UNITED STATES MILITARY ACADEMY
WEST POINT

Lessons from Data-driven Reynolds Stress and Turbulent Scalar Flux Closures

Roles of anisotropy, auxiliary equations, and model extrapolation

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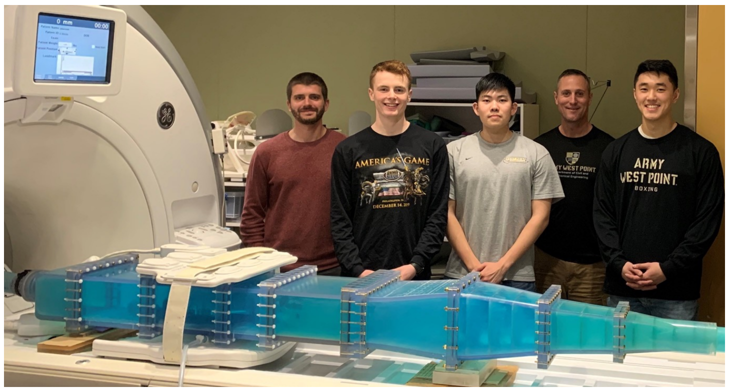
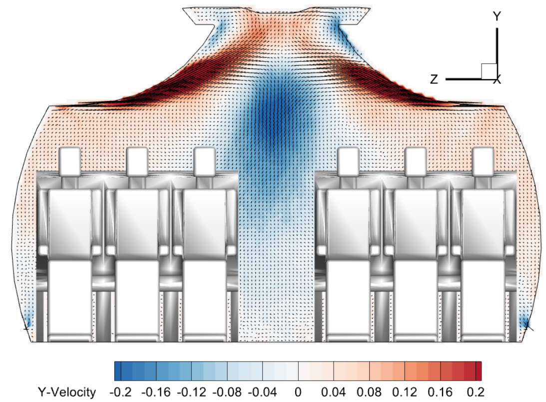
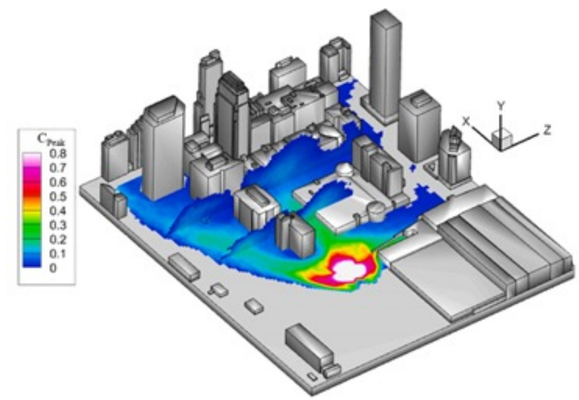
Research Objectives

Understanding heat and mass transfer in complex, 3D flows …

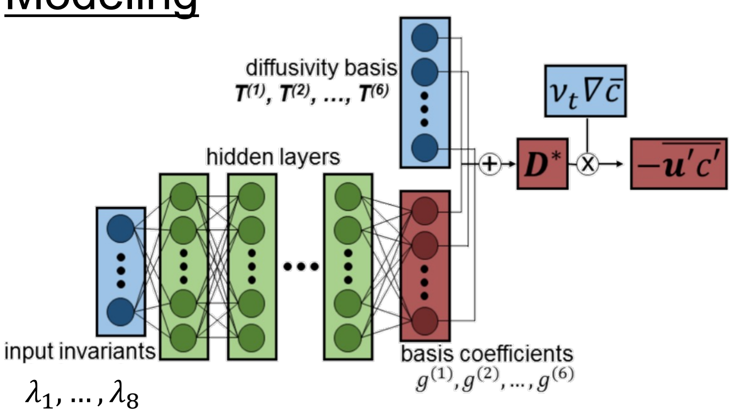
Experiments   3D mean flow measurements using magnetic resonance imaging







Modeling

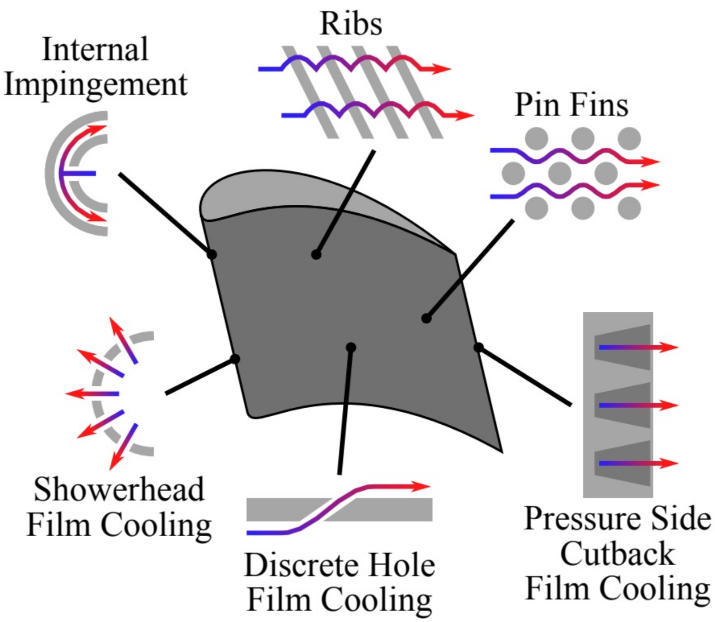


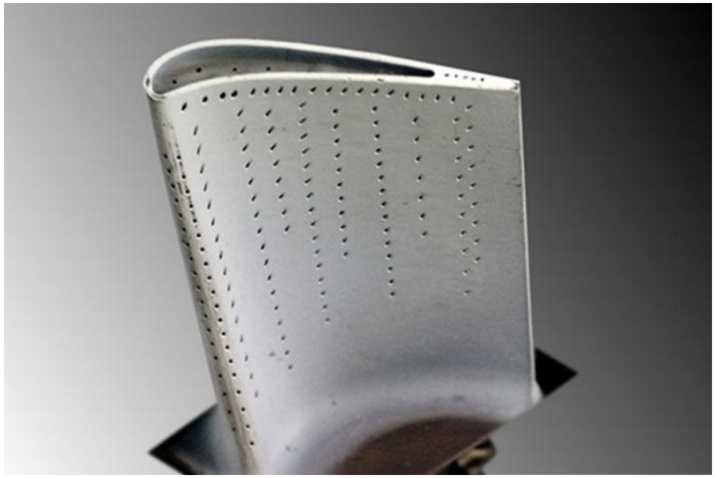
Key Questions

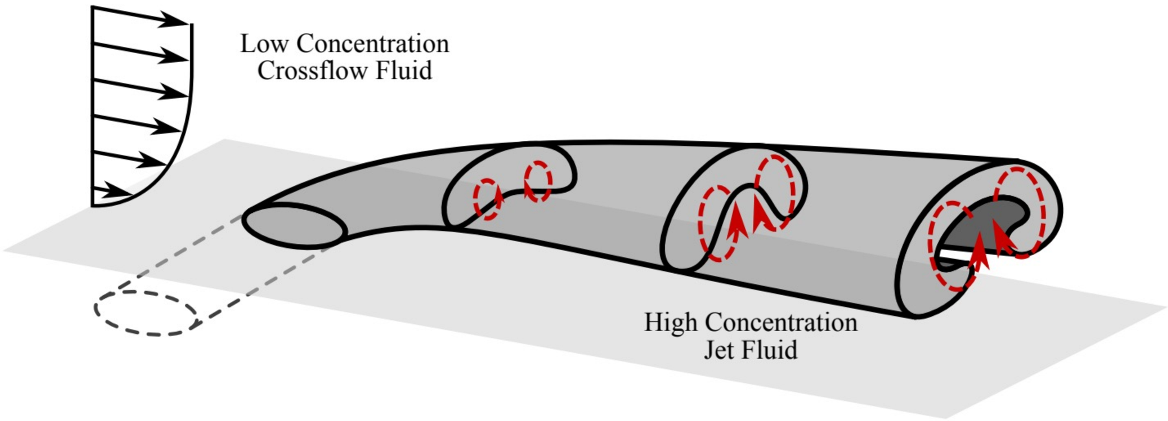
- Relative importance of anisotropy and auxiliary equations
- Performance bounds of model forms
- Data requirements and dealing with extrapolation

