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What is different about transition modeling?

- Many non-local and global parameters
- Interface region with turbulence model is weird
  - ➔ Lots of babies, less bathwater
- Less physics-based, more data-driven (by construction)
- Less theory guidance (e.g. can’t hang our hats on homogeneous turbulence)
- Feature selection is harder, and more empirical/intuitive
- BUT, there is a much higher possibility of running DNS for most (all) regimes of interest

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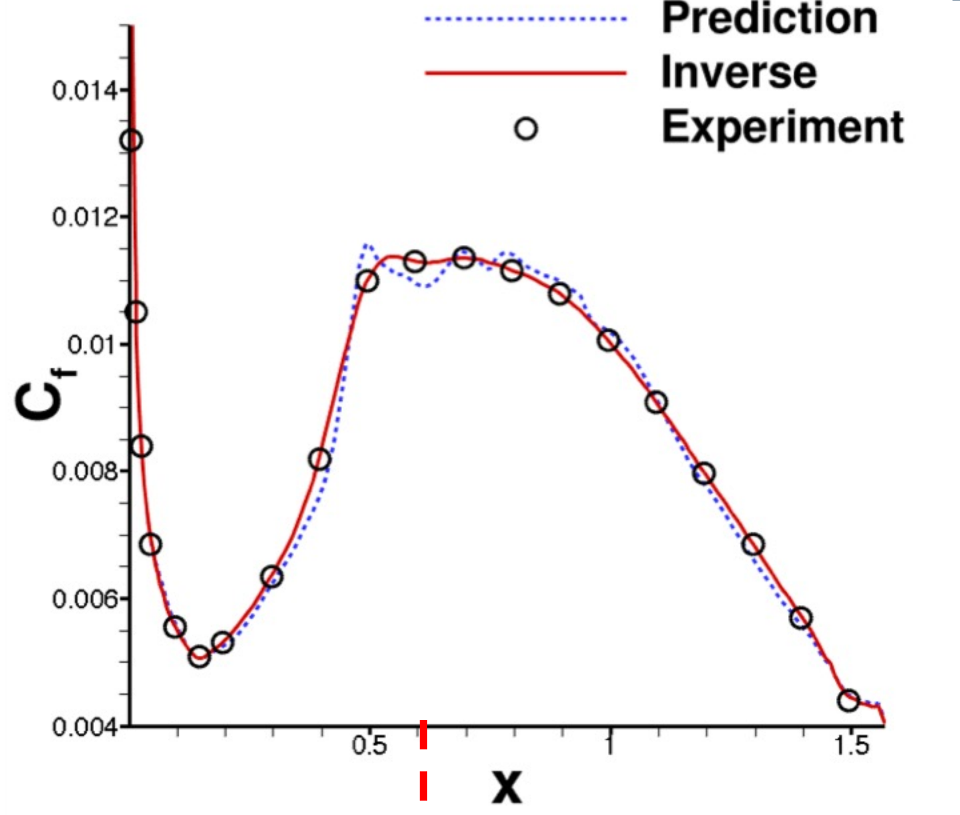
A thought experiment

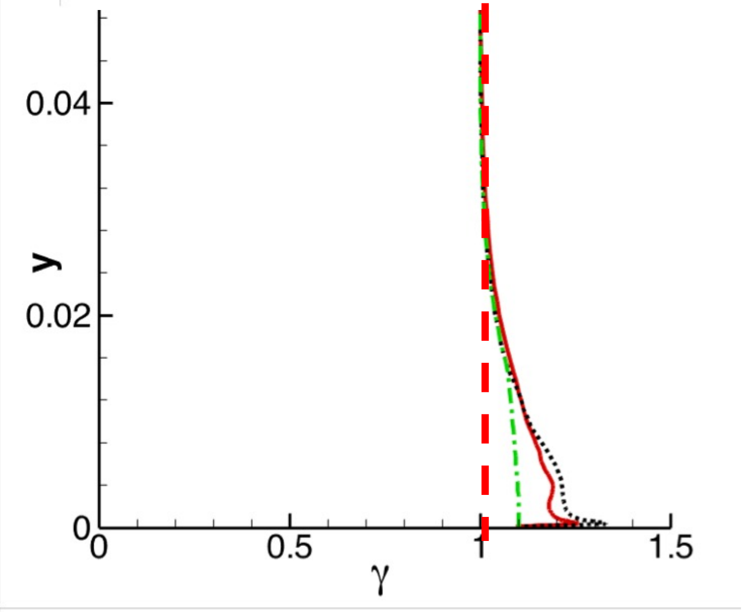
$$\frac{Dk}{Dt} = 2\nu_T|S|^2\gamma - C_\mu k\omega + \partial_j\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\partial_j k\right]$$

