Potential of Data Driven Methods for Reynolds Stress Modeling - A Fundamental View

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Symposium on Turbulence Modeling:

Roadblocks and the Potential for Machine Learning

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† Sadly, Bernhard passed away on the 26th of January 2022, a few days after the completion of this work.

Overview

- Introduction
- Theory
- Reynolds Stress Modeling
- Simulation Results
    - Fundamental Flows
    - Applications
- Potential of Data Driven Methods
- Conclusion

Overview

Introduction

RANS Models

- No limitations on
  - Reynolds number
  - Geometric complexity
- Limitations on
  - Accuracy (separation)

➔ Indispensable in
- Industrial application
- Aerodynamic design

➔ Requirements
- Improved accuracy
- Design: single model for various flow conditions

From theory:
Fundamental conditions ➔ Calibration
Turbulent equilibrium

potential conflict

Data Driven Turbulence Modeling

Idea
- Real data
- Artificial intelligence (machine learning)

➔
- No limitation to canonical flows for learning
- Improved predictions in application

Method
- Optimisation of
  - Model coefficients
  - Functional dependence of coefficients
  - Model form (additional terms)

➔ Unlimited improvement?

Overview

cf. Abid & Speziale (1993)

Theory

Turbulent Equilibrium

- High Reynolds number
- Boundary layer assumptions

➔ Simplification of turbulence equations

$$0 = P_{ij} + \Pi_{ij} - \varepsilon_{ij}$$ Reynolds stress equation

$$0 = P^{(k)} - \varepsilon$$ k-equation.

Turbulent equilibrium

Reynolds Stress Modeling

- Pressure strain correlation (off walls)

$$\Pi_{ij} = \underbrace{\varepsilon A_{ij}}_{\text{slow}} + \underbrace{k M_{ijkl} \frac{\partial U_k}{\partial x_l}}_{\text{rapid}}$$

with $A_{ij}, M_{ijkl} = f(b_{pq})$

functions of Reynolds stress anisotropies

- Dissipation (high Re)

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}$$

isotropic

2D Mean Flow

- Only one velocity gradient

$$\frac{\partial U_k}{\partial x_l} \rightarrow \frac{\partial U}{\partial y}$$

➔
- 3 algebraic equations for $b_{11}\big|_{eq}, b_{22}\big|_{eq}, b_{12}\big|_{eq} = f(C_i)$

  with C_i = coeff. of pressure-strain model
- Independent of velocity profile

  ➔ valid for any 2D flow in turbulent equilibrium

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Reynolds Stress Model

Pressure Strain Correlation Models

- SSG

$$\Pi_{ij} = -\varepsilon\left(C_1 + C_1^*\frac{P^{(k)}}{\varepsilon}\right)b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) + \left(C_3 - C_3^*\sqrt{II_b}\right)\mathrm{k}S_{ij}$$
$$+C_4k\left(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}\right) + C_5k\left(b_{ik}\Omega_{jk} + b_{jk}\Omega_{ik}\right)$$

Full model.

- Simplification 1

$$\Pi_{ij} = -\varepsilon\left(C_1 + C_1^*\frac{P^{(k)}}{\varepsilon}\right)b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) + \left(C_3 - C_3^*\sqrt{II_b}\right)\mathrm{k}S_{ij}$$
$$+C_4k\left(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}\right) + C_5k\left(b_{ik}\Omega_{jk} + b_{jk}\Omega_{ik}\right)$$

Reduce non-linearity

- Simplification 2

$$\Pi_{ij} = -\varepsilon\left(C_1 + C_1^*\frac{P^{(k)}}{\varepsilon}\right)b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) + \left(C_3 - C_3^*\sqrt{II_b}\right)\mathrm{k}S_{ij}$$
$$+C_4k\left(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}\right) + C_5k\left(b_{ik}\Omega_{jk} + b_{jk}\Omega_{ik}\right)$$

Remove dependence on invariants

- Simplification 3

$$\Pi_{ij} = -\varepsilon C_1 b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) + C_3\mathrm{k}S_{ij}$$

Drastic surgery

Reynolds Stress Model

Calibration

- Strategy
  - Consider equilibrium state of original SSG model ➔ invariants/eigenvalues of anisotropy tensor
  - Rotate principal axes of anisotropy tensor to target $b_{12}|_{eq}$ ➔ maintain invariants/eigenvalues

- Equilibrium states

	$b_{11}\|_{eq}$	$b_{22}\|_{eq}$	$b_{12}\|_{eq}$	
Set 1	0.2099	−0.1355	−0.1506	Reduced momentum transfer
Set 2	0.2007	−0.1266	−0.1603	Original SSG model
Set 3	0.1907	−0.1165	−0.1700	Increased momentum transfer

➔ 12 different models (4 model forms x 3 sets of coefficients)
➔ Equilibrium values of II_b, III_b virtually identical

Length-scale equation

- BSL-ω-equation (Menter, 1994)
- Length-scale correction (Eisfeld & Rumsey, 2020)

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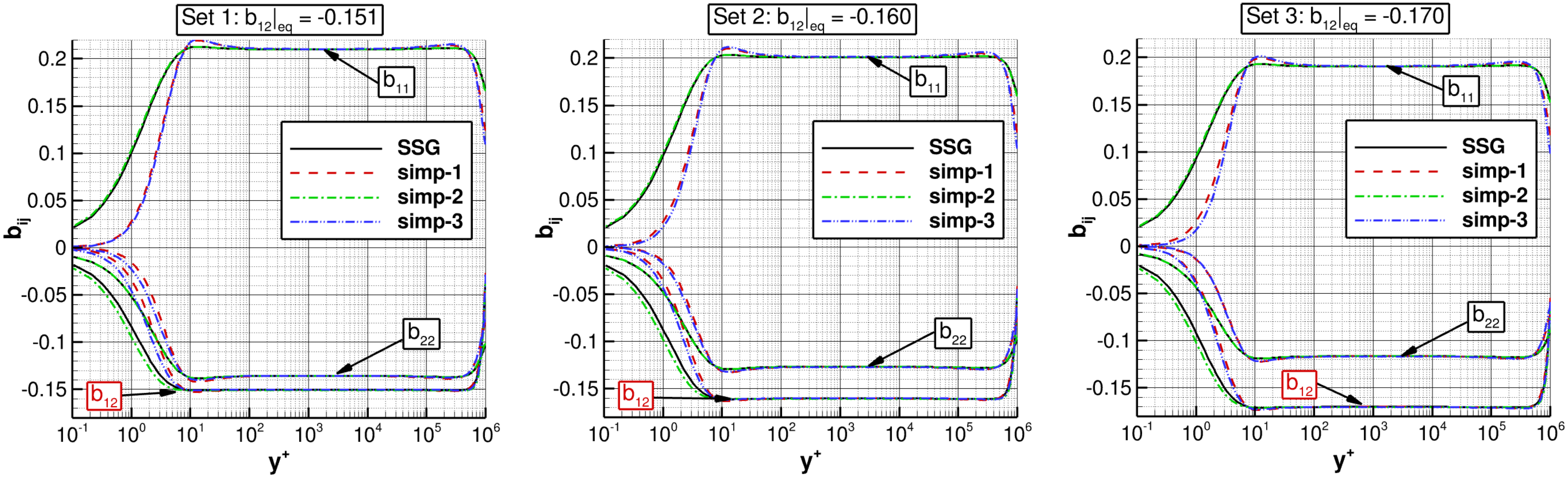
Simulation Results: Fundamental Flows

Channel Flow at Re_H = 80e6 (1)

- Reynolds stress anisotropies



- Wide range of constant $b_{ij}|_{eq}$ = target $b_{ij}|_{eq}$
- Equilibrium state independent of model form