Gas Turbine Film Cooling

Blade surfaces are actively cooled using arrays of round and shaped holes



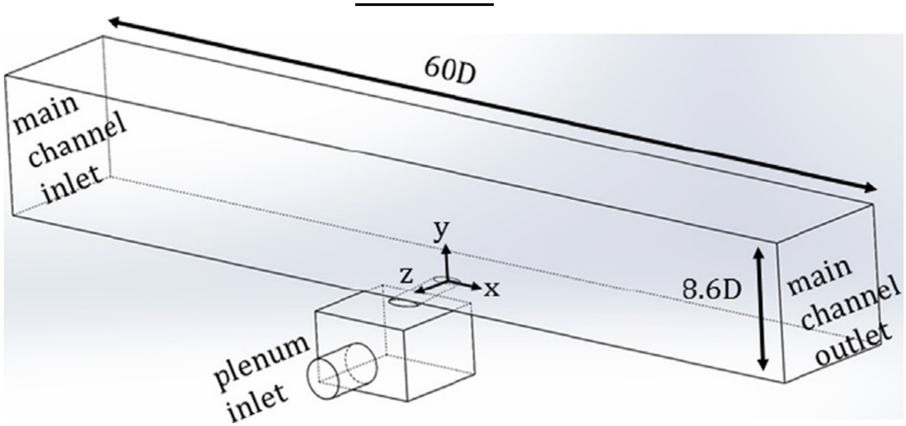




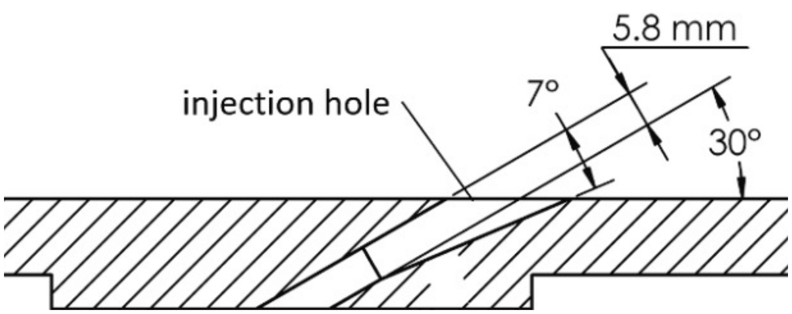
LES Database

- Family of jet in crossflows

Domain

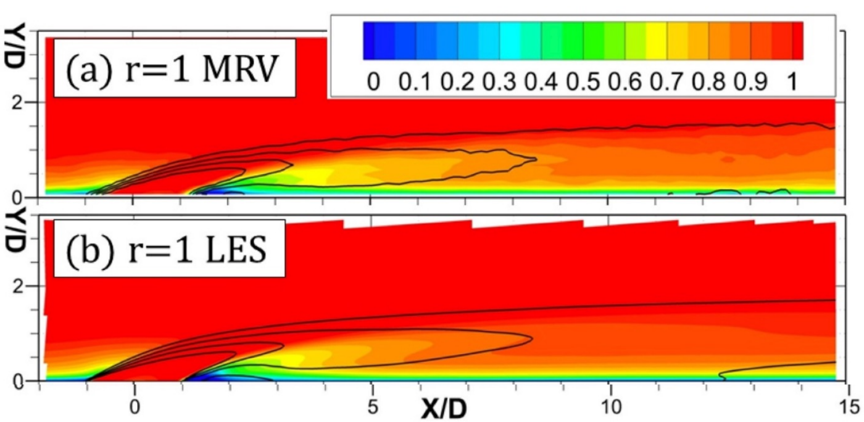


777 Diffuser Hole



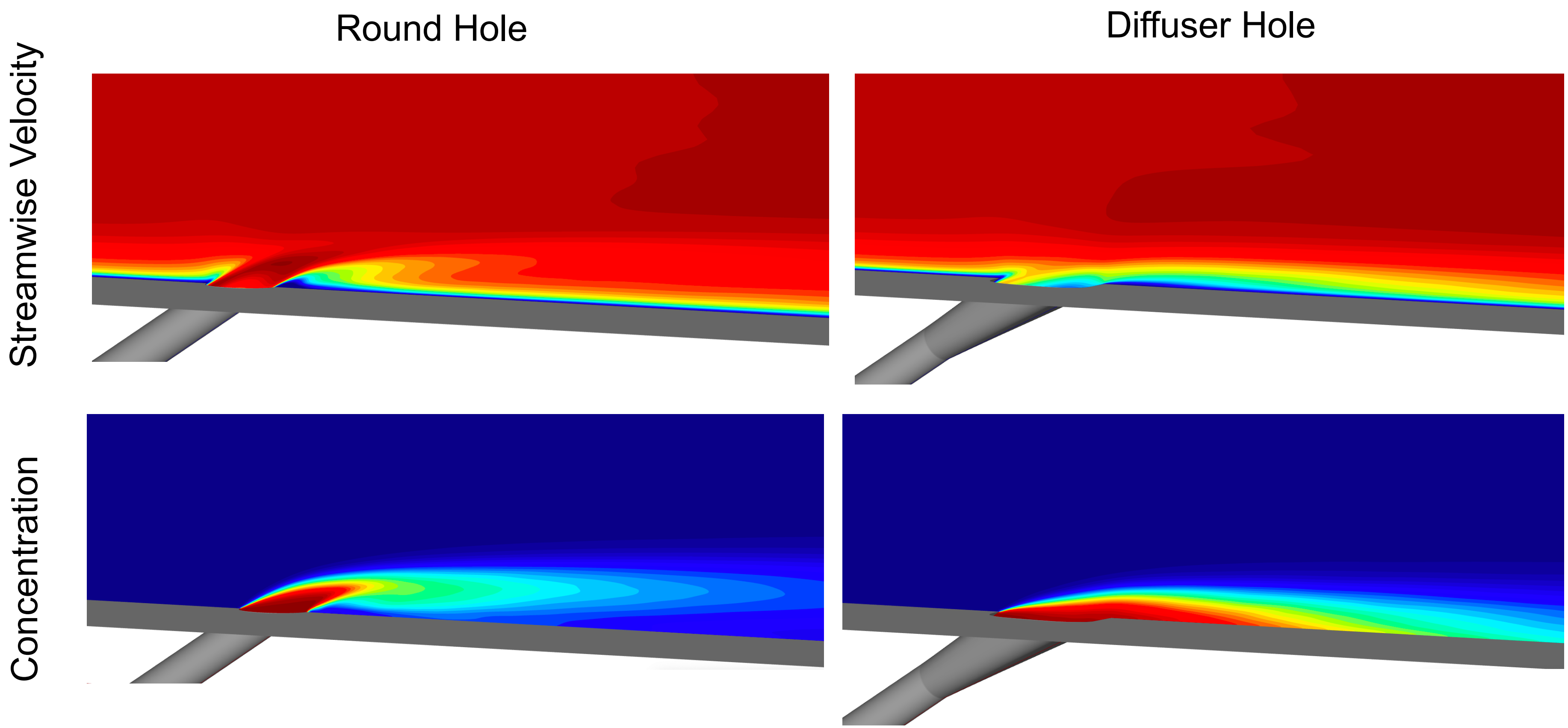
- Conditions
  - Grid: 40M cells, wall-resolved on bottom/hole
  - SGS Model: Vreman, ν_{SGS}/ν < 1
  - Main flow: Re_H = 25,000
    Re_θ = 2,500
    δ_{99}/D = 1.5
  - Jet: BR = 1 (based on metering hole)
    Re_D = 2,900