$$\frac{D\omega}{Dt} = 2C_{\omega 1}|S|^2 - C_{\omega 2}\omega^2 + \partial_j\left[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\partial_j \omega\right]$$

$$\frac{D\gamma}{Dt} = \partial_j\left[\left(\frac{\nu}{\sigma_l} + \frac{\nu_T}{\sigma_\gamma}\right)\partial_j\gamma\right] + P_\gamma - E_\gamma$$

- Variables are more operational
- Interfacing of turbulence and transition models is critical !!





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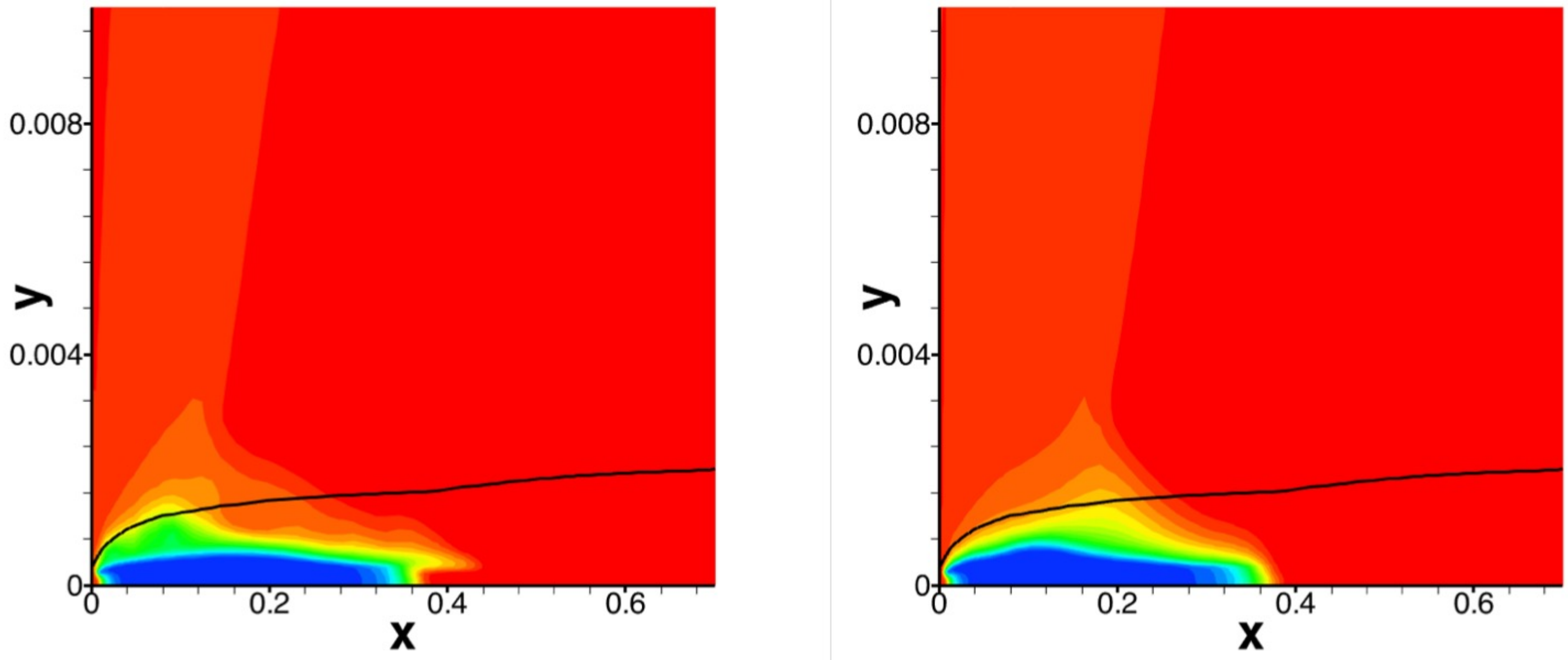
A thought experiment