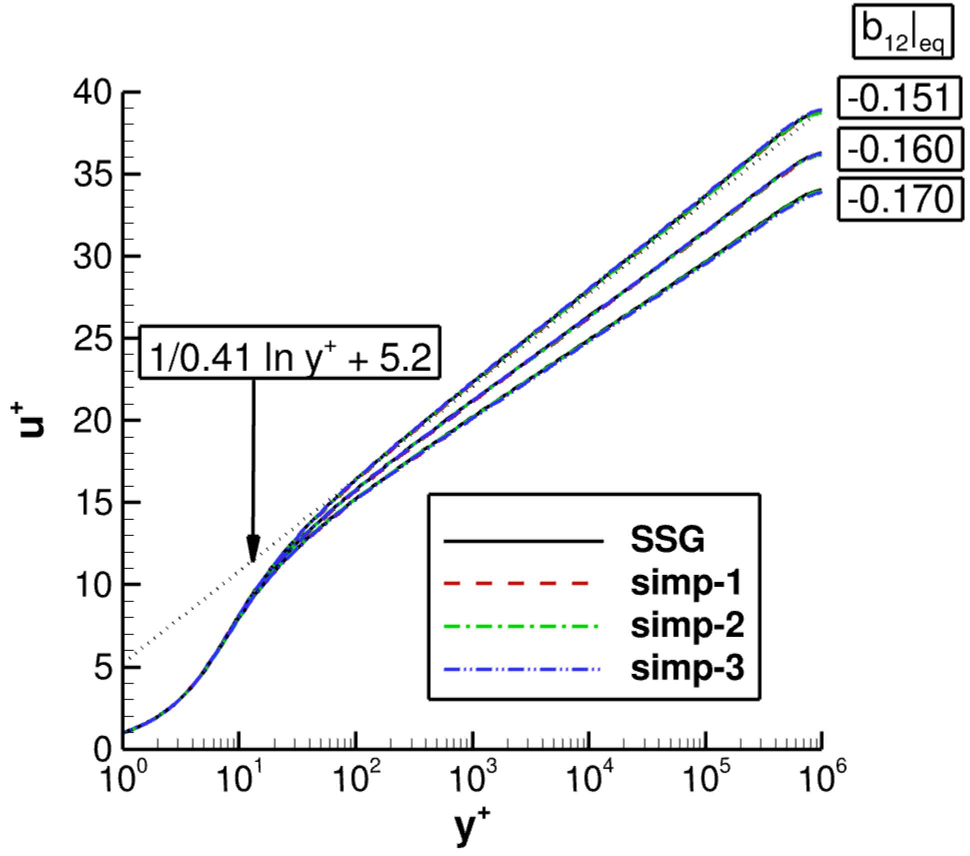
➔ Equilibrium state confirmed
➔ Theory confirmed

➔ Calibration strategy successful

Simulation Results: Fundamental Flows

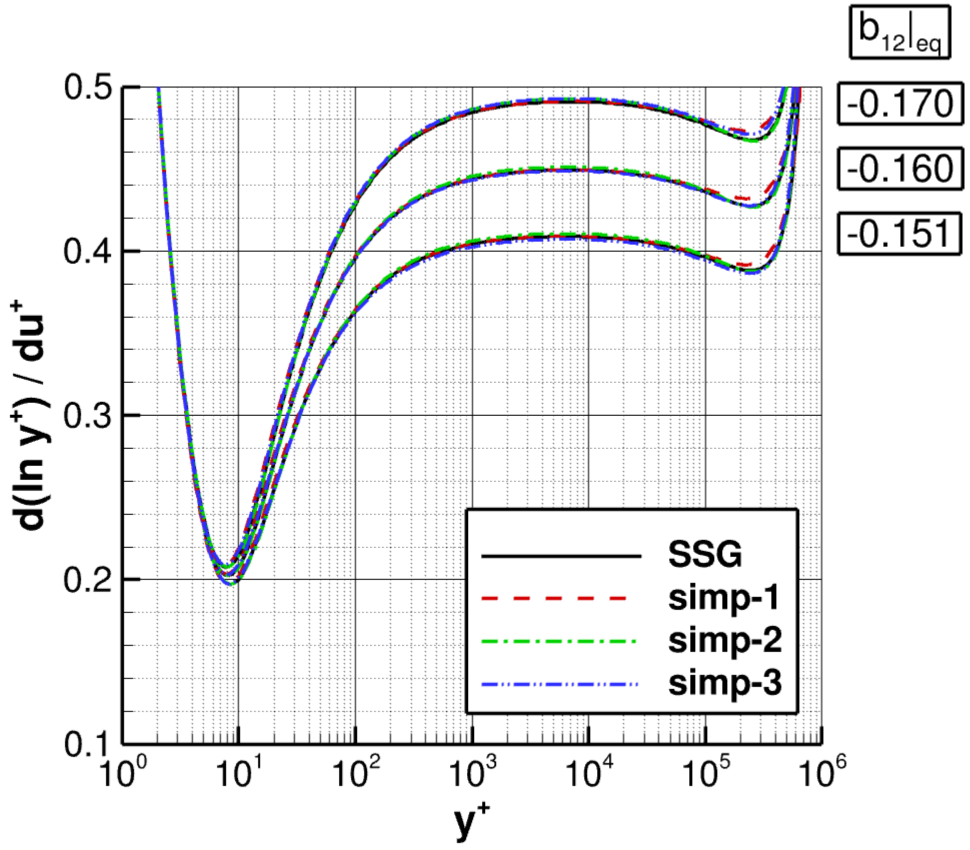
Channel Flow at Re_H = 80e6 (2)

- Log-law



- Determined by equilibrium state
- independent of model form

- Von-Karman constant

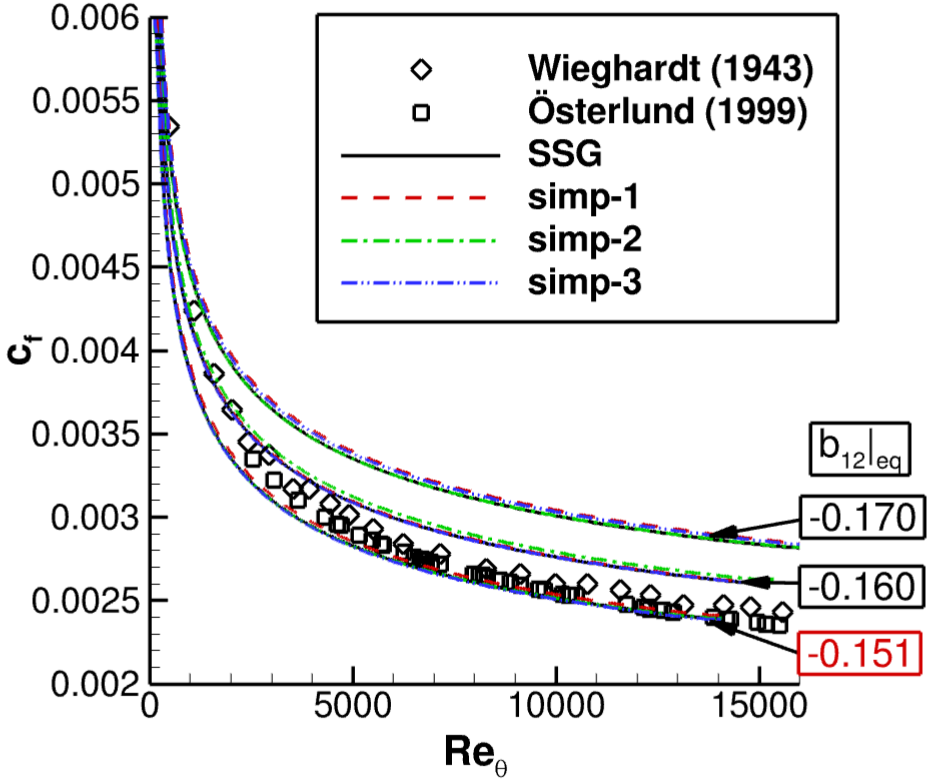


- $\kappa = f(b_{12}|_{eq})$
  cf. Abid & Speziale (1993)