- Validation against 3D MRI data



Flow Field Overview

- Round hole has a strong counter-rotating vortex pair
- Diffuser hole bleeds low momentum fluid adjacent to the surface

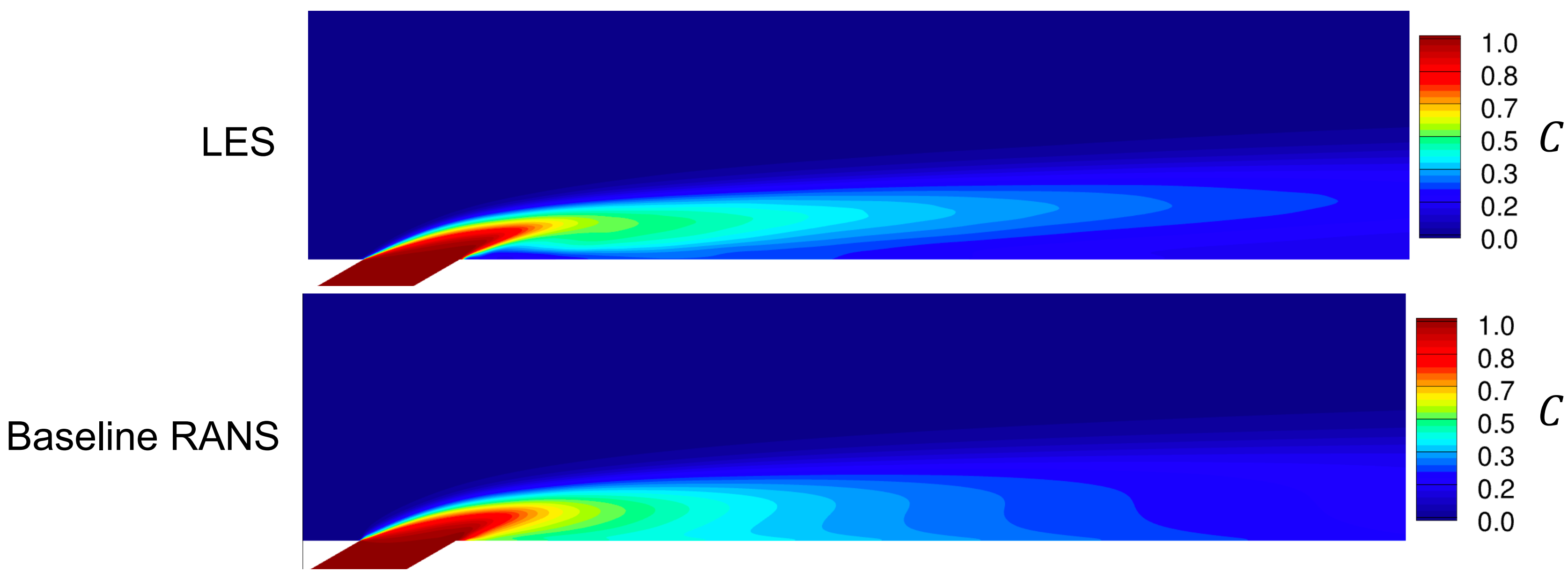


Baseline RANS

- Focus on scalar transport equation with $k-\epsilon$ as a baseline

$$\boldsymbol{U}\cdot\nabla C=\alpha\nabla^2 C-\nabla\cdot\langle c'\boldsymbol{u}'\rangle \quad \text{where} \quad \langle c'\boldsymbol{u}'\rangle=-\frac{\nu_T}{Pr_t}\nabla C \quad \text{and} \quad Pr_t\approx 0.85$$

- Isolate turbulent scalar flux model
  - Same grid as LES
  - Use LES mean velocity
  - Determine k and ϵ by solving their transport equations using the LES velocity field



ML Turbulent Scalar Flux Modeling

- Turbulent scalar flux: $\langle c' \boldsymbol{u}' \rangle = -\nu_T \boldsymbol{D}^* \nabla C$ $\quad \boldsymbol{D}^* = \boldsymbol{F}(\boldsymbol{S}, \boldsymbol{R})$ $\quad \boldsymbol{S}, \boldsymbol{R}$ normalized by k/ϵ

[1]Milani et al., JFM, 2021
[2]Banko et al., In prep.
[3]Zheng, 1994

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- Tensor basis neural network (TBNN-s)[1,2]
  - Input invariants

$$\lambda_1 = tr(\boldsymbol{S}^2) \quad \lambda_3 = tr(\boldsymbol{S}^3) \quad \lambda_5 = tr\left(\boldsymbol{S}^2\boldsymbol{R}^2\right) \quad \lambda_7 = \sqrt{k}d/\nu$$

$$\lambda_2 = tr(\boldsymbol{R}^2) \quad \lambda_4 = tr(\boldsymbol{S}\boldsymbol{R}^2) \quad \lambda_6 = tr(\boldsymbol{S}^2\boldsymbol{R}^2\boldsymbol{S}\boldsymbol{R}) \quad \lambda_8 = \nu_T/\nu$$

  - Tensor basis (derived from vector basis[3])

$$\boldsymbol{T}^{(1)} = \boldsymbol{I} \quad \boldsymbol{T}^{(3)} = \boldsymbol{R} \quad \boldsymbol{T}^{(5)} = \boldsymbol{R}^2$$

$$\boldsymbol{T}^{(2)} = \boldsymbol{S} \quad \boldsymbol{T}^{(4)} = \boldsymbol{S}^2 \quad \boldsymbol{T}^{(6)} = \boldsymbol{S}\boldsymbol{R} + \boldsymbol{R}\boldsymbol{S}$$

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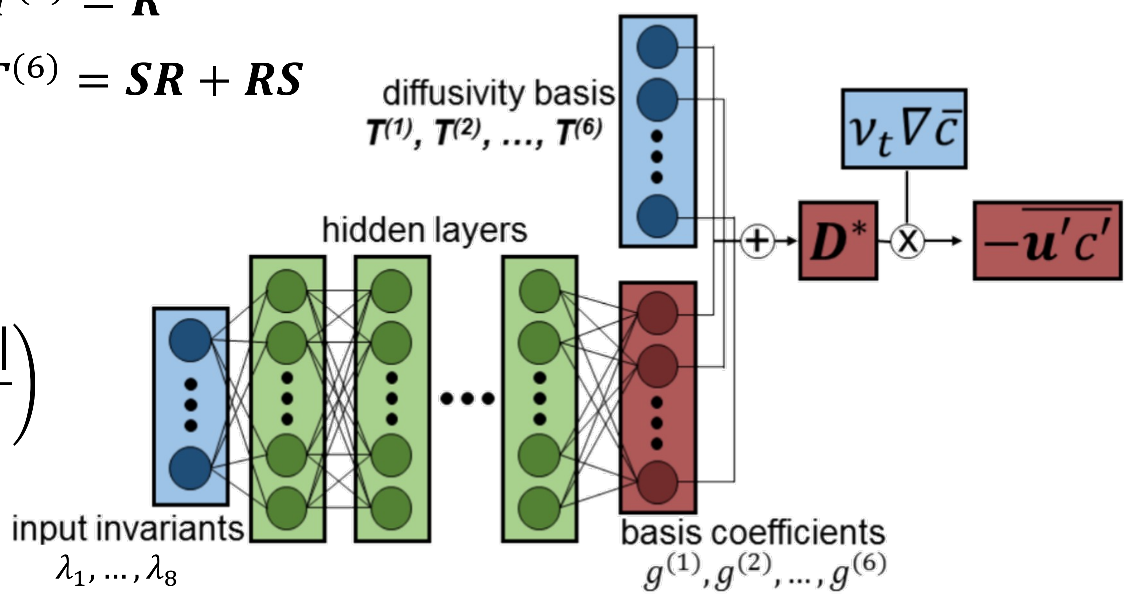
  - 8 layers, 20 nodes/layer
  - Loss function

$$L = log\left(\frac{\|\langle \boldsymbol{u}'c'\rangle_{ML} - \langle \boldsymbol{u}'c'\rangle_{LES}\|}{\|\langle \boldsymbol{u}'c'\rangle_{LES}\|}\right)$$