(c) T3C1: Optimal γ

(d) T3C1: Model γ

Feature to help determine transition onset

$$Re_\Omega = \frac{\Omega d^2}{2.188\nu} \qquad \max_d Re_\Omega \approx Re_\theta$$

Physics-informed choice of features

Praisner and Clark (J. Turbomachinery, 2007) gave the correlation

$$\theta_{\text{tr}} \approx \sqrt{\frac{7\nu}{9\omega_\infty}}$$

Physics-based non-dimensionalization

Then, we have $$\frac{Re_\theta}{Re_{\theta,\text{tr}}} \approx \max_d \frac{Re_\Omega}{U_\infty \theta_{\text{tr}}/\nu} \approx \max_d \frac{\Omega d^2 \sqrt{9\omega_\infty}}{U_\infty \sqrt{7\nu}}$$

Freestream quantities are extracted from a constant wall distance.

Applying a conservative bound

$$\eta_1 = \min\left(\frac{d^2 \Omega \sqrt{9\omega_\infty}}{U_\infty \sqrt{7\nu}}, 3\right)$$

Bounded features

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Reducing extrapolation in feature space via feature design

$$\boldsymbol{\eta} = \{\nu_t, \nu\}$$

$$\boldsymbol{\eta} = \left\{ \frac{\nu_t - \nu_{t,\min}}{\nu_{t,\max} - \nu_{t,\min}}, \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}} \right\}$$

$$\boldsymbol{\eta} = \left\{ \frac{\nu_t}{\nu} \right\} \qquad \eta = \{\frac{\nu + \nu_t}{\nu}\}$$

$$\boldsymbol{\eta} = \left\{ \frac{\nu}{\nu_t + \nu} \right\}$$

$$\boldsymbol{\eta} = \left\{ \frac{\nu^n}{\nu_t^n + \nu^n} \right\}, \quad \boldsymbol{\eta} = \left\{ \left(\frac{\nu}{\nu_t + \nu} \right)^{1/n} \right\}, \quad \boldsymbol{\eta} = \left\{ \frac{\log(\nu)}{\log(\nu_t) + \log(\nu)} \right\}, \ldots$$

Reducing extrapolation in feature space via feature design

$$\boldsymbol{\eta} = \{\nu_t, \nu\}$$

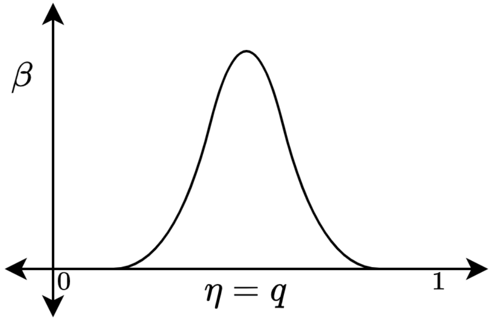
$$\boldsymbol{\eta} = \left\{ \frac{\nu_t - \nu_{t,\min}}{\nu_{t,\max} - \nu_{t,\min}}, \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}} \right\}$$

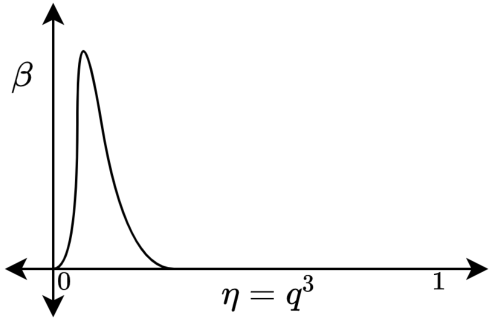
$$\boldsymbol{\eta} = \left\{ \frac{\nu_t}{\nu} \right\}$$