Simulation Results: Fundamental Flows

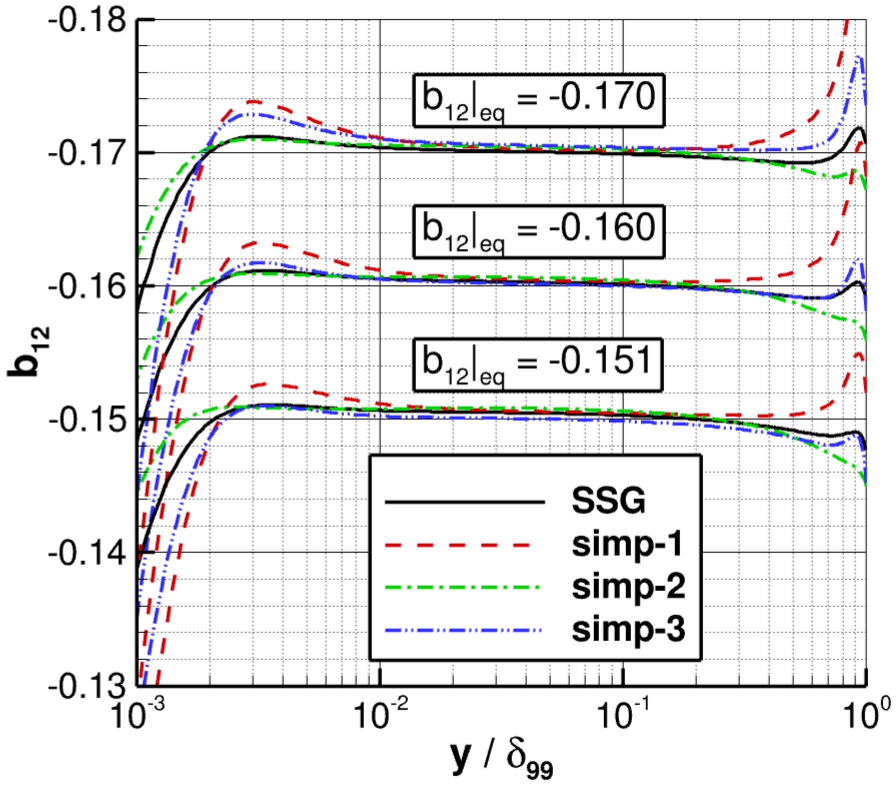
Zero Pressure Gradient Boundary Layer (Flat Plate at Re_c = 10e6)

- Skin friction



- $u_\tau^2 = -R_{12}|_{eq} = -k b_{12}|_{eq}$
- c_f increases with $-b_{12}|_{eq}$

- Shear-stress anisotropy



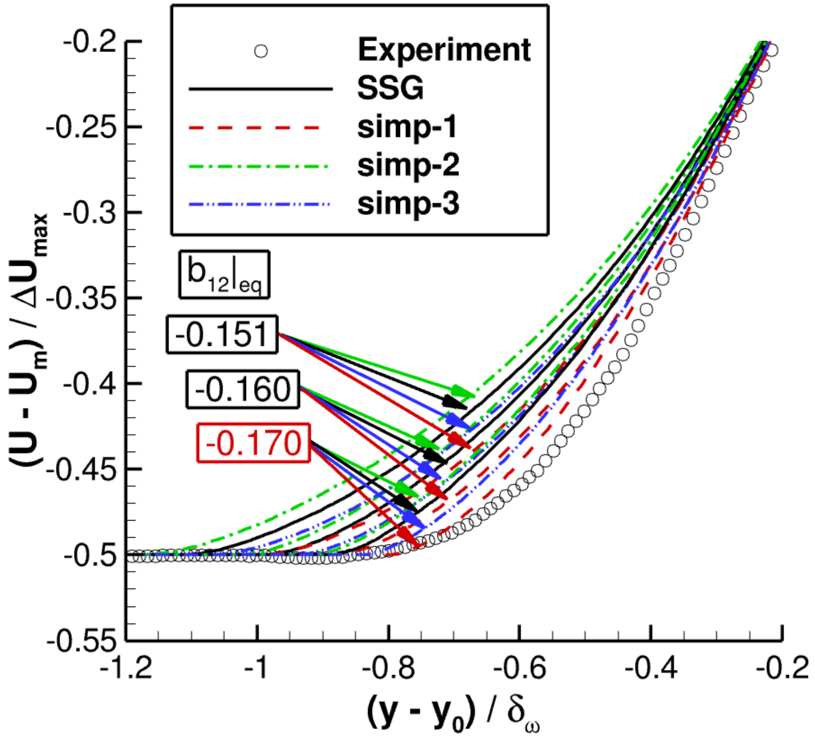
- Target $b_{12}|_{eq}$ reached for $0.01 \leq y/\delta_{99} \leq 0.2$

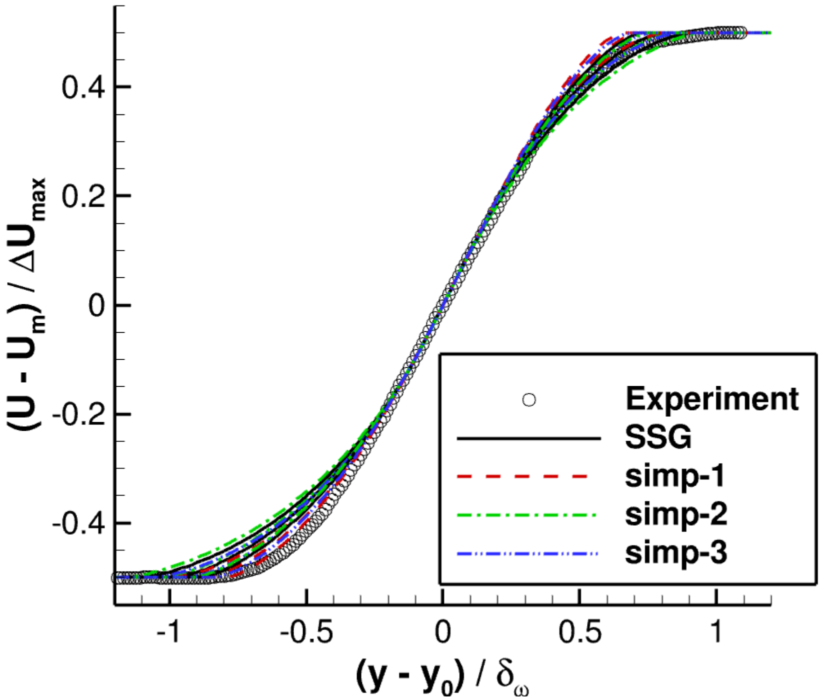
➔ $b_{12}|_{eq} = -0.151$ advantageous at high Re

