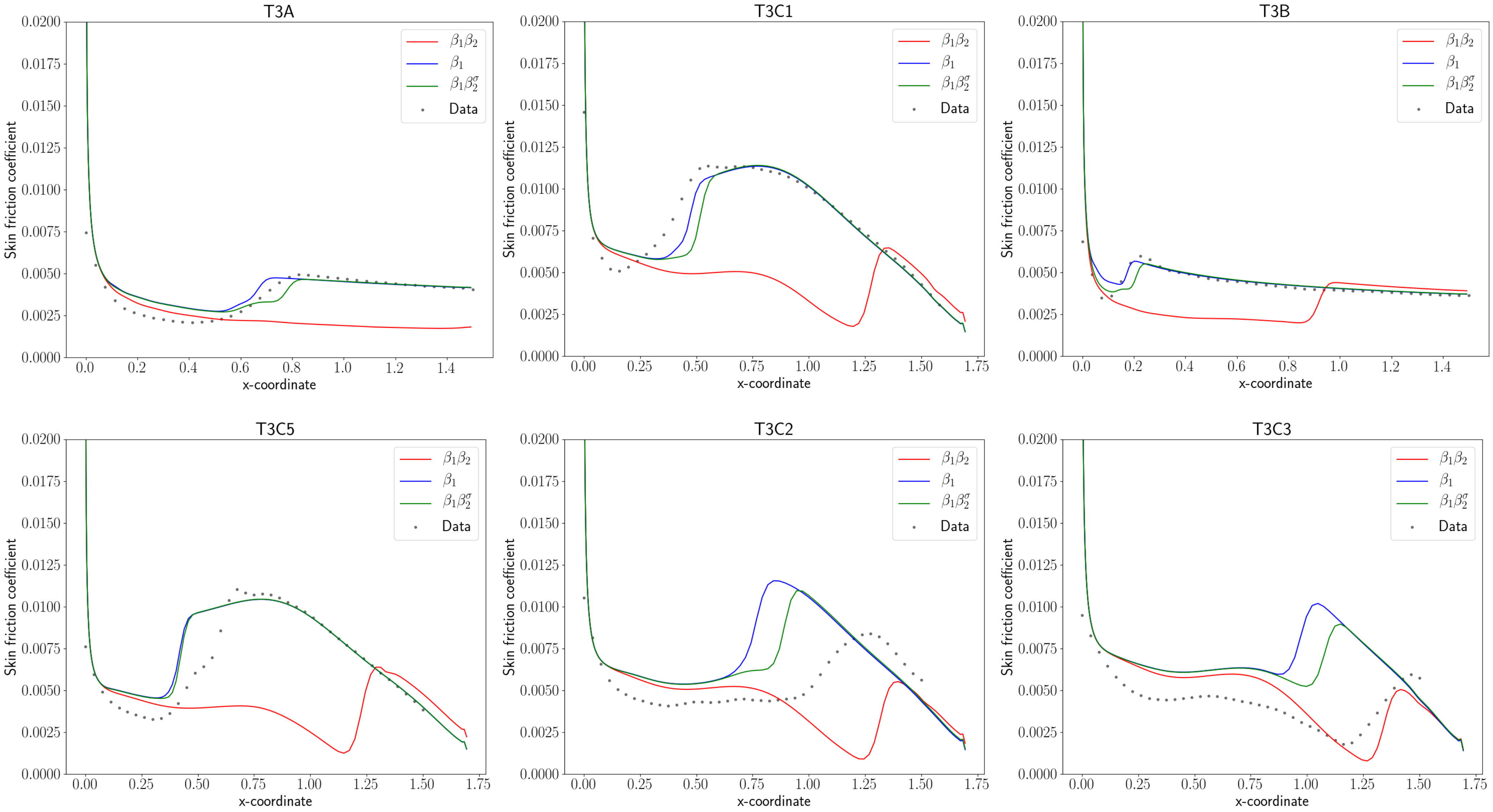
Background

LIFE

Hierarchical Augmentation

Summary

Predictions using the blended hierarchical augmentation



Blending function affects (and slightly improves) predictions for transition in APG regions

AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

Summary

- A C^0-continuous augmentation function provides excellent solver convergence and added implicit regularization
- The LIFE framework was used to infer two augmentations:
  - β_1 inferred from *two flat plate cases*
    - Training cases involved transition of attached flows in zero/favorable pressure gradients
    - Transition predictions generalize to unseen zero/favorable pressure gradient configurations well
    - Transition is predicted significantly upstream in adverse pressure gradient regions
  - β_2 subsequently inferred from *one compressor cascade case*
    - Training case involves separation-induced transition
    - Transition location predictions significantly improve across all test configurations
    - Transition is predicted slightly upstream compared to what is observed from the LES data in some instances
- An appropriate blending function (σ) was designed to shield attached flow regions from the effects of the second (hierarchical) augmentation
- Future work will include exploring purely local feature candidates and the blending function in addition to building a formal framework to optimize hyperparameters for localized learning