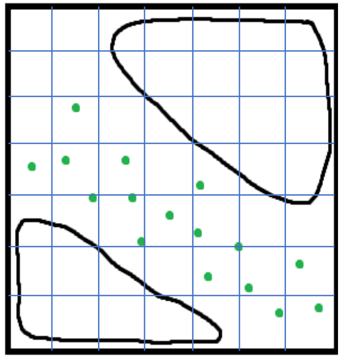
$$\eta = \left\{ \frac{\nu + \nu_t}{\nu} \right\}$$

$$\boldsymbol{\eta} = \left\{ \frac{\nu}{\nu_t + \nu} \right\}$$

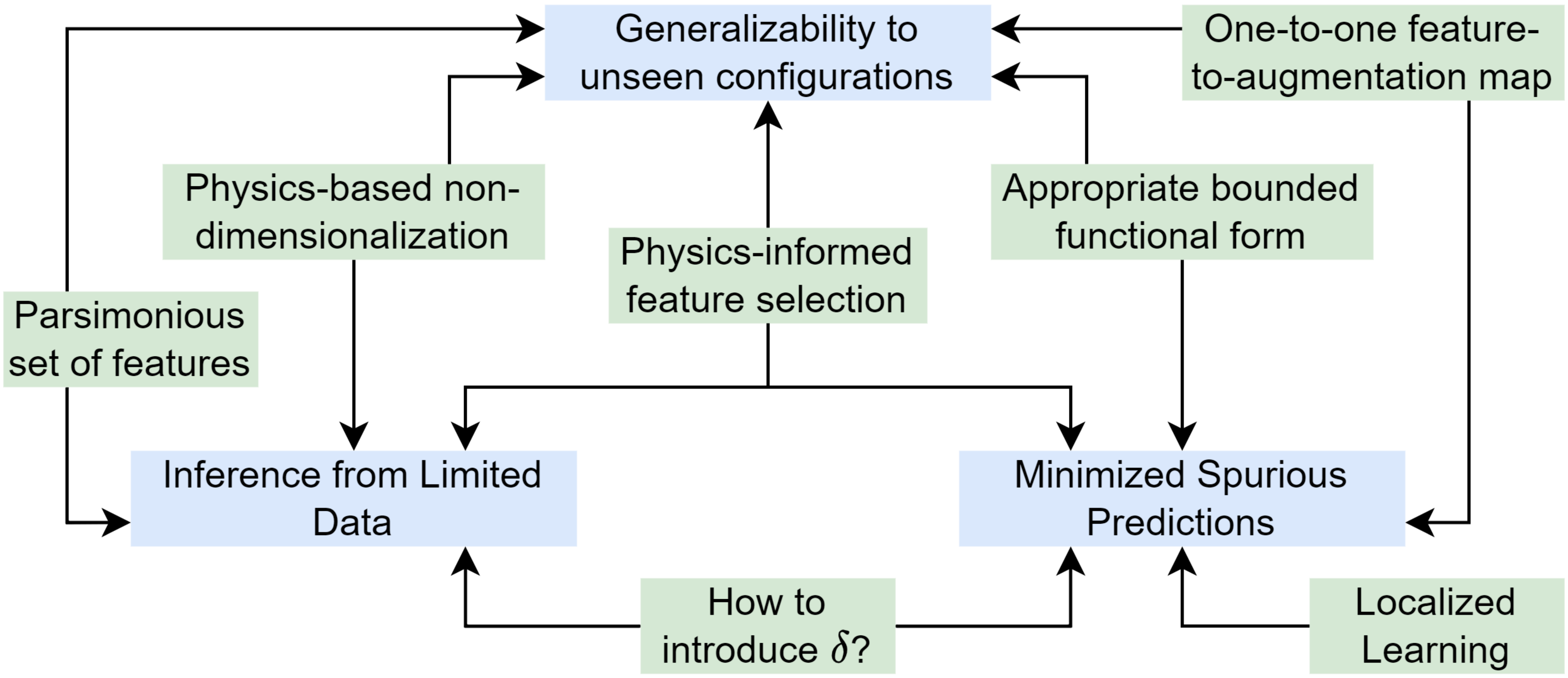
Interpolation in feature space can give you extrapolation in physical space







Learning & Inference Assisted by Feature Space Engineering (LIFE)



Srivastava & Duraisamy, PRF 2021.

Feature(s) to identify laminar/turbulent regions

Compare ν and ν_t. What about the viscous sublayer, though?