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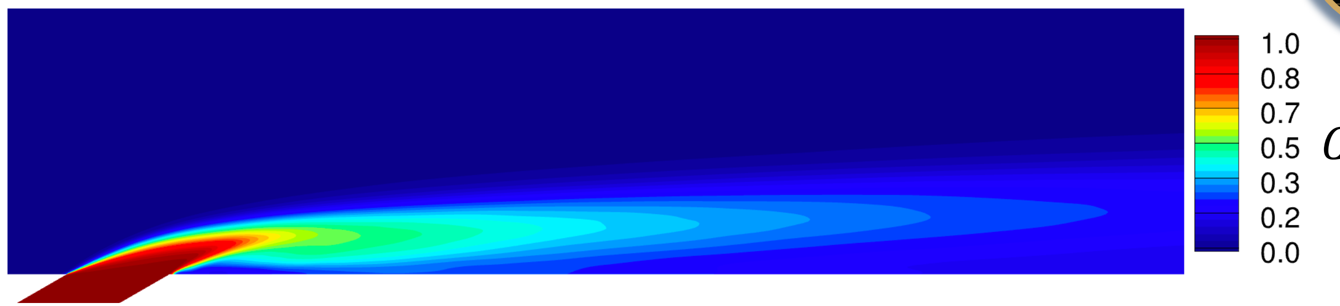
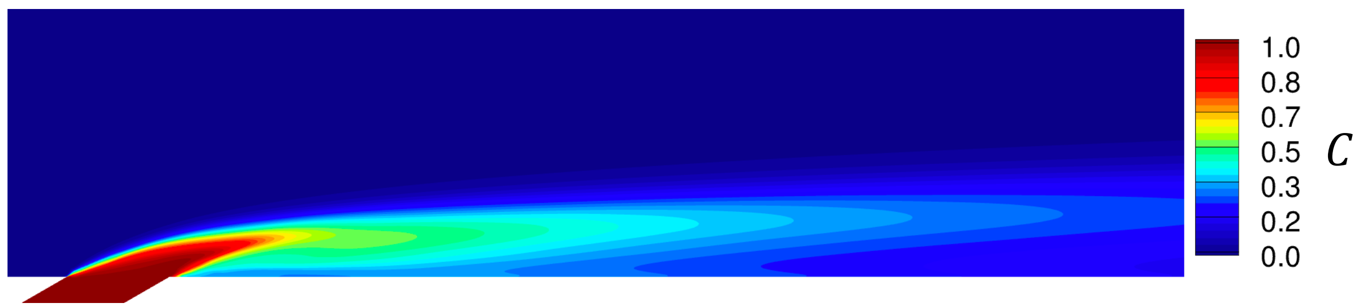
Round Hole Results

