Hybrid closure modeling with laminar to turbulent transition

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2:30-2:45, 29 July

Yin, Ge, Durbin *JFM 915 (2021)*

Can a hybrid— RANS/LES —model, for turbulence simulation, plausibly capture laminar-to-turbulent transition?

If yes, how?

Length scale interpolation

(non-zonal) Hybrid model interpolates between RANS and LES *formulas*

Motive: eddy-resolving simulation for engineering—very coarse (RANS) to fine grids

Simplest is DES (Spalart, 2009): only ℓ is interpolated

$$\ell_{RANS} = \sqrt{k}/\omega \qquad \ell_{LES} = C_{DES}\,\Delta; \;\; \Delta \equiv V^{1/3}$$

c.f., $\min(\ell_{RANS}, \ell_{LES})$

$$\boxed{\ell = (1 - f_d)\ell_{RANS} + f_d \min(\ell_{RANS}, \ell_{LES})}$$

$f_d \to 0,\; y \to 0;\quad f_d = 1$ is the eddy simulation region

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The *shielding function* is from the RANS literature

$$f_d = 1 - \tanh(8r_d)^3; \quad r_d = \frac{k/\omega + \nu}{\kappa^2 d_w^2 \sqrt{|S|^2 + |\Omega|^2}}$$

ℓ^2 - ω Formulation

Reddy, Ryon, Durbin *Int. J. Heat Fluid Flow (2014)*
Yin, Reddy, Durbin *Phys. Fl. (2015)*

$$\nu_t = \ell^2 \omega$$

ω-equation is unchanged; viewed as diffusively filtered S:

$$\frac{D\omega}{Dt} - \nabla \cdot (\nu + \sigma_\omega k/\omega)\nabla\omega + C_{\omega 2}\omega^2 = 2C_{\omega 1}|S|^2$$

Only change to k-equation is production

$$\frac{Dk}{Dt} - = 2\,\boxed{\ell^2\omega}\,|S|^2 - C_\mu k\omega + \nabla \cdot \left[\left(\nu + \sigma_k \frac{k}{\omega}\right)\nabla k\right]$$

Recall $\quad \ell_{LES} = C_{DES}\Delta$

Adapt C_{DES} to turbulence intensity

Computable test filter stress, as in LES

$$L_{ij} = -\widehat{\bar{u}_i \bar{u}_j} + \hat{\bar{u}}_i \hat{\bar{u}}_j$$
$$M_{ij} = \hat{\Delta}^2 \hat{\omega} \widehat{S_{ij}} - \Delta^2 \widehat{\omega S_{ij}} \qquad \hat{\Delta}/\Delta = 2$$

$\widehat{\bullet}$ is test filter; $\bar{u}$ is the resolved velocity field

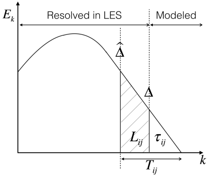
Germano identity is replaced by least squares, which can be wrong – hence lower bound:

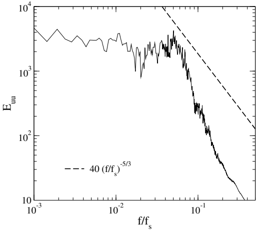
$$C_{dyn}^2 = \max(L_{ij} M_{ij} / 2M_{ij}^2, 0)$$

$C_{DES} = C_{dyn}$ is OK on fine enough grid – but it's awful on fairly coarse grids (hybrid simulation type)

Adapt C_{DES} to grid resolution

There is not enough small scale energy to provide test filter stress

Ideally: 

Coarse grid: 

Make lower bound depend on resolution of eddies:

$$\xi \equiv \frac{h_{max}}{\eta} \quad \text{with } \eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \quad \text{where } \epsilon = C_\mu k\omega$$

$$C_{lim}(\xi) = 0.06(max(min(\xi - 23)/7, 1), 0) + max(min(\xi - 65)/25, 1), 0))$$

$$\boxed{C_{DES} = \max(C_{dyn},\ C_{lim})}$$ adaptive model $\rightarrow$ transition sensor

A simpler formula

Bader, Yin, Durbin; Flow Turbulence and Combustion 2022

To enable transition, without dynamic method,
From the ω-equation in equilibrium, with the standard C_{ω_1}, C_{ω_2}

$$\nu_T = (C_{DES}\Delta)^2 \frac{20}{3\sqrt{3}} \sqrt{|S|^2} \tag{1}$$

Vreman's model

$$\nu_T = 2.5(C_S\Delta)^2 \Pi \tag{2}$$

where

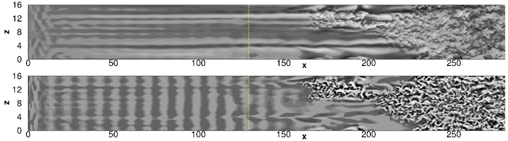
$$\alpha_{ij} = \frac{\partial \overline{u}_j}{\partial x_i}, \quad \beta_{ij} = \alpha_{mi}\alpha_{mj}, \qquad \Pi = \sqrt{\frac{B_\beta}{\alpha_{ij}\alpha_{ij}}}$$

B_β is the second invariant of β_{ij}. In parallel shear flow, $B_\beta = 0$.