Compare d and ℓ_t to see if d is significantly larger. For k-ω model, $\mathcal{O}(\ell_t) = \mathcal{O}(\sqrt{k}/\omega)$

Mathematically bound both the features as:

$$\eta_2 = \frac{d}{d + \sqrt{k}/\omega}$$

$$\eta_3 = \frac{\nu}{\nu_t + \nu}$$

- Too many features over-specify physical conditions and reduce generalizability
- Too few features can result in lower predictive accuracy even for the training cases

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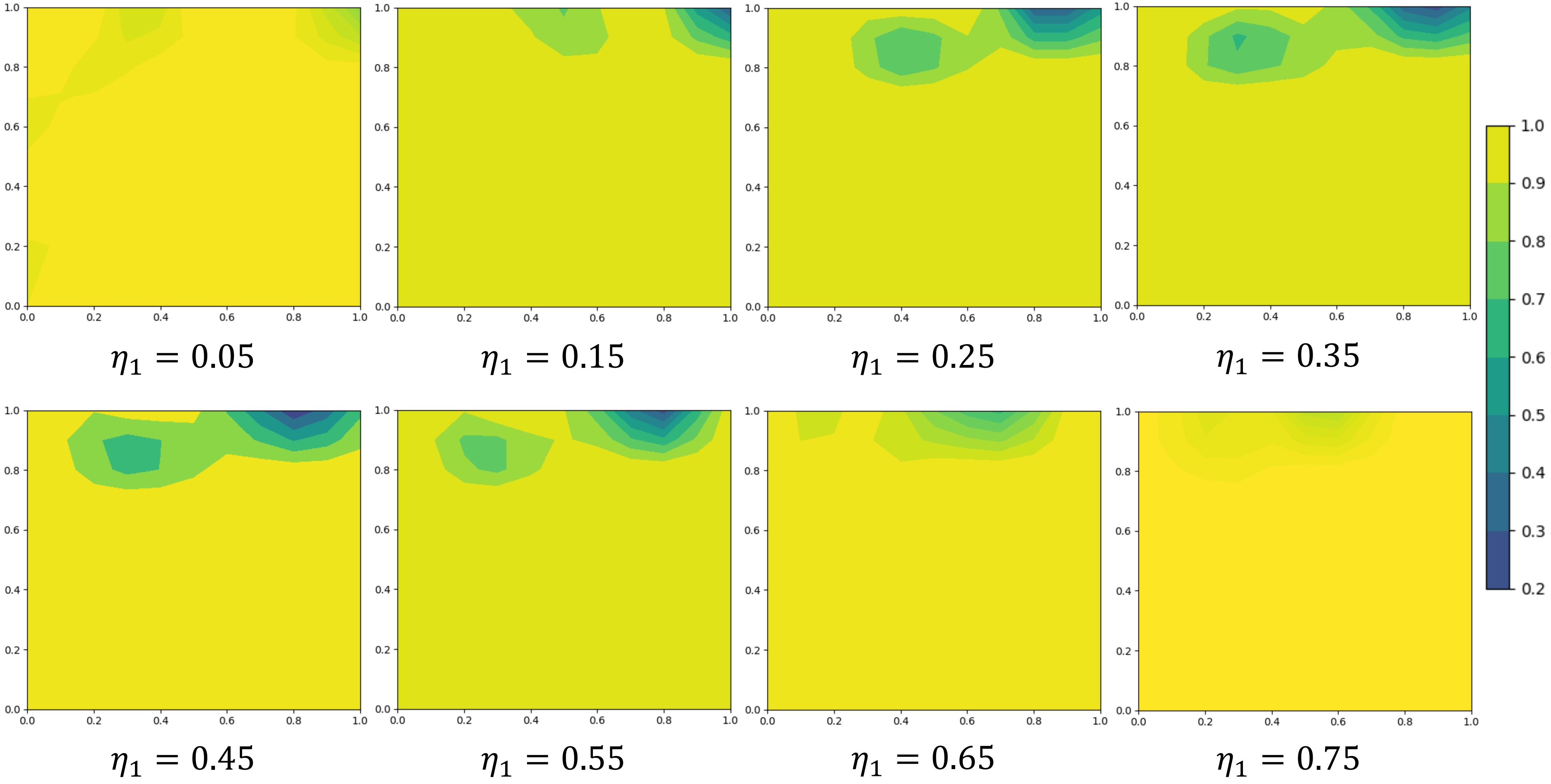
How does the feature-space look like?