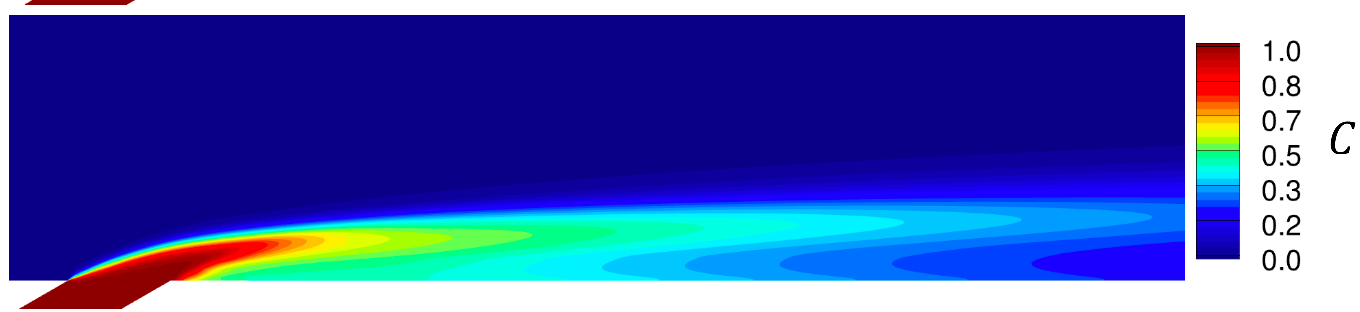
LES



Trained on Round Hole



Trained on Diffuser Hole



Baseline RANS

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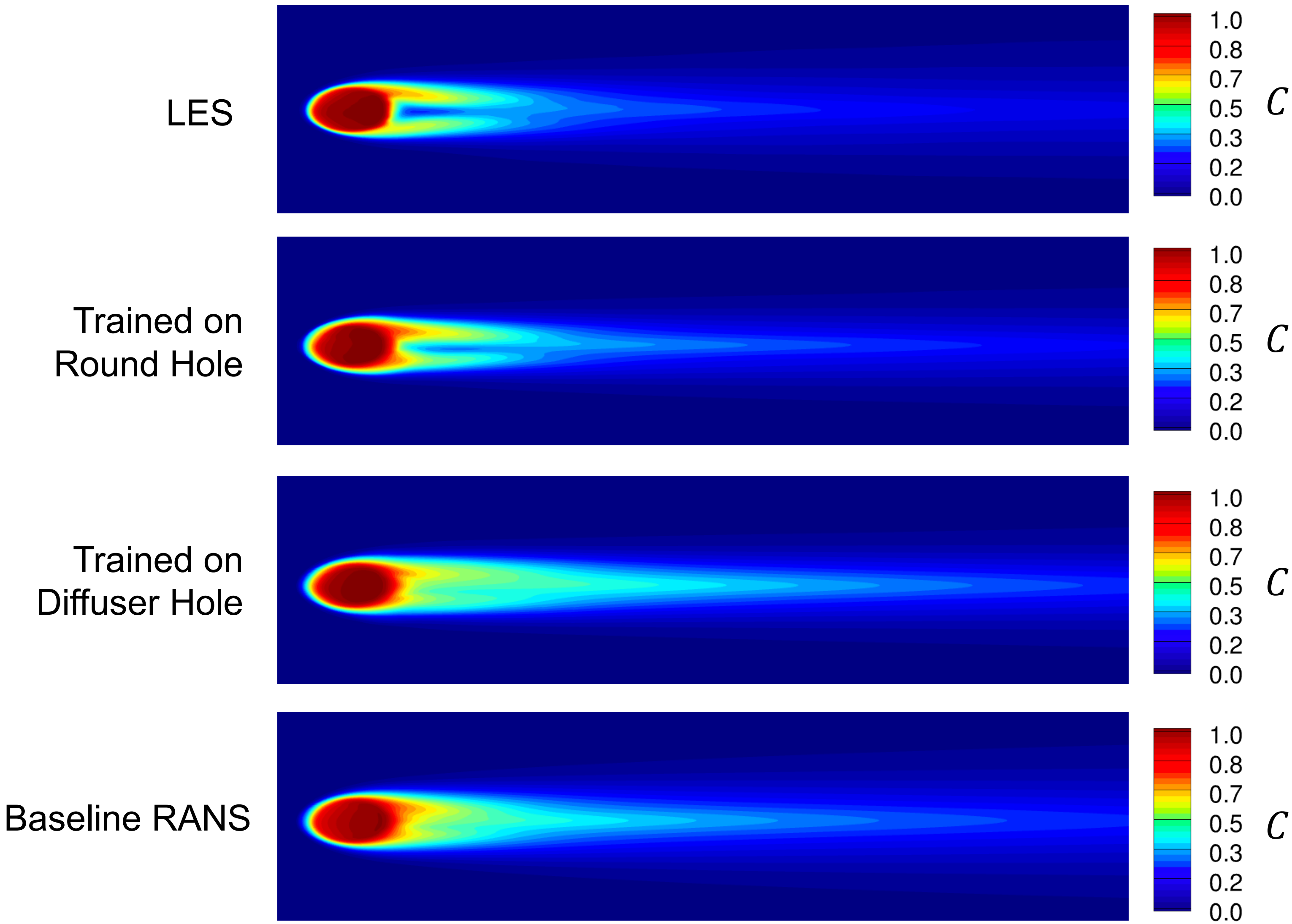
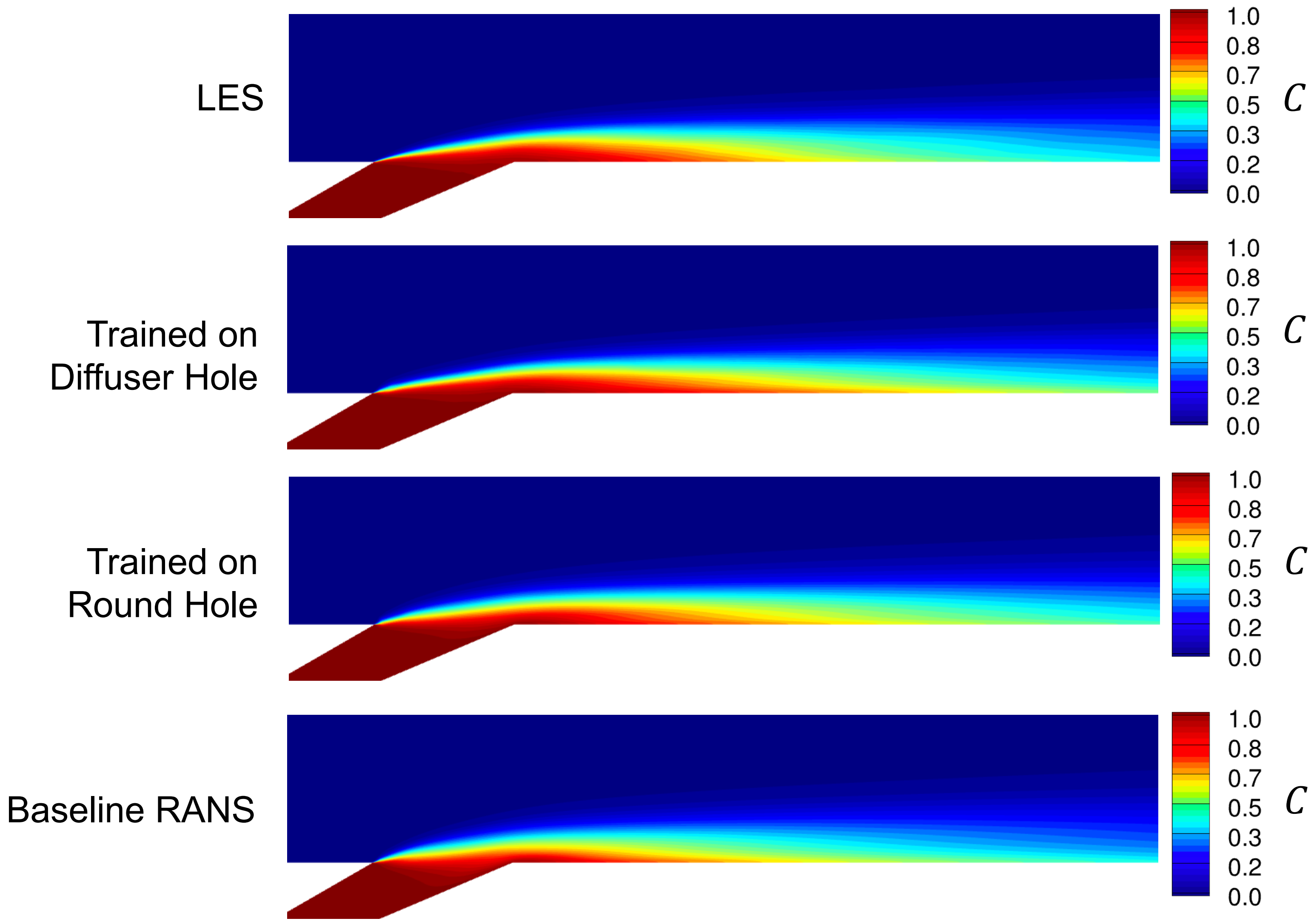
Round Hole Results