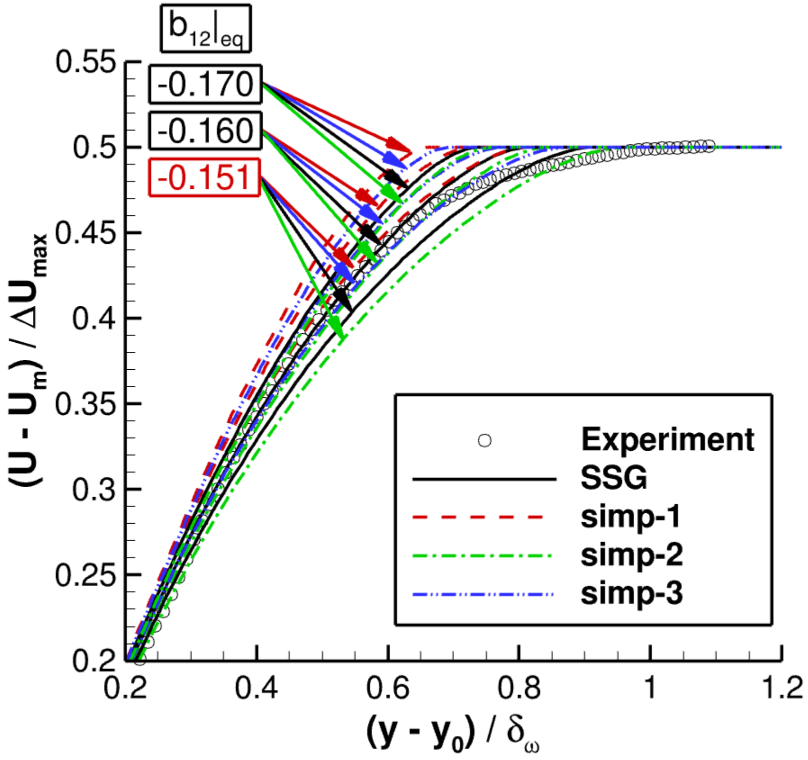
Simulation Results: Fundamental Flows

Mixing Layer (Delville et al., 1989) (1)

- Velocity profile at x = 950mm (most downstream measurement position)







- Lower velocity edge:
- $b_{12}|_{eq} = -0.170$ optimum
  ➔ high momentum transfer

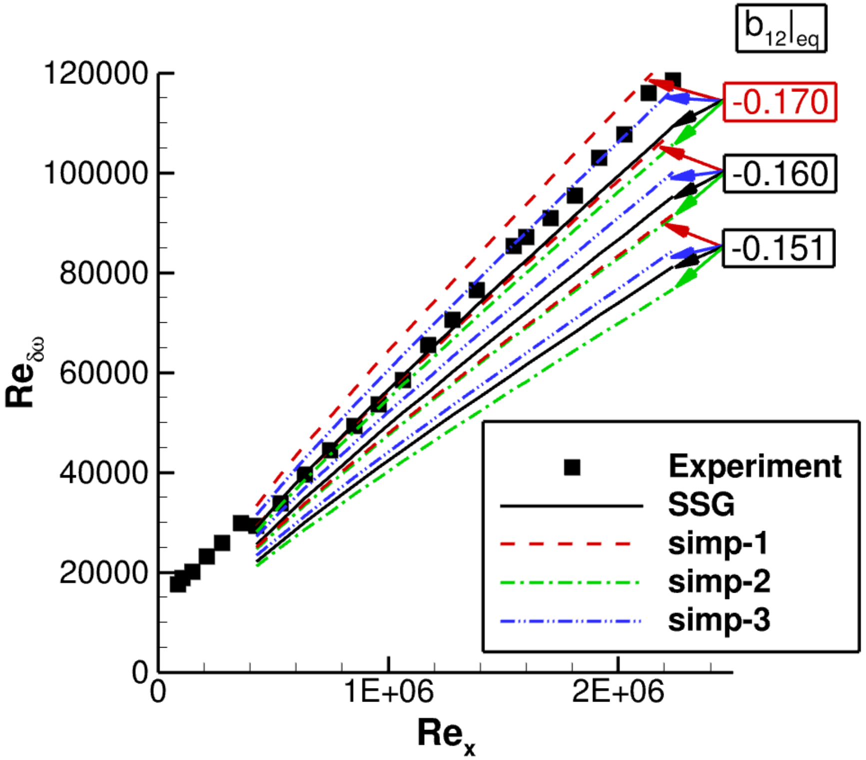
- Central part (equilibrium state): generally good agreement

- Higher velocity edge:
- $b_{12}|_{eq} = -0.151$ optimum
  ➔ low momentum transfer

Simulation Results: Fundamental Flows

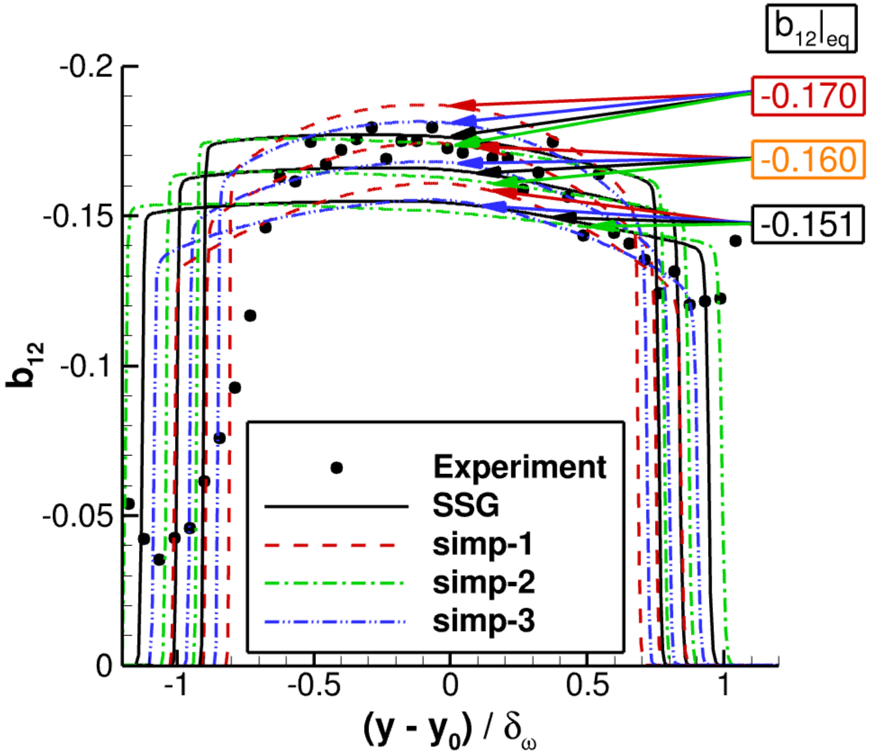
Mixing Layer (Delville et al., 1989) (2)

- Spreading



- Equilibrium state defines spreading rate
- Larger variation due to model form
- $b_{12}|_{eq} = -0.170$ optimum

- Shear-stress anisotropy at x = 950mm



- Target $b_{12}|_{eq}$ reached approximately only
  ➔ equilibrium state not yet reached
  ➔ Re too low?