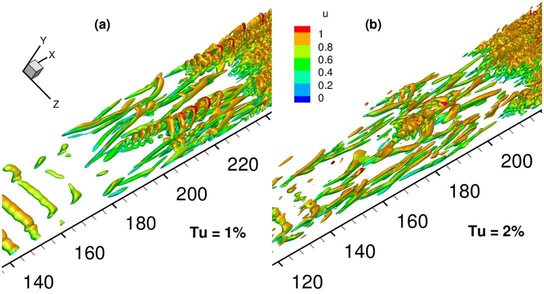
Equating (1) and (2) $\quad C_{DES} = 0.16\sqrt{\frac{\Pi}{|S|}}\quad$ also works for transition.

DNS for comparison

$N_{grid} \sim 100 \times 10^6$
Instability waves with f.s.t. — mixed transition



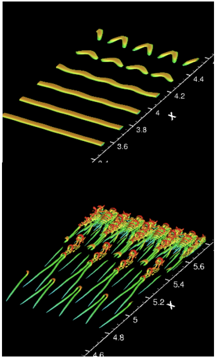
$y = \delta_0/2$



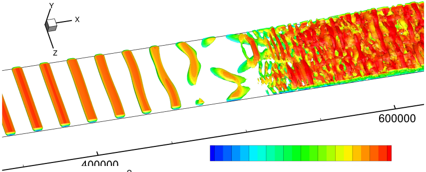
Hybrid Transition: Orderly

H, K-type TS transition

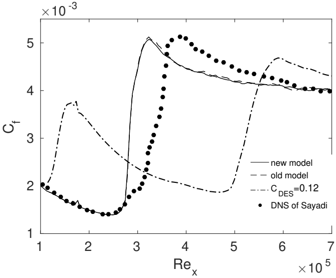
DNS



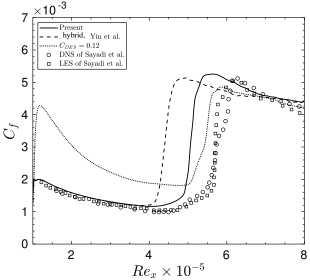
Hybrid simulation

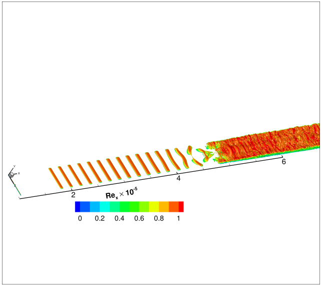


by U/U_∞



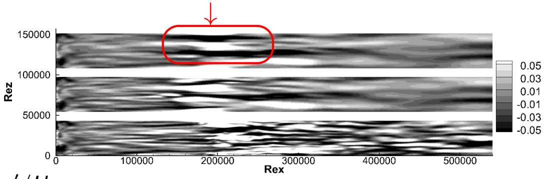
H-type transition using Vreman-based approach