Diffuser Hole Results



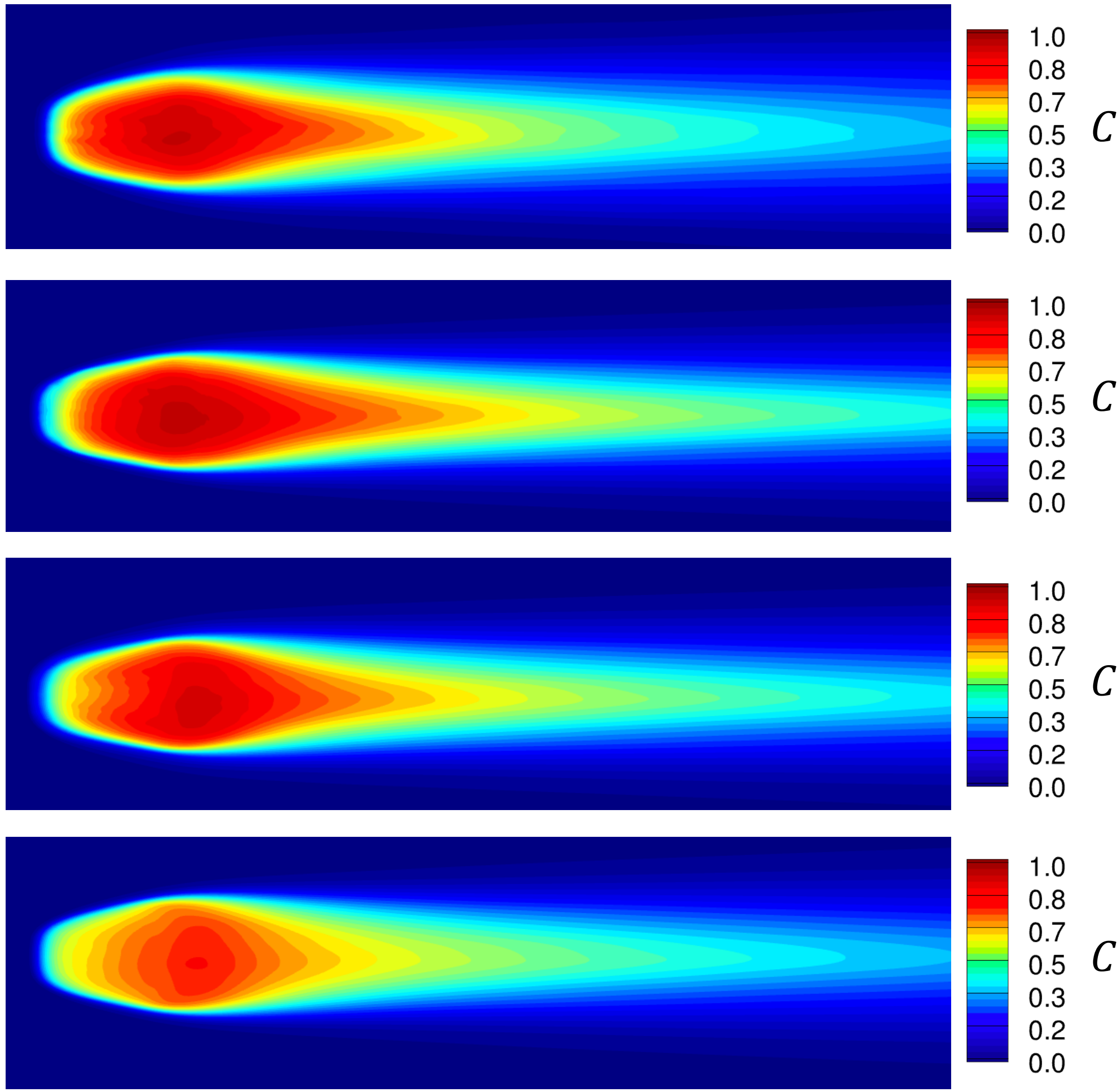
Diffuser Hole Results

LES

Trained on Diffuser Hole

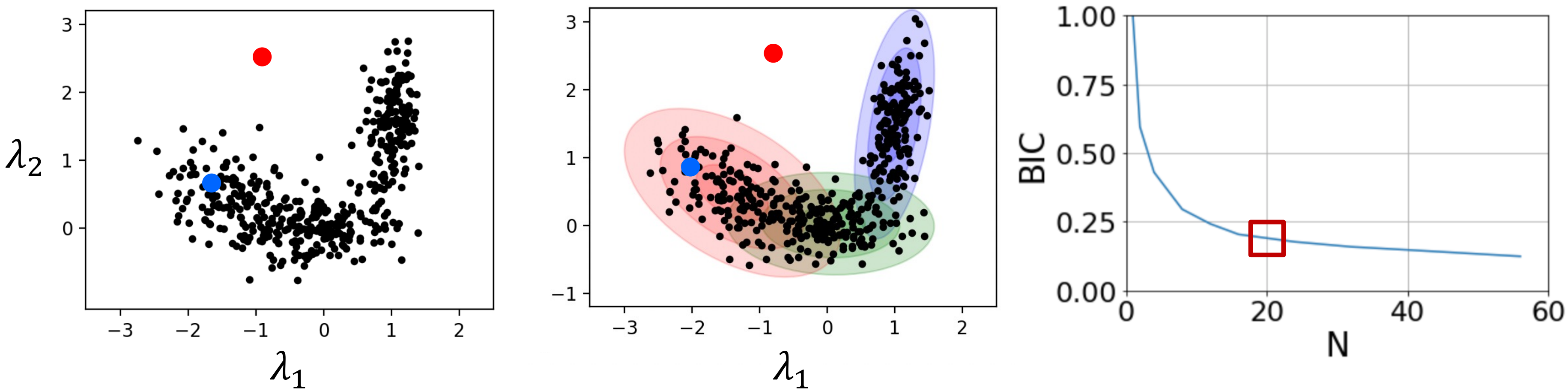
Trained on Round Hole

Baseline RANS



Extrapolation Detection

- Gaussian mixture model



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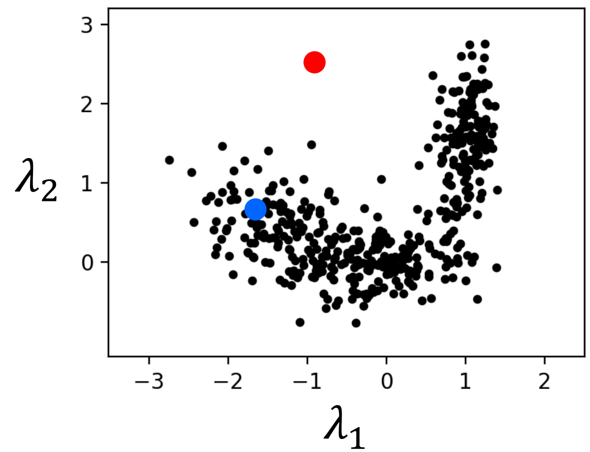
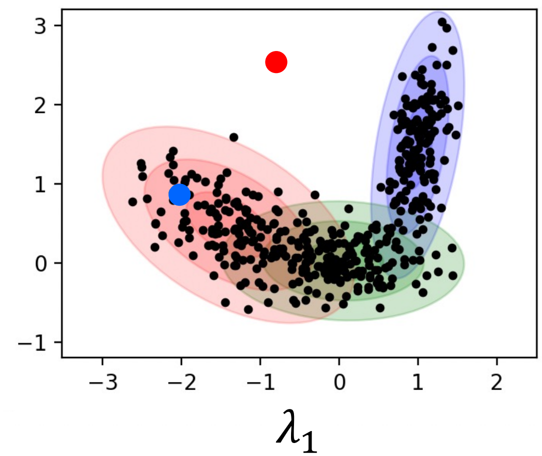
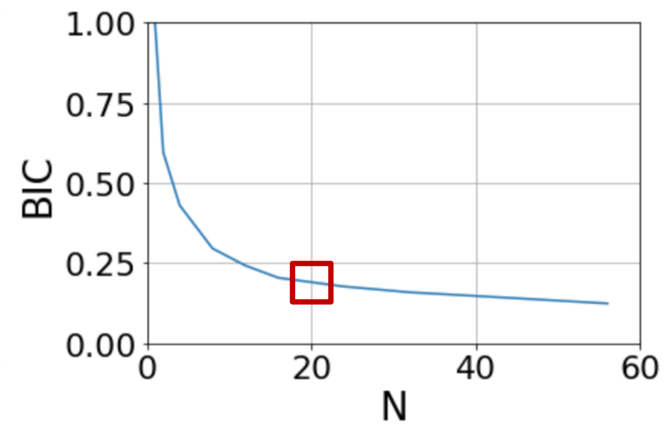
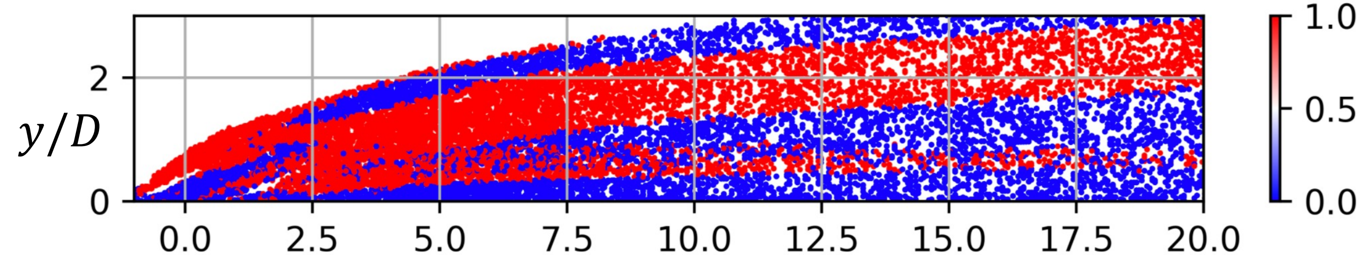