For all plots: X-axes: η_2 (0 to 1) Y-axes: η_3 (0 to 1)

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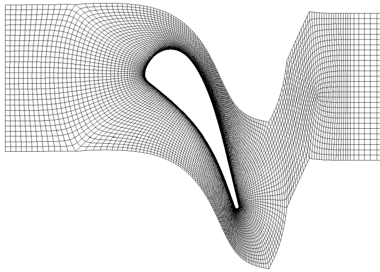
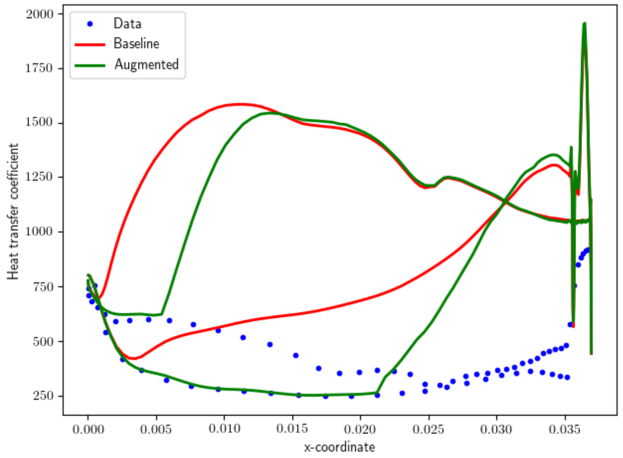
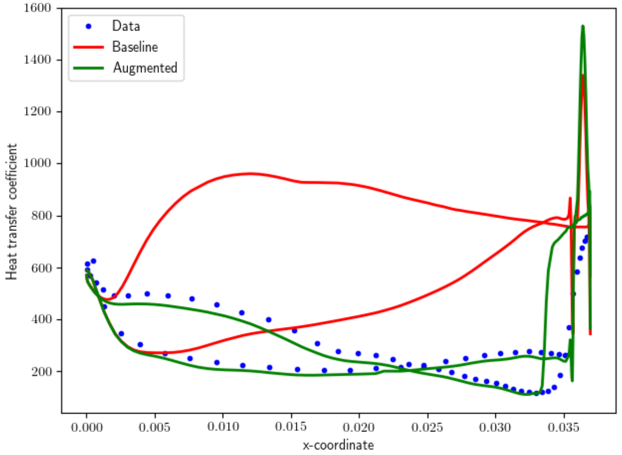
Prediction on turbine cascade (Model trained on only 2 flat plate cases – T3A, T3C1)

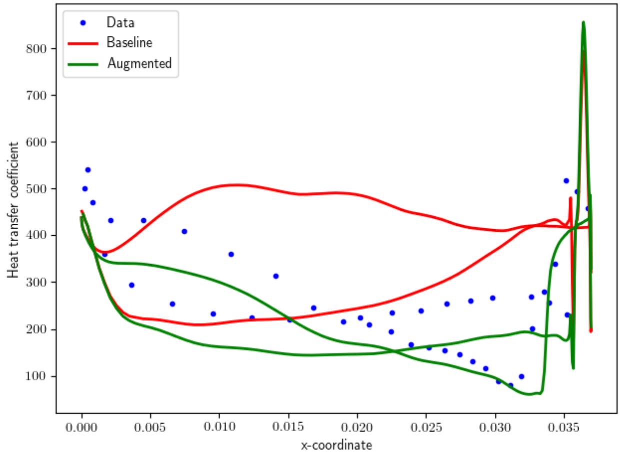




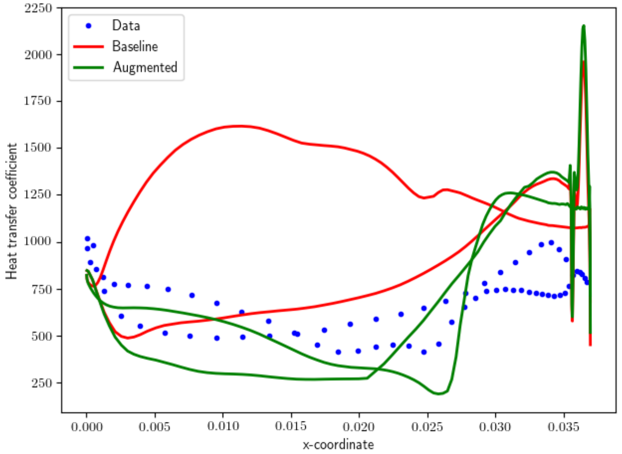
Heat transfer coefficient for MUR116



Heat transfer coefficient for MUR129



Heat transfer coefficient for MUR224



Heat transfer coefficient for MUR241