➔ Equilibrium state incompatible with boundary layer

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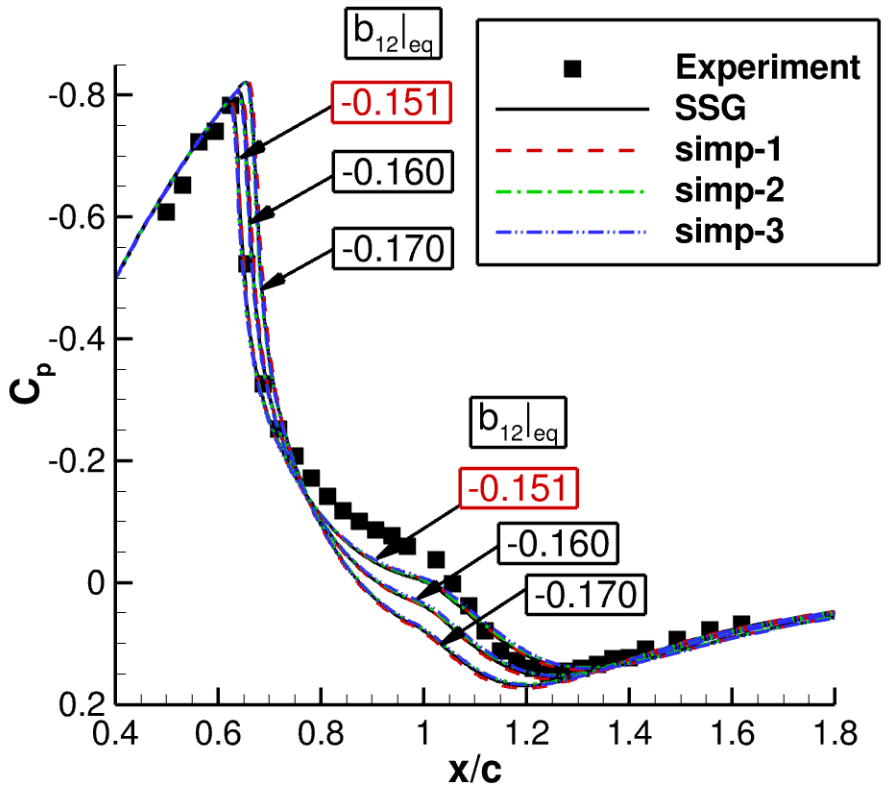
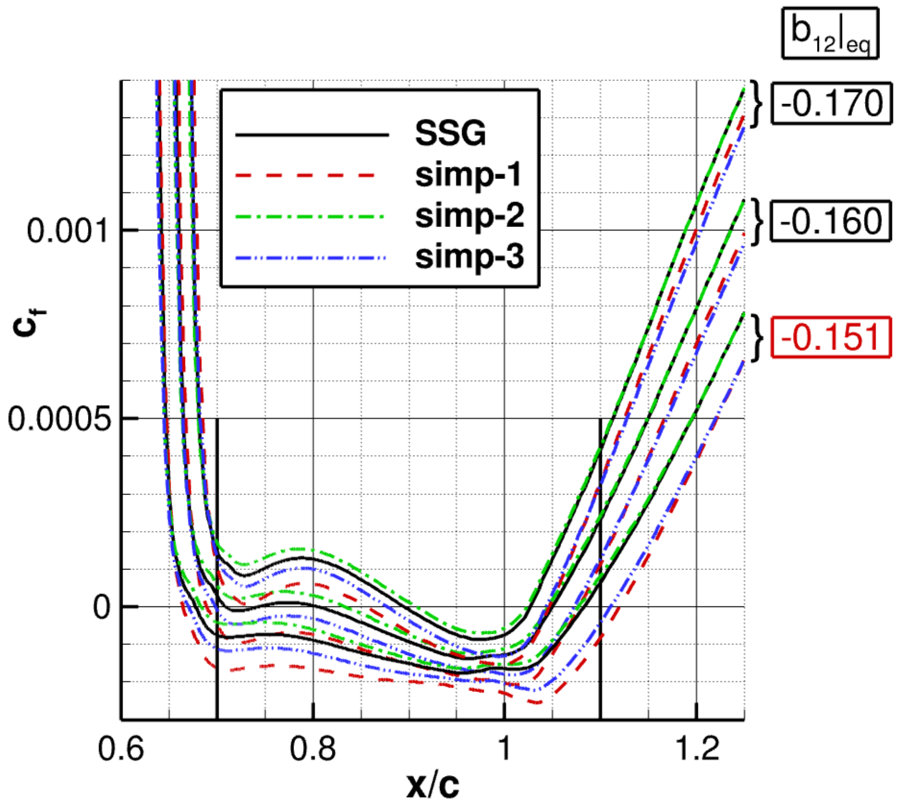
Simulation Results: Applications

Axisymmetric Transonic Bump (Bachalo & Johnson, 1986)

- Pressure distribution



- Skin friction in separation region



- Shock position $= f(b_{12}|_{eq})$
- $b_{12}|_{eq} = -0.151$ optimum (boundary layer)

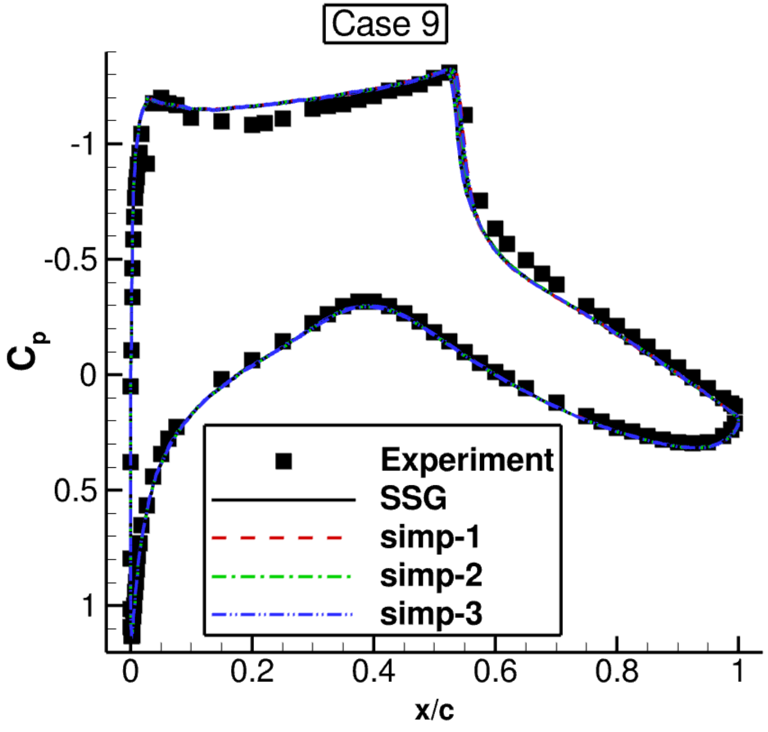
- Minor variation by model form

Results determined by equilibrium state

Simulation Results: Applications

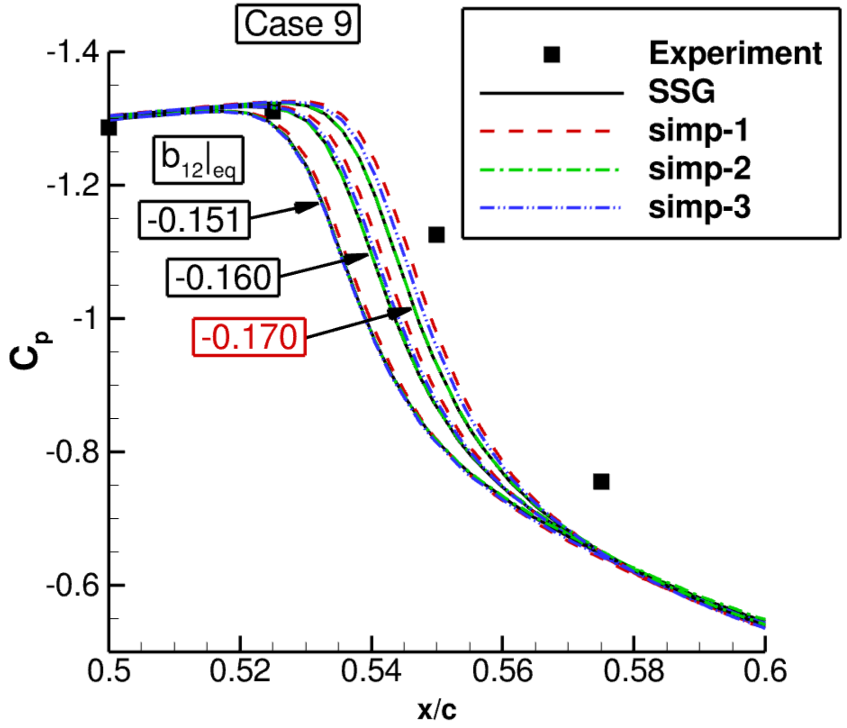
RAE 2822, Case 9 (M = 0.73, Re = 6.5e6)