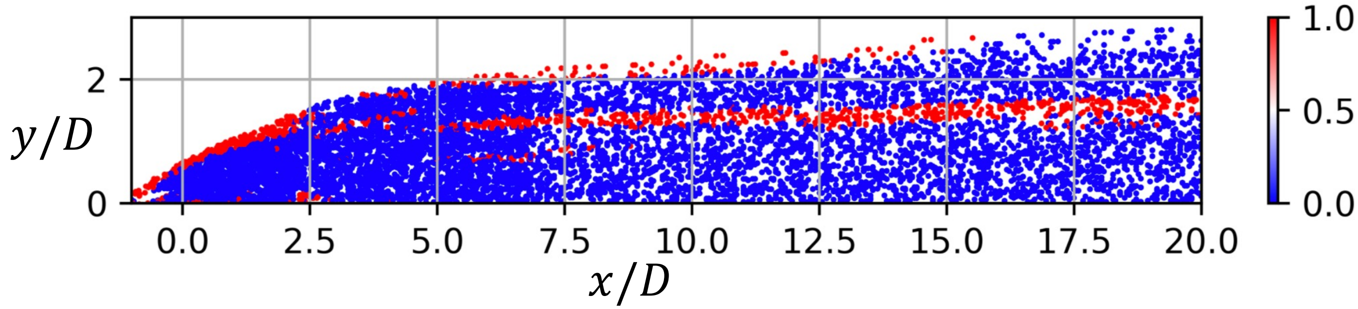


- Extrapolatory points: Blue = interpolating Red = extrapolating

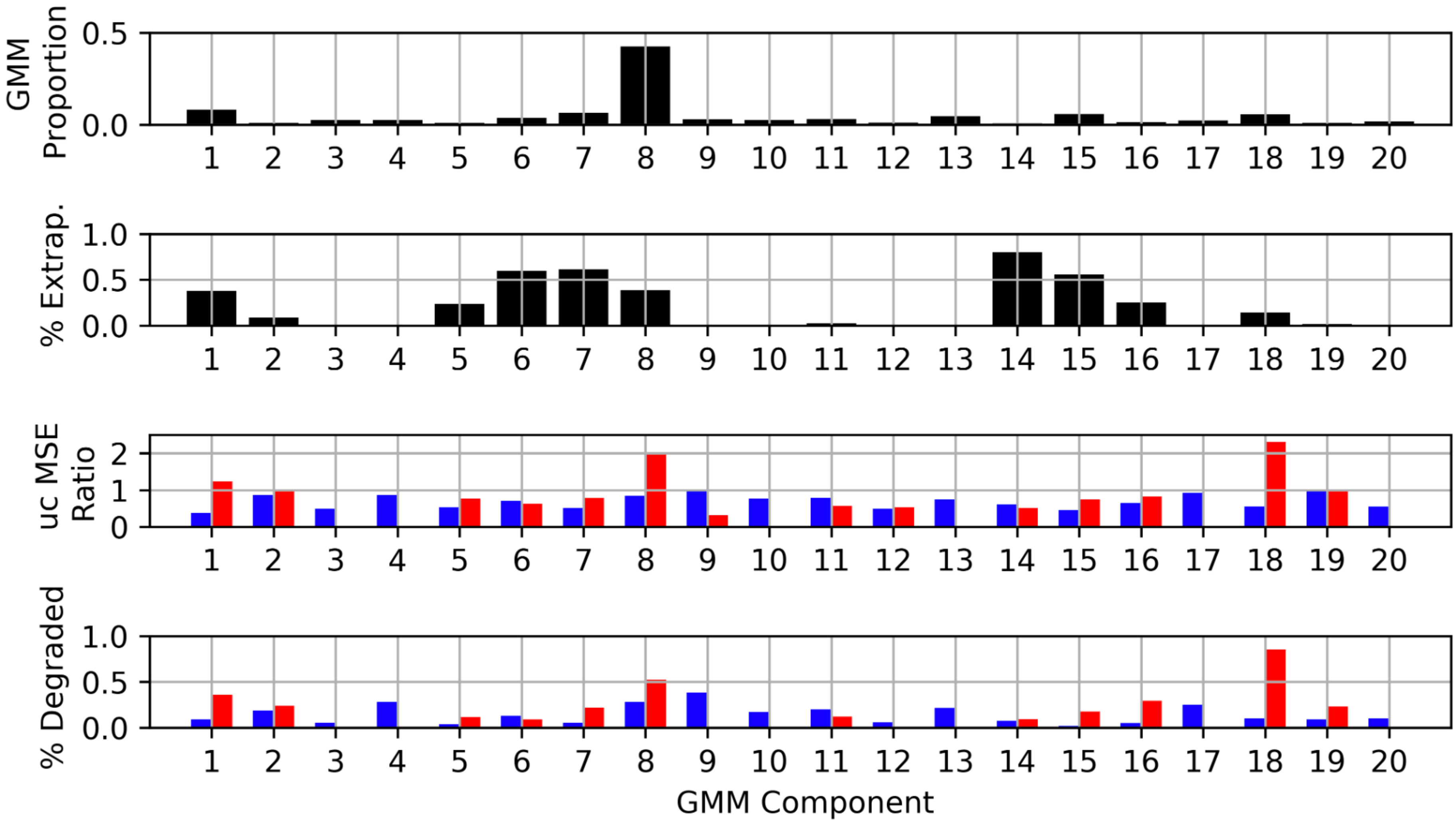
Round Hole
(Train on Diffuser)



Diffuser
(Train on Round)



Breakdown by Gaussian



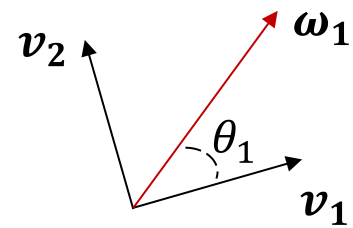
Blue = interpolating Red = extrapolating