Iso-surfaces of Q criterion colored by instantaneous U/U_∞

Hybrid Transition: Bypass



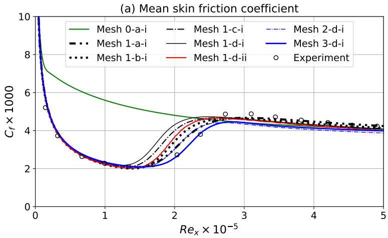
u'/U_∞

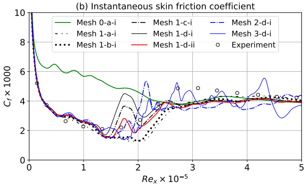
$N_{grid} = 0.48 \times 10^6$

0.72×10^6

1.44×10^6

$Re_y = 180$

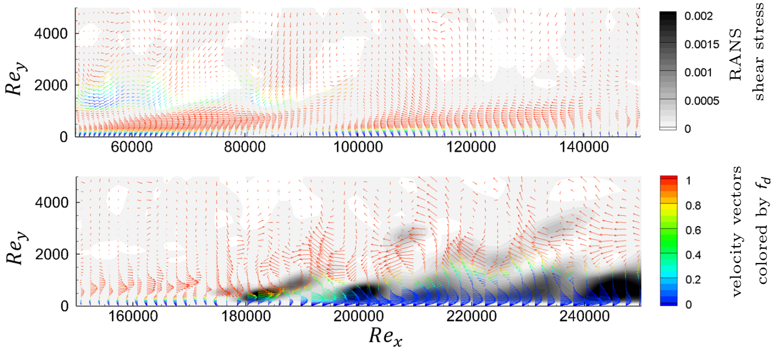




Closer Look

Yin, Ge, Durbin *JFM 915, (2021)*

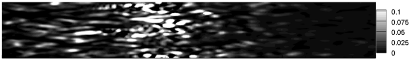
Closer look at bypass transition



Transition sensor

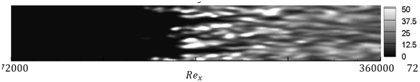
Is the model activated by L_{ij}? No

plane of $Re_y = 720$



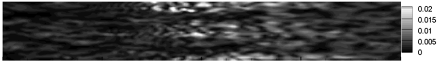
(c) $L_{ij}M_{ij}$

test filter stress



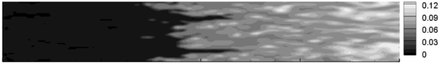
(g) $C_\mu k\omega$

ε



(e) C_{dyn}

Max $C_{dyn} = 0.02$

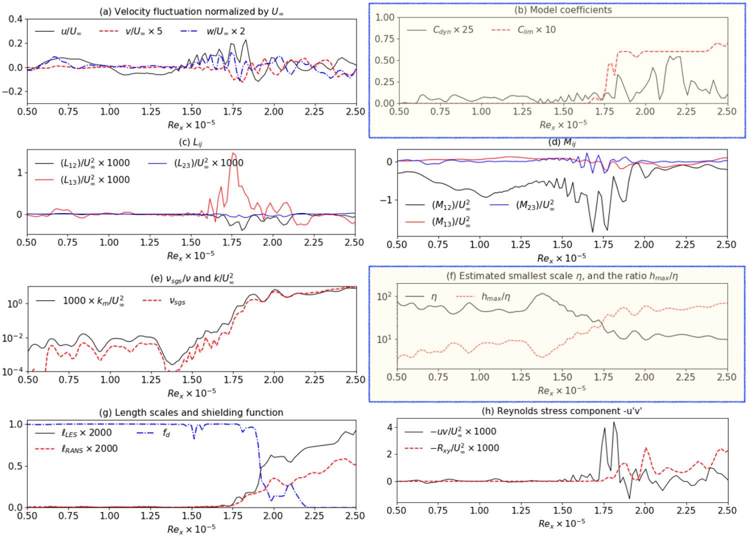


(f) C_{DES}

Max $C_{DES} = 0.12$ $\therefore$ $C_{DES} = C_{lim}$

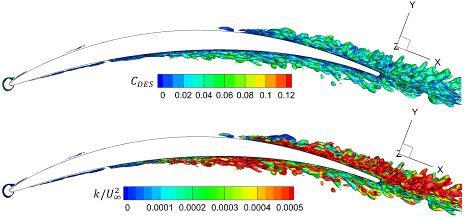
Recall $C_{DES} = \max(C_{dyn}, C_{lim})$

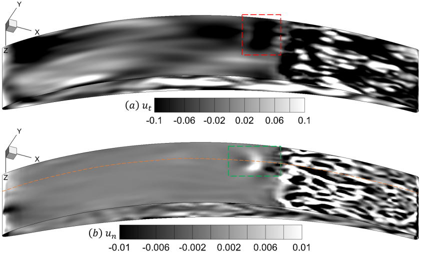
What is the sensor?



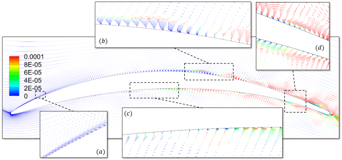
Bypass on turbine blade

Yin & Durbin *J. Turbomach. 144(2022)*

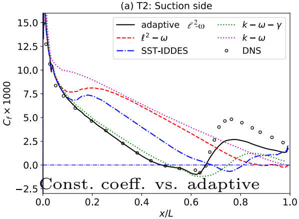


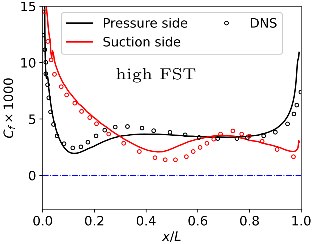


Bypass on turbine blade



T2: Instantaneous perturbation velocity $(x, y$ components), colored by instantaneous modeled kinetic energy k/U_∞^2.





In short

The nature of hybrid transition is:

Precursors to transition are simulated; but, literal breakdown is replaced by activating Reynolds stresses

h_{max}/η is the transition sensor in the $\ell^2 - \omega$ model