- Pressure distribution



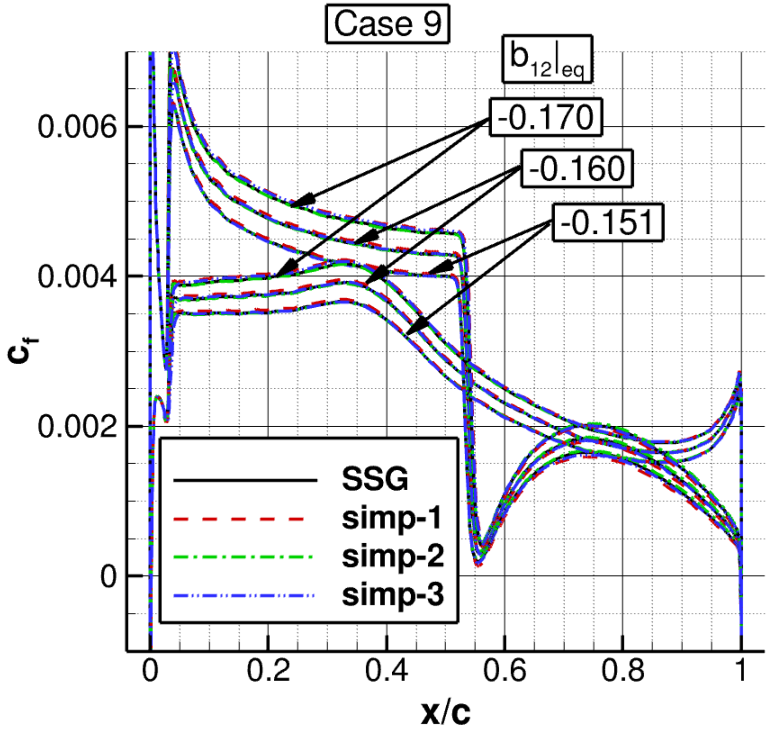
- All results similar

- Pressure distribution, detail



- Shock location = $f(b_{12}|_{eq})$
- $b_{12}|_{eq} = -0.170$ optimum (minor effect)

- Skin friction in separation region



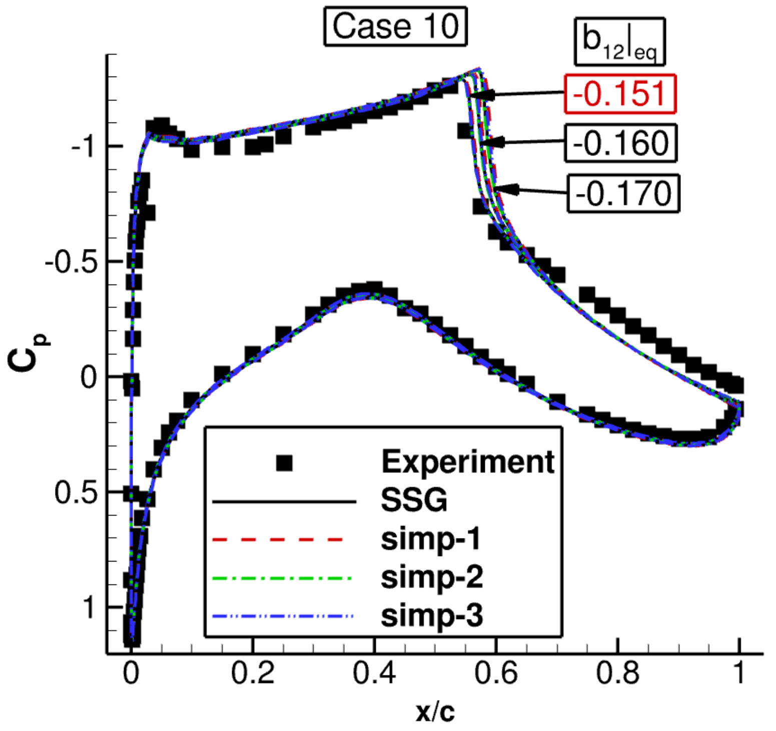
- No separation

Results determined by equilibrium state

Simulation Results: Applications

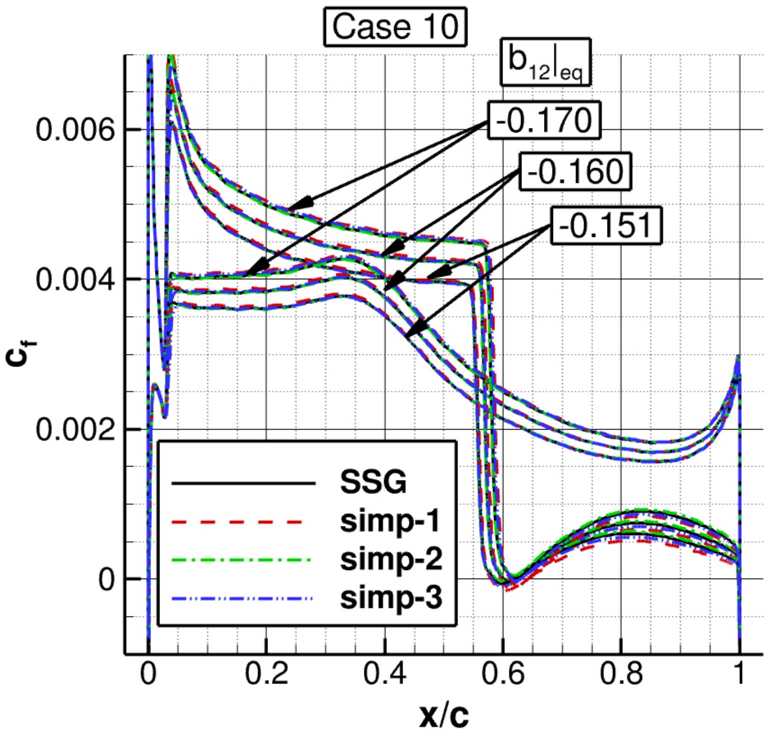
RAE 2822, Case 10 (M = 0.75, Re = 6.2e6)

- Pressure distribution



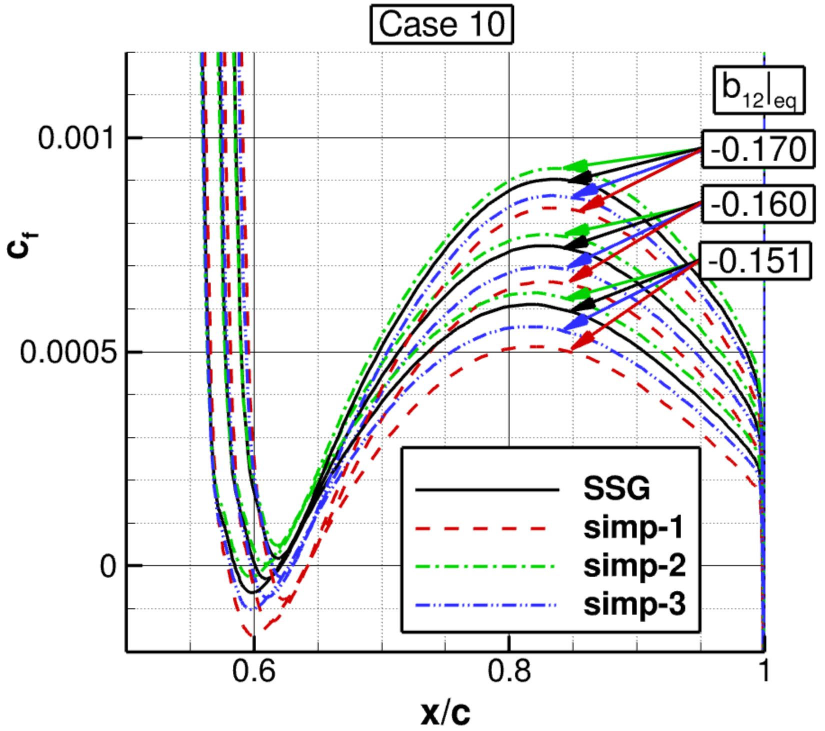
- Shock location $= f(b_{12}|_{eq})$
- $b_{12}|_{eq} = -0.151$ optimum (major effect)