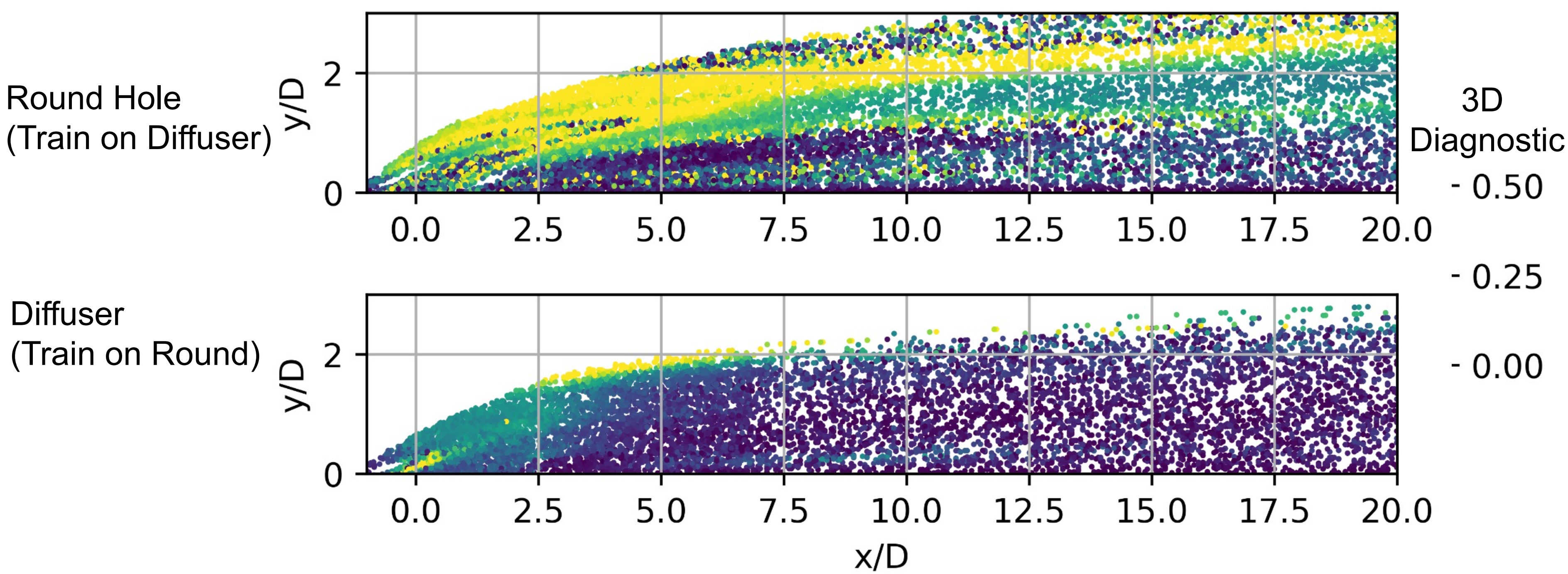
Flow Dimensionality

- Detecting statistically 2D vs. 3D flow

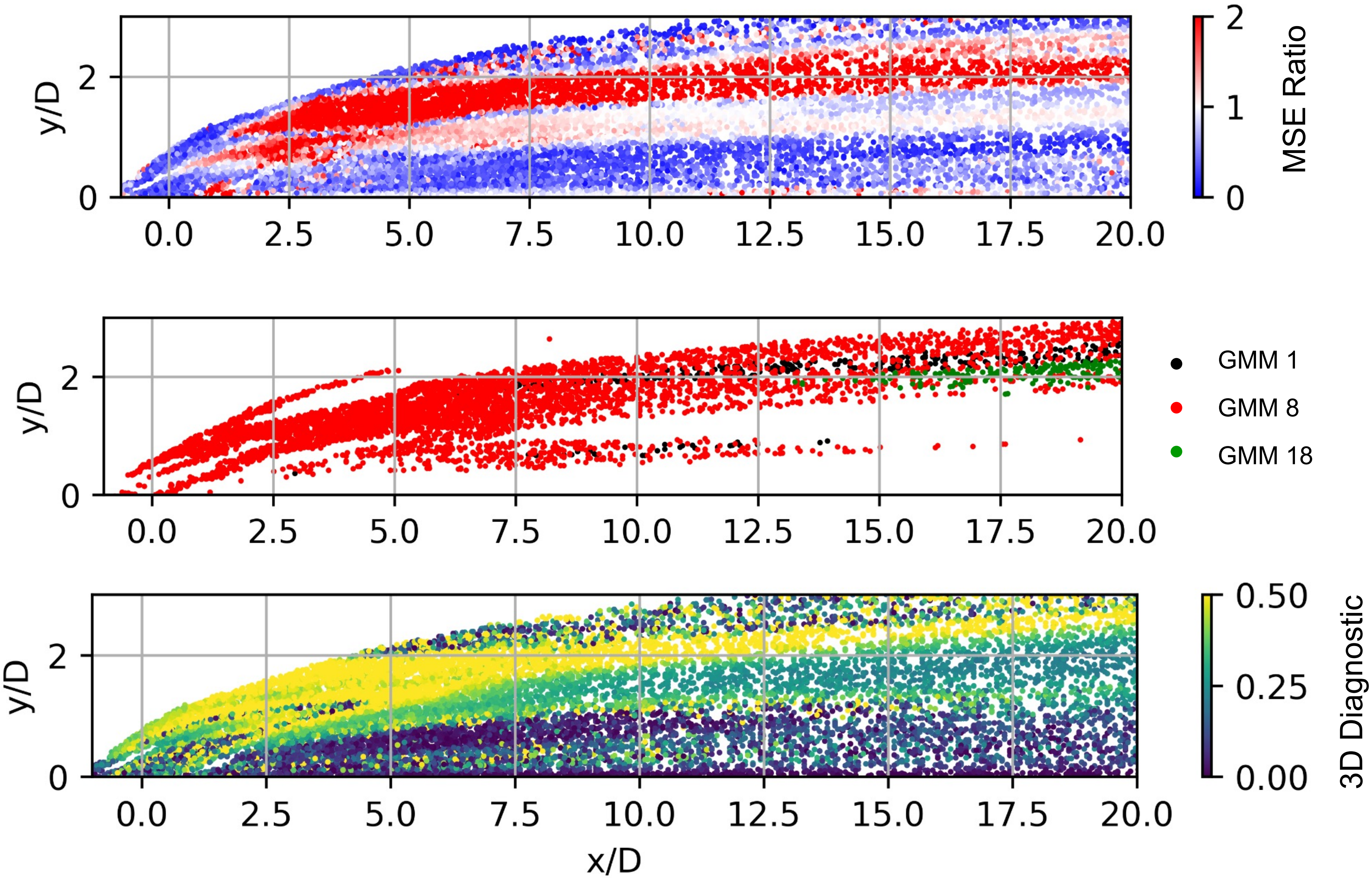
$$\lambda_5 = tr(\boldsymbol{S}^2\boldsymbol{R}^2) = \frac{1}{4}\omega^2 \sum_{i=1}^{3} \cos^2\theta_i\, s_i \;\; = \begin{cases} \neq 0 & \text{if 3D} \\ = 0 & \text{if 2D} \end{cases}$$

$$\boldsymbol{S} = \boldsymbol{V} \begin{bmatrix} \boldsymbol{s_1} & & \\ & \boldsymbol{s_2} & \\ & & \boldsymbol{s_3} \end{bmatrix} \boldsymbol{V}^T$$





"2D"-Trained Network Extrapolates in "3D"



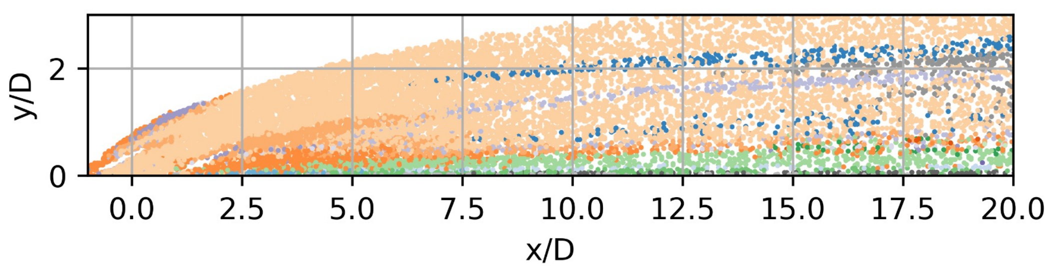
Conclusions

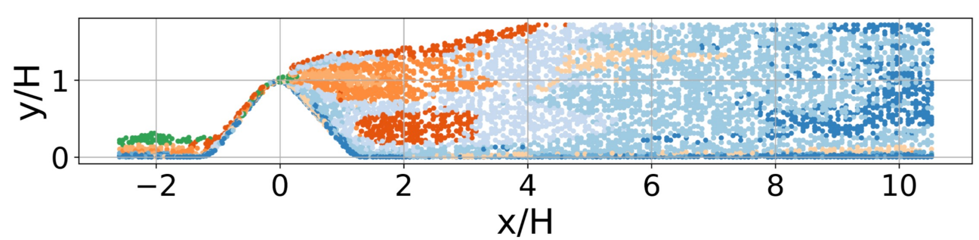
- GMM efficiently characterizes interpolation and extrapolation points
- Predictions were improved on average for interpolation cases

  Caution: High error for interpolating points is possible
- Extrapolation and error were significant when moving from quasi-2D flow to 3D flow

  3D: 5 independent invariants → 2D: 1 indp. inv. → 1D: 1 indp. inv.
- Big picture: an irony of data limitation

40 million CVs
≠
40 millions useful training points





Acknowledgments

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  - National Science Foundation
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