- Skin friction



- Separation $= f(b_{12}|_{eq})$

- Skin friction in separation region



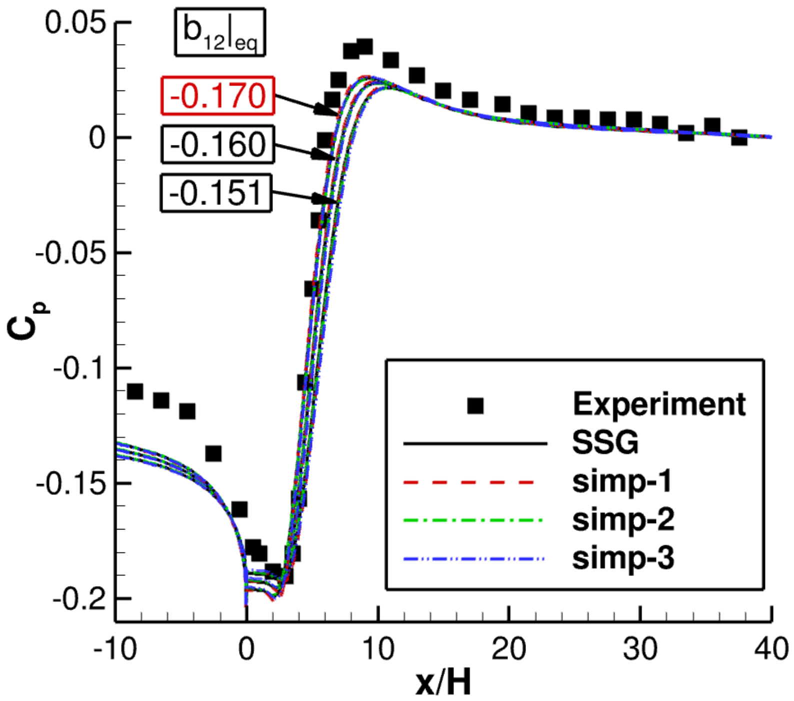
- $|b_{12}|_{eq}|$ increases ➔ separation reduces

Results determined by equilibrium state

Simulation Results: Applications

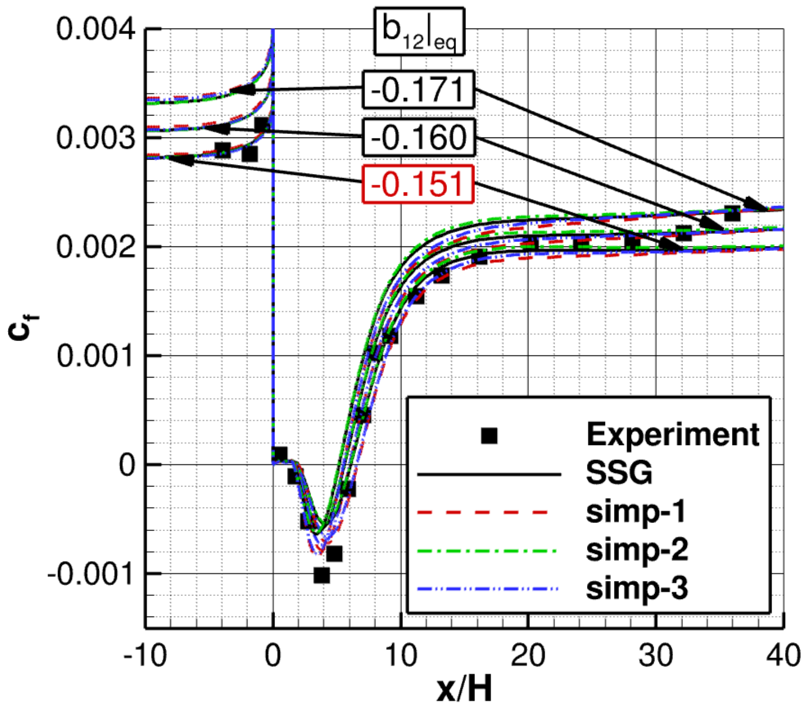
Backward Facing Step (Driver & Seegmiller, 1985)

- Pressure distribution



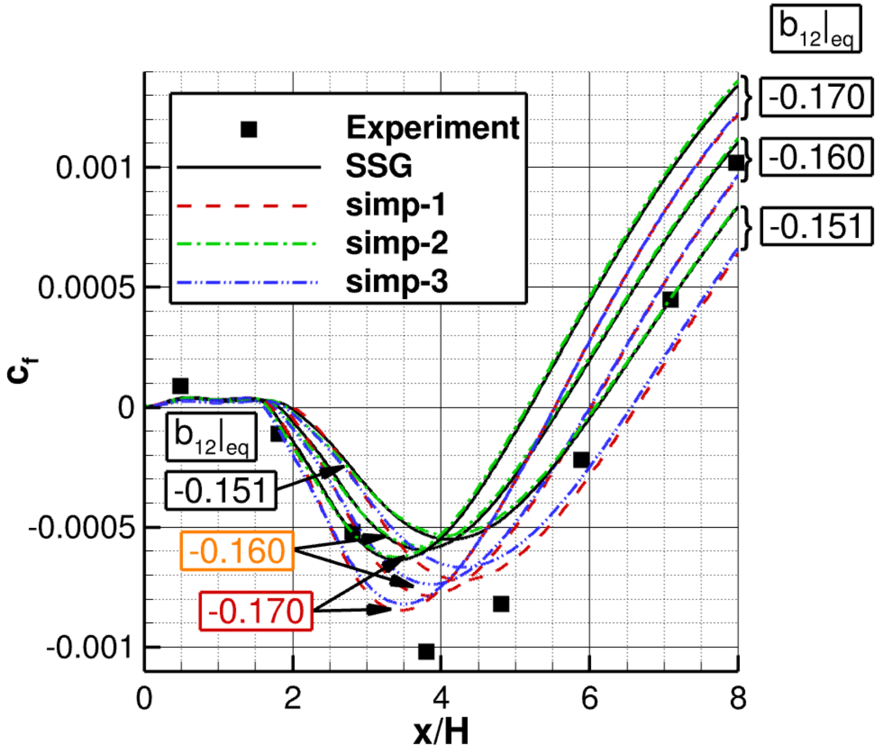
- $b_{12}|_{eq} = -0.170$ optimum
  ➔ mixing layer

- Skin friction



- Inflow/recovery:
  $b_{12}|_{eq} = -0.151$ optimum ➔ BL

- Skin friction in separation region



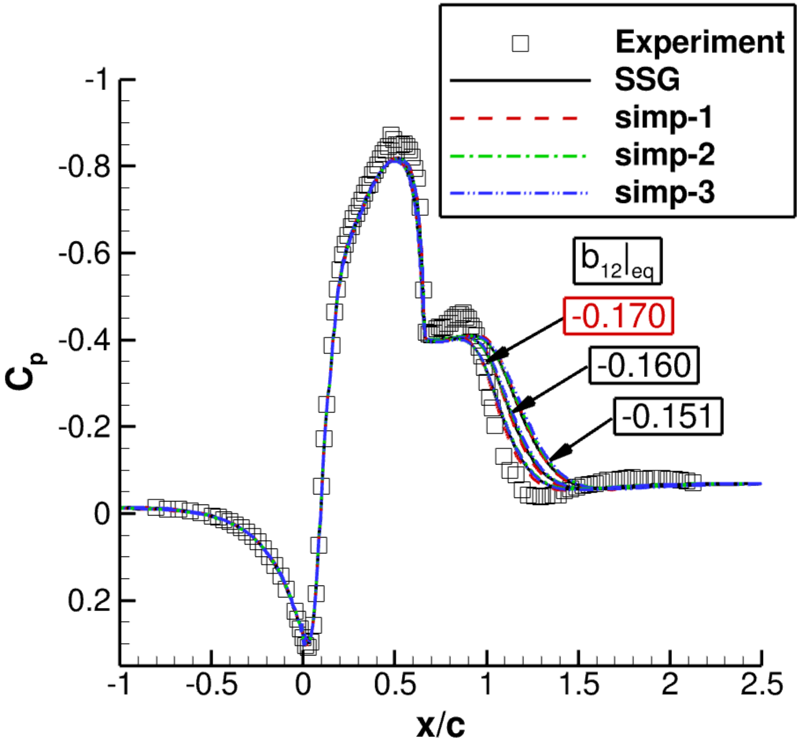
- In bubble:
  $b_{12}|_{eq} = -0.170$ optimum ➔ ML
- Reattachment = $f(b_{12}|_{eq})$

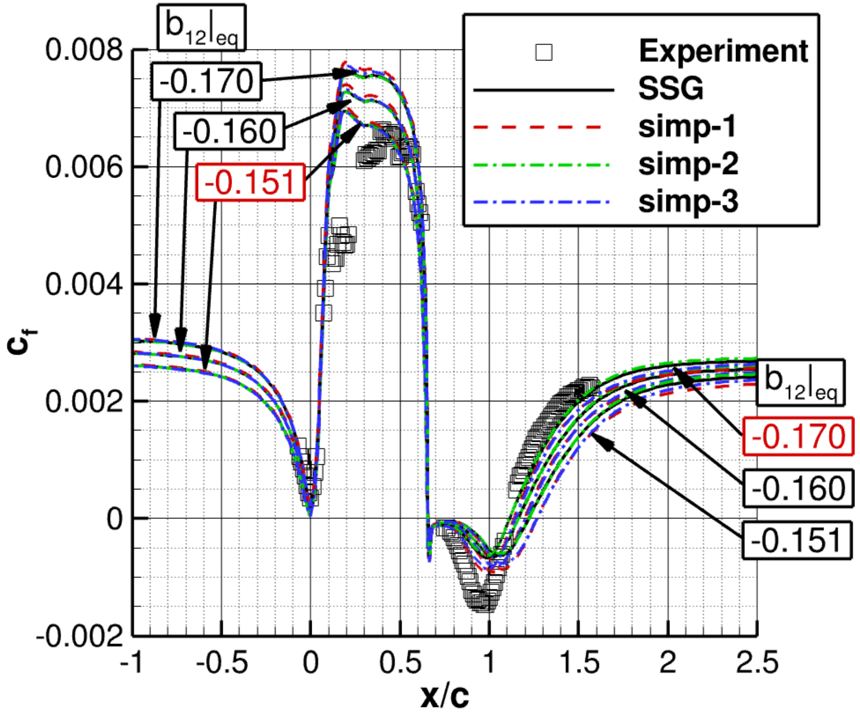
Results determined by equilibrium state

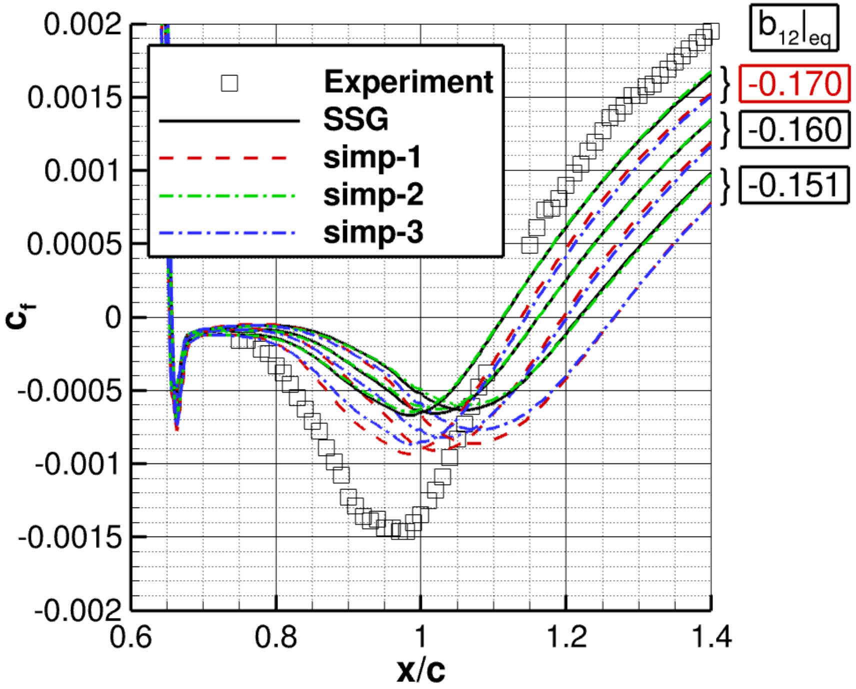
Simulation Results: Applications

NASA Hump (Greenblatt et al., 2006)

- Pressure distribution
- Skin friction
- Skin friction in separation region







- $b_{12}|_{eq} = -0.170$ optimum
  ➔ mixing layer
- Inflow: $b_{12}|_{eq} = -0.151$ opt. (BL)
- Recovery: $b_{12}|_{eq} = -0.170$ opt. (ML)
- In bubble: $b_{12}|_{eq} = -0.170$ opt. (ML)
- Reattachment = $f(b_{12}|_{eq})$

Results determined by equilibrium state

Overview

Potential of Data Driven Methods

Turbulent Equilibrium

- Modeling
  - Calibration condition
  - Determines model predictions
    - Shock location
    - Separation/reattachment
  - Independent of model form

potential conflict

- Physics
  - Equilibrium state depends on flow
  - Boundary layer ≠ mixing layer

Data Driven Turbulence Modeling

- Modifications
  - Model coefficients
  - Model terms
- Reference to application data
  - Change of equilibrium state
    ➔ original calibration deteriorated

➔ No universal solution

Potential of DD/ML Technology

- Protect fundamental conditions (Ph. Spalart!!!)
  ➔ classification of local flow type
  - Implicit (selection of parameters/features)
  - Explicit (supervised/unsupervised learning)
- DD/ML outside protected areas

➔

- Maintains previous achievements
- Improvement beyond fundamental flows (unhampered learning)

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Conclusion

Reynolds Stress Models

- Turbulent equilibrium ➔ calibration condition for pressure-strain correlation
- Equilibrium state determines model predictions

Physics

- Equilibrium state depends on flow type

Potential of Data Driven Methods

- Identification of local flow type (classification) ➔ protection of fundamental conditions
- Modification outside protected areas ➔ improvement beyond fundamental flows

Conclusion

Reynolds Stress Models

- Turbulent equilibrium ➔ calibration condition for pressure-strain correlation
- Equilibrium state (and the corresponding Reynolds stress anisotropies) determines model predictions

Physics

- Equilibrium state depends on flow type (boundary layer vs. mixing layer)
  - ➔ Implication for modelling: Different sets of model coefficients for, e.g., boundary layers and mixing layers

Potential of Data Driven Methods

- Identification of local flow type (classification)
  - ➔ Protection of fundamental conditions (e.g., regions of equilibrium state)
  - ➔ Inside protected areas:
    - ➔ distinguish different equilibrium states with different Rij-anisotropies (boundary layer vs. mixing layer)
      - ➔ adaptation of the model coefficients to the local flow type
- Modification outside protected areas ➔ improvement beyond fundamental flows using ML/DD